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Logic and Law: A Matter of Values Behind Content and Form

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Abstract: This special issue on Logic and Law consists of four research papers and one interview focusing on epistemological reflections on relationships between logic and law, whether in a reductionist or complementary approach. Logic aims to elucidate through formal frameworks, yet it often grapples with the intricate nuances of everyday legal discourse. While law endeavors to delineate permissible conduct within defined jurisdictions, it often encounters challenges stemming from the ambiguity of terms, leading to frequent judicial interpretations and the perception that proliferating exceptions undermines the efficacy of the rule itself.
Keywords: legal reasoning, law, logic, truth.

Rule is rule. Here is a blatant tautology. But, not everyone agrees about the correctness of this or that law. Are they merely legal, legitimate, limited in space and time, imposed by some over the others? Rule is rule, but no one agrees about which one should be obeyed. Justice or equality, reciprocity are such criteria that can help to discriminate between good laws and bad laws. Everything is a matter of values and values to defend other values, accordingly. Truth has its own word as well, and logic with it.

There are legal rules and logical rules. Thinking about the sources of logic and law is like searching for the roots of validity: What makes any argument correct, and according to what authorities? Legal reasoning is grounded on law, and law is not a single thing. Logical reasoning is grounded on truth, and truth is not a single thing. Both may be posited or specified, however, for it is not because a concept is tricky that it cannot be defined and limited by a number of clauses. So what is properly logical into legal reasoning, and how can the former feature contribute to the latter? Four papers are included into the present issue entitled *Logic and Law*, in order to deal with the tricky interconnections between both axiological disciplines from different standpoints.

In the first paper, "How law's nature influences law's logic", Jaap Hage (University of Maastricht) tackles the sources of rule from the domain of social conventions. Ontological assumptions (one-sided direction of fit from world to language, external relations, bivalence) are approached by the author, who claims that the objects of a world depicted by "model-theoretic semantic" betray a biased way of how sentences are true or false. In particular, classical logic yields a world view of material facts that doesn't take the normative dimension of laws and rules into account; their internal relations or interconnections are made silent by classical logic, so they are to

be included into our social reality of duties and obligations by a specific logic of legal reasoning; but the latter still differs from the huge number of non-classical logics that blossomed from the 1950s. In non-classical logics, rules proceed as further truth-values or intensional operators. In Hage's logic, rules are logical individuals "just like persons, organizations, and pieces of furniture". Moreover, legal reasoning deals with rules that may be added or removed, so it has to do with constructivist facts that are to be taken seriously: "constructivist reality is what rationally ought to be recognised as real", and such a mixture affects the usual, classical rules of inference including the property of monotonicity. Such a simple rule as Modus Ponens cannot be applied to legal reasoning, Hage claims, once assuming that rules do not occur as arguments.

Rules are space-time dependent and, thus, hardly reducible to logical laws. Does it mean that reasonings *per analogiam*, *a fortiori* or *e contrario* are not of logical nature, for want of any formal criterion to warrant their truth-values?

The same issue appears in the second paper, "Legal reasoning and logic", where Jan Woleński (University of Information Technology and Management in Rzeszow) shows that the logical virtue of disambiguating ordinary language frequently meets orderline cases in the area of law. To what extent is legal logic a proper "logic", assuming that the latter essentially relates to entailment relations from premises to conclusion?

Paradox of the Court illustrates how a lack of precision leads to difficulties once the sources of a rule are not specified: Who has to pay between Protagoras and Euthalos, assuming that the latter is expected to pay the former once he wins his first court case but suddenly decided to postpone legal practise? If Protagoras pursues his student, then by virtue of their contract Euthalos doesn't pay by losing the trial against his teacher; if Euthalos wins the trial, then he doesn't pay his teacher although he should by virtue of their contract. Assuming that norms proceed as deontic statements (unlike Jaap Hage, accordingly), Jan Woleński points to a number of difficulties logic faces when applied to legal reasoning: the ambiguous meaning of logical constants ("and", "if and only if", "more" and "less") into ordinary legal statements. Two famous laws are cases in point, i.e. *argumentum a contrario* and *argumentum a fortiori*. Wolenski claims that instances of the former are easy to disentangle logically, whereas the latter includes cases in which informal interpretations are necessary to validate formal schemes. There is something that logic cannot control to make its own application safer, consequently. Unless the variety of informal interpretations of formal laws occur as a case for logical pluralism, i.e. the view that there is not only one set of valid laws of logic in any rational context?

This position is advanced in the third paper, "Legal Gap and their Logical Forms". Matheus Gabriel Barbosa (Federal University of Goiás) and Fabien Schang (Lycée Alfred Mézières, Jarny) argue for a many-valued treatment of one of the main arguments against the logical treatment of law cases, viz. legal gaps. Unlike Jaap Hage, both authors assume two non-classical-friendly clauses: rules are sources of law, these occur as a metalogical operator whose flexible authority leads to a number of distinctive truth-conditions; legal reasoning is made of truth-apt objects that enter into schemes of valid entailment (once truth is extended to a finite subset of designated values). Ordinary difficulties like antinomy (excessive norms) and gap (insufficient norms) are both formalized and streamlined into different logical systems that extend from most to least strict in terms of permission and prohibition. Then two different systems of law, Common Law (if something is not prohibited then it is permitted) and Civil Law (if something is not permitted then it is prohibited), are exemplified and treated as asymmetric inferences that cannot be validated by normal modal logic (that assumes symmetry: something is not prohibited if, *and only if*, it is not permitted). For this purpose, the authors show that a many-valued analysis of deontic operators (permission and obligation) overcomes the previous difficulty and claim that a formal treatment of informal issues is made possible by a more open-minded formal semantics. And yet, extending the range of logic to legal reasoning doesn't explain how such a generalization of logical forms may lead to successful assessments in concrete cases including open-textured concepts.

In the fourth and ultimate paper, "Neural Networks in Legal Theory", Vadim Verenich (University of Tallinn) extends the formal ground to naturalist arguments and advocates a unifying

source of rules in terms of neural networks. The author recalls that ambiguity or vagueness are the main reasons to criticize any purely formal treatment of legal reasoning. Nevertheless, logic may be extended to more mathematical devices like the statistical approach of database legal systems and fuzzy, non-monotonic decision processes. Finally, a neutral network model of legal reasoning is preferred over the formal, syllogistic pattern by turning ordered sequences from axioms to theorems into synaptic connections of computational units. The author argues that this view does justice to an evolutionary version of the sources of law. Merits and limits of an such an AI-friendly approach to legal reasoning are scrutinized as well, in order to illustrate the variety of arguments with respect to the relationship between logical and legal systems.

In a nutshell: the present issue is nothing but a new attempt to update the epistemological reflection on logic and law, whether in a reductionist or complementary approach. Logic is expected to clarify by means of formal schemes; but its weak point is an excessively broad analysis of subtle daily-life legal arguments. Law is expected to make sense of what is permitted or not into a delimited area of jurisdiction; but its weak point is a wide range of ambiguous terms that lead to frequent jurisprudence and the resulting impression that multiplying exceptions doesn't make the rule anymore. It is not logic over law, or law despite logic. Let us think about law with logic, assuming that none consists in a standing number of tenets. Both disciplines are living and growing. The reader is pleased to consider herself as Neurath in his boat, eventually.

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How Law's Nature Influences Law's Logic

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Abstract:
Classical logic is based on an underlying view of the world, according to which there are elementary facts and compound facts, which are logical combinations of these elementary facts. Sentences are true if they correspond to, in last instance, the elementary facts in the world. This world view has no place for rules, which exist as individuals in the world, and which create relations between the most elementary facts. As a result, classical logic is not suitable to deal with rules, and is therefore unsuitable to deal with legal reasoning. A logic that is more suitable should take into account that law is a part of social reality, in particular a part that consists of constructivist facts, and that rules play a central role in law. This article gives a superficial description of how social reality exists and of the place of law and legal rules in it. It uses this description to argue that traditional techniques to reason with and about legal rules provide a better logic for law than classical logic. These techniques can be accommodated in a logic that treats rules as logical individuals.
Keywords: classical logic, constructivist facts, rules, social reality.

1. Introduction

The aim of this article is to argue that the nature of law influences the logic of legal reasoning and to give an impression of how this 'works'. The argument consists of three steps. In the first step it is shown how classical logic, which I will take to be first order predicate logic (Priest, 2008, p. xvii), is based on an underlying view of the world. This view has no place for rules and as a result, classical logic wrestles with rules and their role in (legal) reasoning. In the second step, an alternative view of the world is presented. In this view, a central place is taken by social reality and by the role of rules in it. In the third step, it is shown how traditional forms of legal reasoning are better suited than classical logic to deal with legal rules and that these traditional forms better fit the 'social' image of law that was presented in the second step.

2. Classical Logic

A particular version of logic has dominated logical theorizing during the twentieth century and particularly the first half thereof. This version is (first order) predicate logic, or a stripped down

version of it, propositional or sentential logic. Because of its central role, this logic will here be called 'classical logic'. Classical logic is a formal logic, which means that it considers the validity of arguments to depend on the form of arguments only, and not on their substance (what the arguments are about). Although other variants of formal logic were developed – in fact, they blossomed since the 1950s – they typically took classical logic as their starting point.

Let us take a closer look at the world view that underlies classical logic.[1] We can find it in the semantics of the logic, and in particular in the informal interpretation thereof. The starting point of it all is the idea that an argument is deductively valid[2] if and only if it is logically impossible that all the premises of the argument are true, while the argument's conclusion is false. Truth and falsity are in logical theory taken to be semantic notions, and therefore this idea of logical validity is called the semantic notion of validity.

Classical logic uses this semantic notion and therefore relies on the idea of logical (im)possibility: in a valid argument it is logically impossible that the premises are true and the conclusion false. But what does logical possibility mean? To answer this question, the theory of classical logic developed so-called 'model-theoretic semantics'. It is this model-theoretic semantics – from now on 'semantics' – in which the world view (ontology) underlying classical logic is made explicit. I will briefly describe this semantics in the following section, to the extent that is necessary to elucidate the ontological assumptions underlying classical logic.

3. Semantics

The semantics of predicate logic consists of two parts. The first part deals with the truth conditions of elementary sentences, such as 'Four is an even number', 'John is a thief', or 'All thieves ought to be punished'.[3,4] The second part deals with the truth conditions of compound sentences, such as 'John is a thief, and all thieves ought to be punished'.

The world contains zero or more objects, which logicians call 'individuals'. Examples of such individuals are this table (a material object), Iris (a person), the number four (an immaterial individual), or the United Nations (another immaterial individual). The world also contains zero or more sets of individuals. These sets informally stand for classes[5] such as the class of wooden objects, individuals who ought to be punished, even numbers, or governmental organisations. For any individual it holds that it is an element of zero or more of these sets. For instance, this table is an element of the set of wooden objects, Iris and the United Nations are elements of the set of entities which ought to be punished, four is an element of the set of even numbers, and the United Nations are also an element of the set of governmental organisations. Figure 1 gives an impression. In this figure, circles represent sets (thieves and those who ought to be punished), and small letters (a-d) represent individuals.

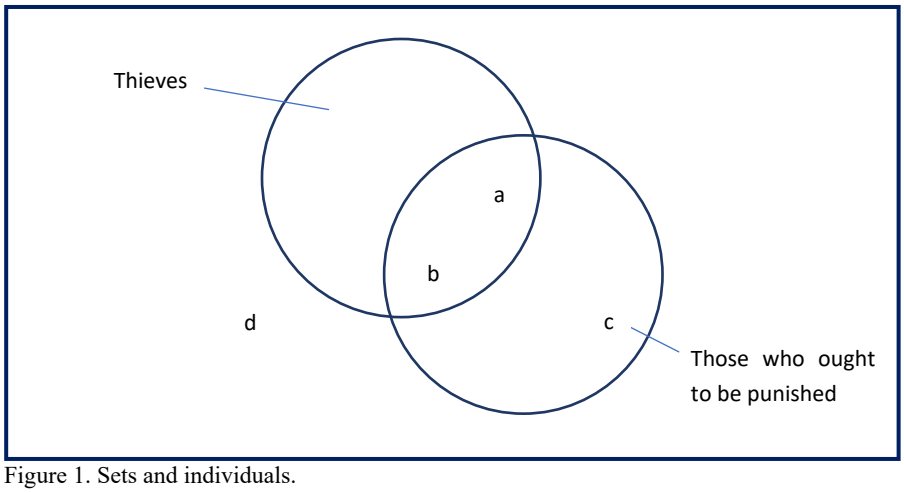

Figure 1. Sets and individuals.

The individuals a and b are both elements of the sets of thieves and of those who ought to be punished. c is only an element of the latter set and d is not an element of any of these two sets.

An important assumption of the semantics is that the assignment of an individual to a set neither depends on, nor influences, the assignment of other individuals to sets. So, the assignment of b to the set of entities that ought to be punished neither influences the assignment of any other individuals to this set, nor the assignment of b to any other set. Moreover, whether b is assigned to the set of thieves has no influence on whether this individual is also assigned to the set of entities which ought to be punished. As we will see, this independence makes classical logic with its underlying ontological assumptions unsuitable for dealing with rules.

There are three kinds of elementary sentences.

- An elementary sentence stating that an individual is an element of a set (e.g. 'a ought to be punished') is true if and only if that individual actually is an element of this set. This is the case in figure 1. However, according to this figure, the sentence 'd ought to be punished' would be false.
- An elementary sentence stating that a set has elements (e.g. 'There are thieves) is true if the set contains at least one individual. Since the set of thieves in figure 1 contains two individuals, this sentence comes out true.
- An elementary statement that all the individuals are elements of one particular set (e.g. 'Everything ought to be punished') is true if indeed all individuals are elements of this set. In figure 1 no such sentence is true, as there is no set which contains all individuals.

It deserves emphasizing that in the semantics of predicate logic, a sentence is either true or false. It is not possible that a sentence is both true and false if, for instance, there are both good reasons why Iris ought to be punished and good reasons why she ought not to be punished. Neither is it possible that a sentence is neither true nor false. And, finally, a sentence cannot be true to some degree. This may be problematic for characteristics which come in degrees, such as baldness, or for general statements which are for most cases but not for all cases, such as 'Birds can fly'.[6]

The truth or falsity, that is the 'truth value', of compound sentences depends on the truth value(s) of the sentence(s) from which it is composed and on the way these sentences are joined together by a logical operator. A sentence with the structure A&B (informally 'A and B') is true, if and only if both the sentences A and B are true. A sentence with the structure A$\vee$B (informally 'A or B') is true, if and only if at least one of the sentences A or B is true.

For this article it is not important how model-theoretic semantics can be used for defining the validity of an argument, but for the sake of completeness I will say a little about it. A logically possible world is an assignment of individuals to the sets that correspond to the predicates of the logical language. Since there may be many ways to make this assignment, there can be many different logically possible worlds. Together, these worlds define what is logically possible. In some of these possible worlds, all the premises of an argument come out true. These worlds are called 'models' of the premises. An argument is valid if and only if the conclusion of the argument is true in all models of the argument's premises (Smith, 2012, chapter 9).

4. Rules in Predicate Logic

Predicate logic distinguishes in its semantics between sentences which are true or false, and the world, consisting of individuals and sets, which makes the sentences true or false. The relation between the world and the sentences is unidirectional: the world gives the sentences their truth values and the sentences do not influence the world. In this simple picture, there is no place for rules, as rules influence the world. Let me explain.

It is often assumed that rules prescribe behaviour. Only in a benevolent interpretation of prescribing, this is sometimes true.[7] Many rules do not prescribe in any sense. Some rules make that things also count as other things, such as the rule that cars count as vehicles for the purpose of the Traffic Act. Other rules assign competences, such as the rule that makes Parliament competent to create statutes. Still other rules assign legal status, such as the rule stating that if the King dies, his oldest daughter becomes the new Queen. And *some* rules impose duties, such as the duty to halt at

red traffic lights, or obligations such as the obligation to compensate the damage caused by one's negligence. Only duty- and obligation-imposing rules can be said to prescribe behaviour, and then only in the indirect sense that the duties and obligations they create are reasons why an agent ought to do something (Hage, 2022, pp. 111-112).

Rules fulfil many different functions, but there is one thing they have in common and that is that they attach facts to other facts. (More on facts in the next section.) A classificatory rule attaches the fact that something is a vehicle to the fact that it is a car. A competence-conferring rule attaches the fact that something has the competence to create statutes to the fact that this something is Parliament. And a duty-imposing rule attaches the duty to halt for red traffic lights to the fact that somebody is a traffic participant. The core function of rules is to attach new facts to existing ones and the semantics of predicate logic has no place for entities with this function.[8]

As a result, predicate logic has a problem with rules. This problem becomes manifest in, amongst others, the phenomena that rules lack truth values and that reasoning with rules is non-deductive. Admittedly, there have been attempts to modify predicate logic to make it possible to represent rule-based arguments. But, first, these modifications had to abandon the simple semantics of predicate logic, for instance by making the truth value of some sentences dependent on more than one possible world and, second, they still make the mistake to treat rules as *reflecting* the facts in these possible worlds, while they should account for it that rules *influence* the worlds in which they exist.

Having diagnosed what goes wrong in classical logic when it has to deal with rules, it is time to make a new start. In the following sections, I will give a brief account of how social reality exists and of the role that rules play in it. This account will function as a new foundation for the logic of rules.

5. Introducing Social Reality

In this section and the following ones, I will attempt to show in some detail how the existence of rules, including legal rules, is *in last instance* a matter of social fact and that the existence of many social facts, including facts about the existence of rules, depends on rules.[9]

In the following, I will use the words 'fact' and 'state of affairs' in a technical sense. My starting point will be the existence of a language which includes statements (descriptive sentences). Statements express states of affairs and are either true or false. For instance, the English language includes the statement 'It is raining'. This statement expresses the state of affairs that it is raining and is true if it is raining and otherwise false. If the sentence is true, the expressed state of affairs is a fact, and otherwise not. In this connection, a fact is an element of the world that makes a declarative sentence (or a proposition) true. The world is then the set of all facts.[10]

People distinguish between what is objective, subjective, and social. The distinction between these three kinds of states of affairs is based on two underlying characteristics which may be present or not. The two characteristics are whether the state of affairs is:

1. mind-dependent; and
2. the same for everybody.

Objective states of affairs are (1) not mind-dependent and (2) the same for everybody. An example would be the state of affairs that Mount Everest is a higher mountain than the Vaalserberg (the highest 'mountain' of the Netherlands).

Subjective states of affairs (1) depend on what individual persons think they are and are mind-dependent, and (2) are therefore not the same for everybody. An example would be the 'fact' that Mozart was a better composer than Brahms. Many people would not call subjective facts 'facts' at all; they reserve the predicate 'fact' for objective and perhaps also social facts.

Social states of affairs are somehow in between objective and subjective: (1) they depend on what the members of a social group recognise and are in that sense mind-dependent, and (2a) they are the same for the members of a group, but (2b) not necessarily the same between groups. One example is the law of a country. The law depends, in a complicated manner, on what the legal

subjects of a country recognise as law, and is the same for these legal subjects. However, different countries may have different laws, and what is the law for a Frenchman may not be the law for somebody in China.

6. Conventional Social Facts

Social facts are either conventional or constructivist. Conventional social facts only (but not always) exist in a group if most members of that group *recognise* that they exist. For instance, Hendrik is the leader of the Maastricht Cycling Club (MCC) if sufficiently many members of that club recognise Hendrik as their leader. A person recognises a fact if she believes that this fact exists and if that person tends to act in accordance with this belief. The simplest case is when recognition is nothing more than mere belief. If Mary believes that yesterday the train to Groningen left at 15h00, she also recognises this fact. However, the recognition of a fact usually involves more than mere belief. To have a leader for a club means that club members believe that some person is the leader, but also that they attach the relevant consequences to this believed leadership. What these consequences are, depends on how the notion of leadership is given content, but there cannot be leadership without any consequences. This means, for instance, that if the leadership of Hendrik involves that club members must do what Hendrik tells them, they will have the disposition to act accordingly.

Sometimes the task of recognition is delegated to one or more specific persons or institutions. A well-known legal example is that the recognition of rules as legal rules is delegated to courts and other 'officials'. Delegated recognition presupposes that the persons to whom the recognition is delegated (the representatives) are recognised as such and that the members of the group tend to recognise what their representatives recognised on their behalf. So, if legal subjects delegated the task to recognise rules as law to the courts, they should recognise courts as their representatives for this purpose and they should normally recognise rules as law for the reason that the courts recognise them.

There is more to the existence of conventional facts than mere recognition. For instance, it should not only be the case that sufficiently many members of MCC recognise Hendrik as their leader; the club members should also believe that sufficiently many other members also recognise Hendrik as leader of the group, and that these other members have the same beliefs about their fellow cyclists. In other words, a group member such as Petra should not only have beliefs about Hendrik, but also about what her fellow group members recognise, including what her fellow group members believe about the beliefs of Petra herself.

A third condition for the existence of conventional social facts is that something can only be a conventional fact if states of affairs of that kind are not considered to be objective, subjective or constructivist. For instance, even if everybody believes that heat consists of calories, and also believes that everybody else believes this, it is still not a social fact. The reason is that the nature of heat is (usually) considered to be an objective state of affairs. For types of states of affairs that are considered to be objective, such as the nature of physical phenomena, the existence of a consensus is not decisive for what the facts are.

To be conventional, a kind of state of affairs should also not be considered as constructivist. For a constructivist in ethics, the mere consensus about a particular moral judgment does not prove the judgment to be correct. Even if 'everybody' agrees that coloured people are inferior, this does not show coloured people to be inferior indeed. Contrast this with being the leader of an informal club, where consensus is decisive.

So, the existence of a conventional social fact requires recognition on two levels: a particular type of state of affairs must be considered social – not objective or subjective – and not constructivist, and a concrete instance of this type must be broadly recognised as existing. For instance, the members of MCC must (1) consider the leadership of their club to be a matter of conventional social fact and (2) they must recognise Hendrik as their leader.

Social facts are the same for all members of a social group, even for those members who do not recognise them. If Petra does not recognise Hendrik as the leader of MCC, she makes a mistake, and other members of MCC may criticise or even sanction her for this mistake.

7. Conventions

Social reality does not only contain facts, but also rules. The most basic form of existence for rules is existence as a convention.[11,12] I will define the existence of conventions as their efficacy. A convention exists in a group if sufficiently many members of the group are disposed to recognise the rule consequence if they believe the facts of the rule conditions and if they tend to justify their recognition by mentioning their belief or the rule.[13] For instance, if most people in Belgium are disposed to recognise the person to whom a property was transferred as the (new) owner of the property and tend to justify their recognition by reference to the transfer or to the rule regulating transfer, then the convention exists in Belgium that the person to whom a property was transferred has become the (new) owner. Notice that legal rules, although they are most often rule-based, can at the same time also be conventions.

Being efficacious is not the only requirement for the existence of a convention. Group members should also believe that most other members of the group recognise the rule consequence if they believe the rule conditions and that they justify this by reference to the rule. Moreover, the other group members should have the same belief. If sufficiently many members of a social group believe a fact and recognise a convention which attaches consequences to this fact, these members will (1) recognise these consequences, (2) believe that the other group members recognise these consequences, and (3) believe that the other group members believe the same about them. *In other words, the rule consequences will be conventional social facts in the group.*[14]

Rules, including conventions, are not statements, although their formulations may look like statements. 'Cars are vehicles', for instance, may be a descriptive sentence, but also the formulation of a rule. Ontological speaking, rules are individuals, just like persons, organisations, and pieces of furniture. It is possible to create them, to destroy (repeal) them, to count them, or to reason about them. The following argument is for instance valid: 'Rule X was made by the legislator. Rules made by the legislator are valid legal rules. Therefore, rule X is a valid legal rule'. Moreover, the conclusion that rule X is a valid legal rule can be used in an argument that applies rule X.[15]

It is worthwhile to emphasize the difference between a conventional fact and a convention. A conventional fact can be expressed by means of a true description, such as 'There must be a fire'. A convention, in contrast, is not a fact, but a connection between facts (as are other rules). The formulation of a convention, such as 'Smoke means fire', is not a statement, but the formulation of a rule of inference.

8. Constructivist Facts

Not all social facts are conventional. There is a second category, constructivist facts, where an existing broad consensus is not the final word on what the facts are. Suppose that the members of MCC take a vote on what was the best cycling trip they made this year. They decide unanimously that the trip to the castle gardens in Arcen was the best trip. Does this mean that the Arcen trip really was the best trip? No, even if all club members agree on what was the best trip, this does not mean that it *really* was the best trip. It remains possible to raise the question of whether all members of the club were mistaken about the best trip.

There seems to be a difference between what most or even all members of the group recognise as the best trip and what really was the best trip. Facts such as the fact about what was the best cycling trip of the year are not objective, because they depend on how people 'feel' about things. Neither are they merely subjective, as it makes sense to argue about them. And, finally, they

do not seem to be conventional social facts either, because a broadly shared belief about them is not the final word. I will call such facts *constructivist facts*.[16]

Constructivist facts are social facts, which are nevertheless open to serious questioning. This combination is possible if the social practice of a group does not only recognise the existence of these facts, but also the possibility to question them. For instance, *prima facie* it may be a social fact in MCC that the trip to the castle gardens of Arcen was the best trip of the year. However, the members of MCC agree and know that the others also agree that, theoretically speaking, everybody might be mistaken. If somebody came up with convincing reasons that another trip was even better, this other trip would be better. Moreover, it would have been better from the beginning, not merely because the members of MCC changed their minds. If an argument makes people change their minds about constructivist facts, they change their minds about what the facts already were.

Constructivist facts are characterized by the possibility to have a *serious* debate about them. 'Serious' means in this connection that the participants in the debate believe that it is possible to disagree about these facts without thereby showing a misunderstanding of what the debate is about and that there is a correct answer to the question what the facts are, independent of what people actually believe it is. For instance, if Joanna and Frédéric disagree about whether red wine is better or white wine, while they believe it is just a matter of taste, they consider the issue at stake to be a merely subjective one. There is no right answer as to what the best wine is[17] and their disagreement is not serious. If two members of MCC disagree about whether Hendrik is their leader, while both know that practically all members of the club recognise Hendrik as their leader, their disagreement is not serious either. The reason is that not believing that Hendrik is the leader while also believing that 'everybody' recognises Hendrik as the leader, shows misunderstanding of the conditions for leadership, which is a matter of convention.[18] The example about the best cycling trip of the year illustrates that it is possible to disagree seriously about what was the best trip. The seriousness of the debate becomes manifest in the assumption of all participants that there is a right answer to some question, even though it is not a matter of objective fact, and that this right answer does not change if people merely disagree about what the answer is.

Which social facts are constructivist, and which ones are conventional? It is impossible to give this question a general answer. The social practice of a group determines which social facts count as constructivist and which ones as conventional. If a broadly shared recognition may seriously be questioned, the social fact is considered to be constructivist; if not, it is conventional. Moreover, it seems that this categorization as conventional or constructivist is itself a matter of constructivist, and therefore also social, fact. People can seriously disagree on whether a particular kind of fact is conventional or constructivist. In legal philosophy, for example, there is a serious debate between hard legal positivists and non-positivists on whether law is conventional or constructivist (cf. Gardner, 2001 and Dworkin, 1986). In ethical theory, there is a similar debate between conventionalists (relativists) and constructivists (Gowans, 1997 and Bagnoli, 2021).

A constructivist fact is a fact that is recognised as a result of the rational reconstruction of the set of objective facts and social facts that are recognised in a social group.[19] Such a reconstruction will often consist of a debate. The debate may be casual, as amongst the members of MCC about the best cycling trip. It may also be more formal, as a debate in science about the best explanation of a newly discovered phenomenon. Rational reconstruction may involve no change for a particular social fact, and then that fact continues to exist as a social fact in the group because it was already recognised. An example would be that the members of MCC group believe that the cycling trip to the castle gardens of Arcen was the best trip of 2020 and that this belief survives a rational reconstruction of their belief set. Then the belief that the cycling trip to the castle gardens of Arcen was the best trip is an element of the rationally reconstructed belief set, because it was already in the original belief set and nothing changed in this respect.

Reconstruction may also involve the inclusion of a particular social fact, and then that fact exists as a social fact in the group because it ought to be recognised according to the rational reconstruction. An example would be that the members of MCC initially did not have the rule that members of all religious convictions should be treated equally, but that the existence of this rule is

included in the rationally reconstructed set and the rule therefore already existed as a matter of constructivist fact.

Finally, reconstruction may involve the removal of a particular social fact, and then that fact did not exist as a constructivist fact in the group because it ought not to be recognised according to the rational reconstruction. An example would be that the members of MCC group ought not to have recognised the trip to Arcen as the best one. Then the belief that the cycling trip to the castle gardens of Arcen was the best trip is not part of the rationally reconstructed belief set and the trip to Arcen was, all things considered, not the best trip.

Rationally reconstructing a set of recognitions or beliefs leads to a judgement on what ought to be recognised, given the original beliefs. The recognitions in the reconstructed set are what the believer of the original set ought to recognise. Moreover, as the example of the best cycling trip illustrates, the facts that rationally ought to be recognised are also the 'real' facts, because we are speaking of constructivist social facts. The members of MCC who argue about what was really the best cycling trip argue about what really was the case. Constructivist facts are the conclusions of the best possible arguments. These arguments determine what ought to be recognised, but *ipso facto* they also determine that part of social reality. Perhaps this is the most important thing to remember about constructivist facts: *constructivist reality is what rationally ought to be recognised as real*.

What counts in this connection as rational? Is there an objective, mind-independent standard for rationality, identical or analogous to the standard of classical logic? The proliferation of logical systems in the last, say, 70 years, suggests the opposite (Priest, 2008 and Walton, 2008). To cut a potentially long argument short, I will assume here that rationality is a matter of constructivist fact. Social conventions form the starting point in determining the standards of rationality, but they are not the last word. The debate on what counts as rational is to be conducted at the hand of standards which are themselves subject to debate.

9. Why Legal Facts Are Constructivist

Let us assume that law is a part of social reality and that this also holds for legal facts such as the fact that Iris is punishable, that John must stop for the red traffic light, or that this statutory rule is valid law. Then the question arises of whether these social facts are constructivist or conventional. Assuming, for the sake of argument, that the answer is the same for all legal facts, the best view seems that legal facts are constructivist.

Remember that whether a kind of state of affairs is conventional or constructivist depends on whether a broadly shared view is the last word, both in the sense that conventional facts are what 'everybody' recognises them to be and in the sense that if there is no broad consensus, there is no conventional fact. If legal facts were conventional, this would mean that where there is a lack of consensus on what the law is there is no law. Hard cases would be cases where there is a gap in the law because of a lack of consensus. If the conventional view of law would be correct for legal facts, gaps would be a common phenomenon. In contrast, if the constructivist view would be correct, gaps would only occur if a rational reconstruction of what is broadly recognised would not give an answer. If this could occur at all[20], it would happen only occasionally. Legal decision-makers seldom seem to assume that there is a gap in the law and to decide a case on the basis of moral or policy considerations only. So, it seems that these officials recognise more law than the conventional view claims there are. Since the views of these officials are decisive for whether legal facts are conventional or constructivist, it would seem that they are constructivist.

A similar argument starts from the observation that even if there is a broad consensus on what the law is, lawyers sometimes continue to argue as if this consensus is wrong. Such arguments can only be taken seriously if law is considered to be constructivist. This also pleads for the view that legal facts are constructivist.

A third argument is that the idea of legal sources only makes sense on a constructivist view of law. The idea of legal sources is that rules that can be traced back to a source of law are for that reason valid legal rules and – a less convincing addition – that rules that cannot be traced back to

some legal source, are for that reason not legal rules. On a conventional view of law, the only reason why a rule is a valid legal rule is that it is broadly recognised as such. If a legal source plays a role in this connection, that maybe an interesting observation, but the source does not make a legal rule valid. On a constructivist view, on the contrary, sources can be crucially important, because legal rules are valid if and only if they rationally ought to be recognised as such. If a rule rationally ought to be recognised as valid law, it is valid law, even if it is not (yet) broadly recognised as such. This makes sense on a constructivist view of law.

A fourth argument is the argument from legal interpretation. Legal debates on the correct interpretation of a legal source are debates on whether a rule can be traced to this source. Such debates are broadly recognised in legal practice as making sense. This is another argument why legal practice treats legal facts – this time facts about what are valid legal rules – as constructivist. And if legal practice treats these legal facts as constructivist, they are *prima facie* constructivist.[21]
Even on the constructivist view of law, the starting point of legal debates in which views on the content of the law are rationally reconstructed are the rules that are broadly recognised as law and the conclusions these rules attach to facts situations (cases). Therefore I will take this rule-centred approach to law as the starting point for an overview of techniques of legal reasoning. Together, these techniques are the best possible view of the logic of legal reasoning.

10. Contributory Reasons

Even though rules take a central place in the most frequent forms of legal reasoning, I will start my description of legal reasoning techniques with contributory reasons. Reasoning with rules cannot be fully understood without an understanding of how contributory reasons 'work'.

Contributory reasons are either constitutive or epistemic. A contributory reason for a conclusion c is a fact r (or a combination of facts) which pleads for the existence of c, in which case it is a constitutive reason. Or it is a reason for believing that c, in which case it is an epistemic reason. For example, the facts that some object has a flat surface and one or more legs supporting this surface are together a constitutive reason why this object is a table. The fact that the rooster is crowing is an epistemic reason to believe that soon the day will begin. Both constitutive reasons and epistemic reasons are important for law, but here I will only focus on constitutive reasons.[22] In the following, when I write about 'reasons', I mean 'constitutive contributory reasons'.

There can not only be reasons pleading for a conclusion, but also reasons pleading against a conclusion. For example, the fact that the surface of an object cannot support other objects is a reason why the object is not a table. A conclusion based on reasons is always a conclusion on the basis of balancing all the reasons for and against this conclusion. Often the set of reasons against a conclusion will be empty and then the conclusion 'follows' – that is: the fact of the conclusion exists – if there is at least one reason pleading for it. Suppose that an object has a surface supported by legs and there are no reasons why this object is not a table, then the object is a table.
If there are both reasons for and against a conclusion, additional information about the relative weight of these reasons is necessary. This weighing knowledge is ordinary knowledge (not meta-knowledge) which can be the conclusion of another argument. For instance, the fact that something is a caravan is a reason why it is movable. The fact that it is attached to the sewage system is a reason why it is immovable. These two reasons need to be weighed (or balanced) to determine whether the object is movable. For example, if there is a court decision that such a caravan is immovable, this decision is a reason why being attached to the sewage system outweighs being a caravan with regard to the issue of movability (Hage , 2005, pp.101-134).

This example is also an example of how legal reasoning works if there are no rules. Prior to an eventual court decision, there is no rule that determines whether caravans attached to the sewage system are movable or immovable. Let us assume that it is broadly recognised that being a caravan is a reason for being movable and that being attached to the sewage system is a reason against being movable (for being immovable). These are colliding reasons with regard to the potential conclusion that the caravan is movable and to deal with this collision weighing knowledge is required. Assume

that there is no broadly recognised view about this weighing knowledge. Then it is necessary to produce reasons with regard to the issue which of the reasons for and against movability outweighs its competitor. If such reasons cannot be found, the weighing knowledge must be introduced by cutting the knot and will become an unfounded premise of the argument.

11. Reasoning With Rules: Applicability, Application, Classification and Interpretation[23]

The most common case of reasoning with legal rules is when a case satisfies the conditions of a legal rule, and the rule attaches its conclusion to the case. This kind of reasoning resembles an argument of the form Modus Ponens and that explains the misguided attempt to model rule-applying arguments as arguments of this form. In this subsection I will pay attention to some details of simple rule-applying arguments and show why the Modus Ponens analysis does not even fit these simple cases.

It is convenient to have a technical term to express that the facts of a case match the conditions of a rule. I will use the term 'applicable' to this purpose. A rule is said to be applicable to a case if and only if the rule exists as a legal rule (is valid) and if the facts of the case satisfy the conditions of the rule. Take for instance the rule that immovable goods can be the objects of a mortgage. We have an object that is immovable and this object therefore *prima* facie satisfies the condition of the rule. Therefore, *prima facie*, the rule is applicable to this case.

Why only *prima facie*? Because a rule has not only conditions that are mentioned in the rule formulation, but also 'scope conditions'. If the mentioned rule is a rule of Belgian law, most likely it can only be applicable to immovables in Belgium. This is an example of territorial scope. Rules also have a temporal scope, determining during the time span during which the rule can be applicable. This time span typically more or less coincides in time with the validity of the rule, but the operation of rules may be retro-active or postponed to cases in the future. And then there are rules with a personal scope, such as rules of religious law which only apply to adherents of the religion, or to persons of a particular nationality. And there are rules which have a scope determined by their subject, such as rules of contract law that only apply to international trade contracts.

Hopefully, the readers have already noticed that both speaking about the applicability of a rule and speaking about ordinary and scope conditions of a rule and the division of the burden of proof with regard to the rule conditions does not treat rules as descriptive sentences. Logically speaking, rules are individuals rather than descriptive sentences or propositions. Since objects cannot function as premises of arguments, the Modus Ponens analysis of rule-applying arguments does not work, not even for the simplest of cases. From here on, I will not even mention the relevance of classical logic for rule-applying or reason-based arguments anymore; this relevance is non-existent.

Even if a rule is applicable to a case, this does not guarantee that the rule conclusion is attached to the case. It remains possible to make an exception to a rule, for instance if application of the rule would be against the rule's purpose, if the rule conclusion would for some other reason be unacceptable, or if the rule conflicts with another rule.[24]

Before continuing the argument, it is easy to have another technical term available. If a rule attaches its conclusion to a case, I will say that the rule *applies* to the case. If we have an immovable object and there are no special circumstances, the rule that immovables are susceptible to a mortgage applies to this case and attaches its conclusion – that the object is susceptible to a mortgage – to the case. Through its application, the rule creates a 'new' fact, namely that the object can be mortgaged. Notice that this operation of the rule is on the level of facts, not only on the level of language. It is rational to conclude that the object is susceptible to a mortgage and since – we assume – this kind of fact is constructivist, it is also the case that the object is susceptible to a mortgage.

Having the notion of rule application available, we can indicate what the relevance of a rule's applicability is: if a rule is applicable to a case, this is a reason why the rule should apply to this case, that is: why the rule attaches its conclusion to the case.[25] The applicability of a rule as reason for its application is in itself decisive if there are no reasons against application. However, if

there are also reasons against application, it is necessary to balance the reasons to determine whether the rule applies. The usual story about weighing knowledge is relevant here.

Before moving to non-standard cases of reasoning with rules, I need to say something about classification. A rule can only be applicable to a case if the facts of the case satisfy the rule conditions. To determine whether this is so, the facts need to be classified in terms of the rule conditions. For instance, if the rule is that thieves can be punished, the facts of the case must mention a thief. If John took away Jane's car without permission, this event can be classified as theft and John as a thief. Classification is just another form of legal reasoning, and all the theory of legal reasoning is relevant for it. It is worthwhile to notice that classification of case facts can be distinguished from the interpretation of a legal source. Interpretation plays a role in the step from legal sources to the legal validity of a rule in some formulation. Classification, in contrast plays a role in the step from one description of case facts to another description that matches the conditions of some rule.

12. The Legal Validity of Rules

A rule can only be applicable to a case if it exists; in traditional legal terminology: if it is valid. Moreover, it must exist as a legal rule, not 'merely' as, for instance, a moral rule. Most legal rules are considered valid because they can be traced back to a broadly recognised source of law, such as a statute, a treaty or convention, or a court decision. Most rules that have this pedigree will also be broadly recognised, directly – by the officials – or indirectly – by those who recognise the officials and the division of recognition labour. For instance, a rule that was adopted in an earlier court decision because it underlies the *ratio decidendi* of the earlier case will directly be recognised as a valid legal rule by courts to the extent that they feel bound by precedents, and indirectly by legal subjects who recognise courts as experts on what the law is.

A rule that can be traced to a source of law will normally be considered a valid legal rule. However, if legal facts are seen as constructivist, the source is not the final word even if it is the first word. It is possible to defend the view that a rule that is based on a source of law is nevertheless not valid law. Possible reasons are that the rule is highly unjust (Radbruch, 1945, Alexy, 1992 and Alexy, 2002), or that the rule systematically[26] conflicts with a 'higher' or more recent rule, or with a human right. Other possible reasons are that the alleged rule is not the proper interpretation of the text of the source, that the author of the statute, treaty of judicial decision was incompetent to make this rule, or that the source was created in an invalid manner.

Not only rules that can be traced back to a source can count as valid legal rules. It is also possible that some rule is broadly recognised as a legal rule without a recognised legal source to support this. Customary law is a case in point, as is 'unwritten law' such as the standards for the lawfulness of behaviour that are used in liability law.[27] If one adopts the constructivist view on law, such rules will exist as a matter of constructivist fact. They exist prima facie if they are broadly recognised as existing, but it is possible to have a serious disagreement on whether such a rule was rightly recognised.

13. Analogy, and Arguments *a Fortiori* and *e Contrario*

If a rule is not applicable to a case, this is a reason why the rule does not apply to the case. However, sometimes a case to which the rule is strictly speaking not applicable resembles cases to which the rule is applicable to such an extent that it is within the purpose of the rule that it should apply. In such a case the purpose of the rule provides a reason why the rule should apply. This reason may outweigh the non-applicability of the rule and if it does, the rule applies. Because of the resemblance to cases in which the rule applies because of its applicability, application because of similarity is called *analogous rule application*. For example, there is a rule that owners of a home are not allowed to have a tree on less than two meters distance from the garden of a neighbour.

There is good reason to also apply this rule to people who lease their home, rather than own it. So, in cases of analogous rule application, the rule actually applies to a case, even though it is not applicable.

An argument *a fortiori* is a special case of analogous rule application: the facts of the case resemble the facts of cases to which the rule is applicable but provide even more reason to apply the rule than the latter facts. If a rule that allows pretrial detention is applicable to cases of involuntary manslaughter, this *may* be a reason to apply it *a fortiori* to cases of intentional manslaughter. Whether it actually applies to such cases may depend on whether there are also rules for detention in cases of intentional manslaughter.

Normally, if a rule is not applicable to a case, this is only a reason not to apply the rule. If there are no reasons for application – and this will normally be the case – the rule does not apply and does not attach its conclusion to the case. However, sometimes the facts of a case which make the rule inapplicable provide a reason why the opposite of the rule conclusion should be attached to the case. If this reason leads to this opposite conclusion, it is sometimes said that the rule is applied *e contrario*. Take for instance the scope-defining rule that criminal law for minors, rather than ordinary criminal law, should be used for criminal suspects younger than 16 years. Then, arguably, the fact that some criminal suspect is 16 years or older is a reason why this special branch of criminal law should *not* be used.

14. Conclusion

Classical logic is based on an underlying view of the world, according to which there are elementary facts belonging to one of three types and compound facts, which are logical combinations of these elementary facts. Sentences are true if they correspond to, in last instance, the elementary facts in the world. The elementary facts, which hold that an individual has a particular characteristic, are independent of each other. This world view has no place for rules, which exist as individuals in the world, and which create relations between the most elementary facts. As a result, classical logic is not suitable to deal with rules, which manifests itself in several phenomena, including that:

- Rules lack a truth value and can therefore not be premises or conclusions in valid arguments.
- Classical logic cannot deal with exceptions to rules, or with rules about rules.
- Classical logic has no way to deal with analogous rule application, or arguments in which rules play an unusual role, such as arguments *per analogiam* or *e contrario.*

A logic that is more suitable for legal reasoning should take into account that law is a part of social reality, in particular a part that consists of constructivist facts, and that rules play a central role in law. This article has given a superficial description of how social reality exists and of the place of law and legal rules in it. It used this description to argue that traditional techniques to reason with and about legal rules provide a better logic for law than classical logic. These techniques can be accommodated in a logic that treats rules as logical individuals.

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Notes

1. I will focus on the semantics of first order predicate logic only. For propositional logic, the story is different, but not in a manner that influences the argument of this article.
2. Classical logic focuses on deductive validity and in that connection 'valid' therefore means the same as 'deductively valid'.
3. I assume here that 'ought to be punished' is an ordinary predicate, different from, but just like, 'is punished'. This means that I assume no logical relation between, for instance, the sentences that John ought to be punished and that John is punished.
4. To keep the argument as compact as possible, I will focus on sentences with an object-predicate structure and ignore sentences which describe relations, such as 'James is the father of Mary', or 'feature-placing' sentences such as 'It's raining'. This focus does, in my opinion, not misrepresent the ontological assumptions of predicate logic.
5. I mention the ugly construction of sets representing classes because sets are defined by their members (that is: extensionally) and not by a characterising property such as being a governmental organisation.
Nevertheless, informally the sets do stand for classes of things which are defined by a common characteristic. Therefore, from here on, I will write about the set of governmental organisations, and so on …

6. These problems have all been addressed, often in dedicated logics such as quantum logic, paraconsistent logic, fuzzy logic, or nonmonotonic logic. However, these logics have in common that their underlying ontology is not the ontology that underlies predicate logic.
7. A more extensive discussion of this topic can be found in (Hage, 2018, Chapter V, Hage, 2022 and Hage, 2022).
8. If rules would have to be represented in the semantics of predicate logic, they should be individuals which influence the assignment of individuals to sets. However, this would be such a gross violation of the assumptions underlying traditional model-theoretic semantics that it would not be the same semantics anymore.
9. Because of space limitations, this section and the following ones are highly condensed. Interested readers can find more extensive accounts of how social reality exists in (Hage, 2022 and Hage, 2022).
10. These definitions make facts and the world dependent on, amongst others, a language, and the descriptive sentences it can express. For a discussion, see (Hage, 2018, pp. 32-34).
11. Here I use the notion of a convention in a way that is close to conventional facts. A convention in this sense is related to, but not identical to the Lewisian (Lewis, 1969) notion of a convention as a solution to a coordination problem. See also (Rescorla, 2011).
12. Another mode of existence is as a rule-based rule. It is more convenient to explain this after the introduction of constructivist facts. See section 9.
13. Notice that the efficacy of social rules is *not* defined in terms of compliance. The definition should apply to *all* rules and not only to rules that impose duties or obligations and can therefore be complied with.
14. Strictly speaking, the group members should also recognise that the rule conclusion is a conventional type of fact.
15. Notice that according to this account, a statement about a rule – that the rule is valid, or that it exists – and not merely the rule formulation, is used in a rule-applying argument This has everything to do with the assumption that rules are not statements, but logical individuals. An advantage of this approach is that there is no issue (confusion of object- and metalanguage) with argument chains that combine reasoning about rules and reasoning with these same rules.
16. There are close connections between these constructivist facts and constructivism (intuitionism) in the philosophy of mathematics (Iemhoff, 2020) and constructivism in moral philosophy (Rawls, 1980, Bagnoli, 2021).
17. An Italian friend of mine, who is more knowledgeable about wines than me, seriously disagrees.
18. Of course, it is possible to have serious discussions on the issues of whether Hendrik is a good leader or whether Hendrik ought to be the leader. However, these discussions would address another issue than whether Hendrik *is* the leader.
19. There is no room in this article to further develop the notion of a rational reconstruction. As a very short alternative, I suggest that rational reconstruction of a set of beliefs and recognitions is making the set integratedly coherent (Hage, 2005, pp. 33-68; Hage, 2013 and Hage, 2016).
20. Early in his career, Dworkin (Dworkin, 1986) claimed that it would not occur. Legal questions would have one right answer.
21. It is only *prima facie* because the issue of whether legal facts are constructivist is itself a matter of constructivist fact.
22. The 'logic' of epistemic reasons is not unlike the logic of constitutive reasons, and much that is written below about reasoning with constitutive reasons also applies to epistemic reasons.
23. The content of this section is an adaptation of the theory of (Hage, 1997, chapter III). It was strongly influenced by discussions with Henrique Marcos and Antonia Waltermann.
24. This brief list of cases in which an applicable rule may not apply seems to cover the most important situations but is not intended to be exhaustive.
25. Notice the identification of the facts that a rule *should* apply to a case and that the rule actually applies to the case. This identification is possible because the application of a rule is a constructivist fact. See section 8.
26. If a rule systematically conflicts with a higher, or otherwise superior, rule, this is a reason against the validity of the former rule. If the conflict is only incidental, this is only a reason not to apply the rule in the specific case.
27. An example of such a standard is the 'Learned Hand rule' that was formulated in *U.S. v. Carroll Towing*, 159 F.2d 169 (2d Cir. 1947).

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Legal Reasoning and Logic

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Abstract:
This paper investigates the basis arguments of so-called legal logic and their relation to logic in its standard meaning. There is no doubt that legal arguments belong to logic in the wide sense (*sensu largo*), but their reduction to schemes of formal logic (*logica sensu stricto*) is a controversial issue. It can be demonstrated that only some legal arguments fall under explicit rules of formal logic, that is, having a deductive character. Most such reasoning is fallible, and its correctness depends on appealing to extra-logical principles taken from legal norms. For instance, if we say, "If it is permitted more, then it is permitted less" (*argumentum a maiori ad minus*), we assume that the concepts expressed by the words "more" and "less" are already defined.
Keywords: argument, premises, conclusion, legal principle, entailment.

Relations between law and logic were discussed in antiquity and persist until today. The *Talmud* contains many examples of reasoning used in solving concrete legal problems (Schumann, 2017). Protagoras, the leading Sophist, had a student Euthalos. Both established by a contract that the student would pay the master for his teaching after Euthalos won his first court case. However, Euthalos decided not to perform legal practise but to enter politics. Protagoras decided to sue Euthalos for the payment. He argued that if he won the case, he would be paid on the basis of the sentence, but if Euthalos won the case, Protagoras would be paid according to the original contract, because Euthalos would have won his first case. Euthalos, however, answered that if he won, then he would not have to pay by the sentence, but if Protagoras won, then Euthalos would not be obliged to pay, because he lost the case. The ancient sources do not say how this controversy was solved, but even a provisional analysis shows that something is lacking as a premise in the argumentation in question. It seems that one must add a principle asserting what has legal priority — a court's sentence or a contract in the case of a behaviour not occurring that activates an obligation. This example shows that so-called legal logic has two ingredients: schemes of reasoning applied in law and the general principles which instruct how to solve inconsistencies or ambiguities stemming from formulations occurring in legal texts. The status and scope of legal logic are central in legal theory and philosophy. Some authors (Sartor, 2005, p. XXV) say that fundamental oppositions in theoretical jurisprudence, such as those between natural law theory and legal positivism or legal functionalism and legal formalism, have their explicit reference to problems of

argumentation employed in law. Independently, whether this opinion is (fully) correct or not, the schemes of legal logic deserve attention. The extensive literature (Armgardt, Canivez & Chassagnard-Pinet, 2015; Hage, 2005; Klug, 1966; Perelman, 1977; Prakken, 1977; Rahman, Armgardt & Kvernenes, 2022; Weinberger, 1970) confirms this suggestion. In what follows, I will concentrate on logical aspects of legal logic.

How is legal logic related to logic in its standard understanding? Clearly, it depends on how logic is understood. Omitting details (a more extended analysis is found in (Woleński, 2007) we can distinguish two understandings: narrow (*sensu stricto*) and wide (*sensu largo*). The former identifies logic with formal logic, which is a collection of logical systems (e.g., standard propositional logic, predicate logic, modal logic), based on the concept of logical consequence. Logic *sensu largo* covers logic in the narrow, semantics (semiotics), and methodology of science. If someone decides to think about legal logic as formal logic, he or she immediately is confronted with a serious problem. Law consists of norms as linguistic items. Now, there is a big controversy whether norms are true or false, that is, whether they can function in premises or conclusions of correct inferences. More specifically, since the concept of logical consequence essentially employs truth and falsity (if *B* is a logical consequence of *A*, then if *A* is true, then *B* is true by logical necessity), what is a semantics foundation of normative logic, if norms are neither true nor false? In what follows, I will not discuss this question and adopt a simple (simplified, if you like) assumption that normative statements are reducible to deontic ones (i.e., forms) like "it is obligatory that *A*", "it is prohibited (forbidden) that *A*", "it is permitted that *A*", etc. However, the Protagoras–Euthalos case suggests that legal logic cannot be reduced to logic *sensu stricto* because it employs some other principle. Without entering into details, we can say that legal logic uses legal semantics and legal methodology. The adjective "legal" is important here because it refers to specific arguments (reasoning) performed by lawyers. For instance, most procedures in forensic science consist in drawing conclusions from empirical data and do not belong to legal logic, but they can still be considered as belonging to legal methodology in the wide sense. Perhaps one forgotten fact illuminates the conceptual situation. The term *deductio* had a special meaning in medieval Latin. It referred to the way of arguing by a party before a court. So, Protagoras performed a deduction, and Euthalos deduced his claim as well. This meaning later disappeared, and today we say that *B* is correctly deduced from *A* if and only if *B* is a logical consequence of *A*.

I do not suggest that standard logic has no application in law. Sometimes it helps in the process of legal interpretation. Consider the regulation

(1) The candidate for a position *P* can be pointed by *X* and *Y*.

How do we understand this norm? The word "and" suggests that it is a conjunction, but legal understanding dictates the use of "or". In fact, it was a controversy in Poland whether candidates for the Constitutional Tribunal are pointed out by the Presidium of Sejm and the group of 50 deputies or by the first or second subject. Defenders of the first solution understood "and" as a conjunction, but the latter view argued that we have to go with a disjunction, frequently expressed in the conjunctive form; they argued that a frequent legal stylistic custom uses "and" as a mark of disjunction. The controversy in question was resolved by a new regulation which explicitly employed "or". Now consequences of both interpretations are far-reaching, because the former states much stronger conditions for the procedure of pointing out candidates for the Constitutional Tribunal than the latter. Of course, logic by itself does not solve the problem of interpretation of (1), but it helps in the evaluation of consequences of adopting a particular understanding of "and".

Argumentum a contrario has a typical deductive structure. Consider article 127.3 of the Constitution of the Republic of Poland. It says "Only a Polish citizen who, no later than the day of the elections, has attained 35 years of age and has a full electoral franchise in elections to the Sejm, may be elected President of the Republic. Any such candidature shall be supported by the signatures of at least 100,000 citizens having the right to vote in elections to the Sejm". The word "only" is important and immediately suggests that this regulation establishes necessary conditions for being a

candidate for the Polish presidency. Consequently, we can derive from this norm several consequences, for instance, "if a person is not a Polish citizen, he or she cannot be elected President of the Republic of Poland". The formal structure of this argument is captured by the following scheme:

(2) $(A \text{ (only)} \rightarrow B) \rightarrow (\neg A \rightarrow \neg B)$,

which is equivalent to

(3) $(B \rightarrow A) \rightarrow (\neg A \rightarrow \neg B)$

and

(4) $A \leftarrow B) \rightarrow (\neg A \rightarrow \neg B)$,

where the symbol $\leftarrow$ refers to the reverse implication. A characteristic feature of the operation $\leftarrow$ is that it allows simple transposition (Perelman, 1977), that is, without changing the order of arguments, contrary to $\rightarrow$, where we must move the antecedent to the position of consequent and reversely.
Another example of *argumentum a contrario* is displayed by the following:

(5) An action *D* is a crime if and only if it is prohibited by the penal code during the period of its validity.

Perhaps it is interesting to observe that (5) expresses one of the most fundamental legal principles, namely *nulla poena sine lege* (no penalty without law). It decides that penal illegality is a sufficient and necessary condition of qualifying an action as a crime. Yet some additional comments are in order. Firstly, penal codes frequently state additional constraints for crimes, for example, that an action must be socially dangerous. In such a case, criminal illegality is a necessary condition of considering an action as criminal, but sufficient. A practical consequence of this assertion is that the necessary condition cannot be omitted. If someone says that extremely immoral deeds should be considered as legal crimes, he or she does not understand the principle *nulla poena sine lege*. It is interesting to observe that the prohibition of analogy against interests of accused persons is justified by the principle in question: assume that a penal norm *A* is extended by analogy. It can happen that an action formerly not qualified as a crime can be considered as legally penalised. Clearly, it might be at odds with *nulla poena sine lege* and *nullum crimen sine lege* (no crime without law). Secondly, (5) is not fully adequate because there are circumstances in which a given action is, so to speak, formally a crime, but it is not qualified as a crime because, for instance, the person who committed it acted in necessary self-defence, or he or she did not attain 18 years of age (or another age, depending on the code). Consequently, (5) should be rewritten as:

(6) For every action *D* that does not hold circumstances excluding being penalised, *D* is a crime if and only if it is prohibited by the penal code during the period of its validity.

In (6), restricted universal quantification is used. There is a discussion whether excluding being penalised also eliminates criminality, but I skip this question as transgressing legal logic.
Logical analysis of *argumentum a contrario* is relatively easy, but things appear differently in the case of *argumentum a fortiori*. It has two forms, namely:

(7) *argumentum a maiori ad minus*.

(8) *argumentum a minori ad maius*.

More specifically, both can be written, respectively, as:

(9) If it is permitted more, then it is permitted less.

(10) If it is forbidden more, then it is forbidden less.

Due to standard deontic logic and the definition $FA =^{df} \neg PA$ (*A* is forbidden if and only if it is not true that *A* is permitted), (9) and (10) are logically equivalent and, thereby, they might be considered as two formulations of the same argument. However, this nice picture must be supplemented by a closer analysis. First of all, since the words "more" and "less" do not express logical constants, schemes (9) and (10) are not logical theorems. Consider the following cases:

(11) If it is permitted to vote, it is permitted to abstain from voting.

(12) If it is prohibited to drive at a speed of 100 km/h, it is prohibited to drive at a greater speed.

Clearly, if we define voting as something more than abstaining from participation in elections, (11) is a sound inference. Similarly, deciding that a greater speed is something more than a slower one, (12) is suitable. Supplementing (11) and (12), we obtain, respectively:

(13) If it is permitted to vote, it is permitted to abstain from voting, and since the second behaviour is less than the first, it is permitted to abstain from voting.

(14) If it is prohibited to drive at a speed of 100 km/h, it is prohibited to drive at a greater speed, and since driving at a speed of 150 km/h is more than driving at a speed of 100 km/h, then driving at 150 km/h is prohibited.

Yet, such arguments can be fallacious. Assume that rules for driving require that there must be a minimal speed, say 100 km/h. So, driving at 100km/h is permitted, but a speed less than 100 km/h is prohibited. Intuitively, killing is something more than injuring, but an executioner can kill doing an execution, yet he cannot injure (as a final effect) a convict. Thus, applying arguments *a fortiori* requires taking into account several regulations that contribute to the legal understanding of "less" and "more".

The above considerations suggest that in the evaluation of schemes of legal logic, their formal structure is not the only criterion, perhaps with the exception of some simple cases of *argumentum a contrario.* In more advanced cases, like *argumentum a fortiori*, it is necessary to take into account more informal aspects. Note, however, that if we compare a legal argument with an ordinary one, the former refers to formal aspects constituted by legal norms. Thus, we should distinguish between formal in a logical sense and formal in a legal sense. For instance, that a reasoning is a deduction or not refers to its logical formality, but that legal norms prohibit analogy against interests of accused persons appeals to legal formality. Perhaps this fact is partially responsible for the opinion that legal logic is formal, because it appeals to forms established by legal prescriptions.

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Legal Gaps and their Logical Forms

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Abstract:
The concept of legal gap is tackled from a number of logical perspectives and semantic methods. After presenting our own goal (Section 1), a first introduction into legal logic refers to Bobbio's works and his formalization of legal statements (Sections 2 and 3). Then Woleński's contribution to the area is taken into account through his reference to the distinction between two juridical systems (viz. Common Law vs Civil Law) and the notion of conditional norms (Section 4). The notion of reason is also highlighted in the case of Raz's legal logic, thereby leading to a future connection with von Wright's logic of truth and an analogy made with an anti-realist reading of truth-values and norms (Section 5). Our personal contribution is introduced through a reflection on how logic should deal with the logical form of norms (Section 6), before entering a number of crucial definitions and distinctions for the concepts of norm, legal statement, and promulgation (Section 7). The final point is a proposed semantics for legal statements, which is both many-valued and gap-friendly (Section 8). A distinction between a number of requirements for permission and forbiddance leads to a set of non-classical juridical systems in which non-permission and forbiddance are not equivalent with each other any more; this does justice to Woleński's former distinction between Common Law and Civil Law, also leading ultimately to a non-classical square of legal

oppositions in which several legal operators may collapse into other ones (Section 9).
Keywords: forbiddance, juridical system, legal gap, legal statement, permission.

1. Introduction

There are two situations in a juridical system that generates century-old discussions. The first one is as follows: a child is drowning in a lake and a passerby is seeing the drama. He has to react accordingly: Ought he to save the child? Yes: there is an obligation to save someone in mortal danger, otherwise one is accused of failure to render assistance to a person in danger. No: it is forbidden to swim in this lake, according to the local *juridical norm*. Does this mean that the passerby is both obliged and forbidden to enter the lake in order to save the child? The other situation is subtly different. Like the first situation, there is a child drowning and a passerby. This time, however, there isn't any legal obligation, neither to save a person in mortal danger nor to refrain from swimming in the lake. In this case, what is to be done from the juridical logic perspective? Does anything goes, or is there some kind of constraint? The first case is a classical example of an *antinomy*, which correlates to legal *inconsistency*, while the second is an example of a *legal gap*, which corresponds to what is called legal *incompleteness*.

Our questioning is twofold: What does *legal gap* mean in a formal context? And how to deal with it from a logical perspective? The first perspective is the *positivist* approach stating that there are no *real legal gaps* but only *ideological gaps*, insofar as law is understood as a juridical system that is taken to be consistent and complete. The second perspective is the rhetorical approach stating that there exists real legal gaps; now they are all solvable only by rhetorical means, since law is not a logical system but an *almost-logic* one. The third perspective states that gap is to *paracomplete* juridical reasoning what legal *glut* is to *paraconsistent* juridical reasoning. Our point is that there are at least three juridical answers in front of this state of affairs. Either there is always a criteria (e.g. hierarchical, chronological, specialty, competency etc.) that decides which juridical norm should be used to give only one deontic normative value for an action, i.e., the passerby is either obliged or forbidden to save the child. Or there is no such criteria, by which a conflict between two different deontic normative values arises. This conflict is one of the following two: the passerby is both obliged and forbidden or the passerby is neither obliged nor forbidden to save the child in the eyes of general law. What of this legal *pluralism*?

A many-valued system of juridical logic is proposed to account for a number of problems related to philosophy of law, especially the case of legal gaps. While a number of papers have been devoted to the case of paraconsistent legal logic (through the issues of inconsistent data bases and *defeasible reasoning*), the following wants to stress on legal gap as a case for paracomplete juridical reasoning. We propose a general framework for this purpose: AR_{4L}, which is a 4-valued juridical system including the aforementioned *juridical systems* as particular sublogics. In the vein of Von Wright's truth-logic, it consists of a formal language of juridical statements S*p*, to be read 'There is a juridical norm that states (the action described by the sentence) *p*' (where the action expressed by *p* is not indifferent in accordance with the law). Then negation may be prefixed to either S or *p*, leading to a set of 16 juridical situations on the basis of the 4 basic ones: S*p*, ¬S*p*, S¬*p*, and ¬S¬*p*. A deontic interpretation of S depends upon which kind of juridical system is mentioned with its correspondent *rule of legal closure*, whether it be Common Law ('If something is not prohibited, then it is permitted') or Civil Law ('If something is not permitted, then it is prohibited'). In the former, a *promulgation* entails that doing what a juridical norm states is permitted; whereas in the latter, a promulgation entails

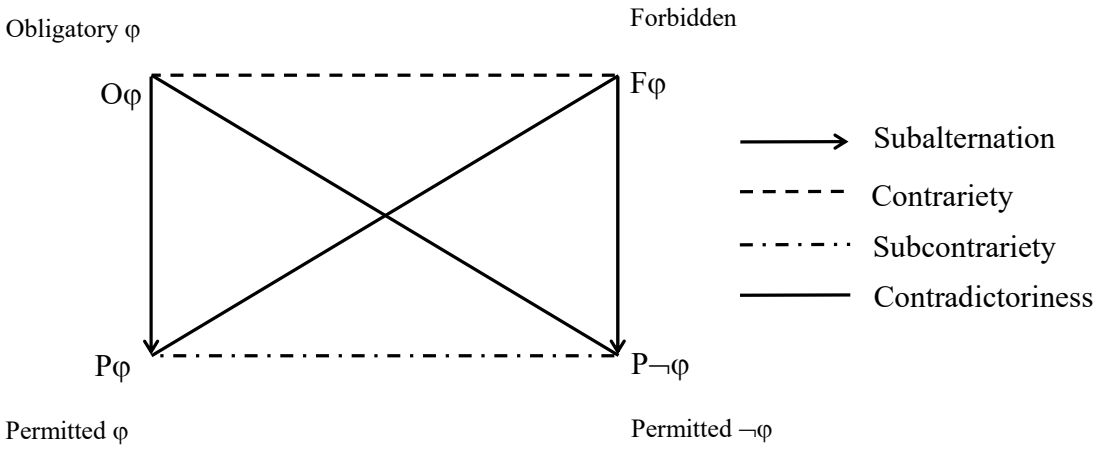
that doing what a juridical norm states is forbidden. Permission will be viewed as the basic deontic normative value, in the following.

A semantics for juridical statements is an interpretation assigning deontic normative values to statements *p*, and juridical pluralism stems from the plurality of assignment conditions of them: (1) There is a promulgation *p*, and there is a promulgation $\neg p$; (2) There is a promulgation *p*, and there is no promulgation $\neg p$; (3) There is no promulgation *p*, and there is a promulgation $\neg p$; (4) There is no promulgation *p*, and there is no promulgation $\neg p$. (1) and (4) correspond to legal gluts and gaps, and we will defend a many-valued treatment of these after a survey of the relevant literature in legal logic.

2. The Differentiation Between Antinomy and Legal Gap

An intuitive way to differentiate the problem of inconsistency from the problem of incompleteness in law is to say that the former is represented by an abundance of mutually incompatible solutions, while incompleteness represents the absence of any compatible solution (Nino, 2015). In law, the problem of consistency relates to *antinomy* and the problem of completeness relates to *legal gap*.

Bobbio's analysis of antinomies involves the use of the deontic square of oppositions to demonstrate the relationships between *juridical norms*[1] in Law (Bobbio, 1999).



- **Contrariety**: two norms are contrary when it is the case that they cannot both be true, but they can both be false.
- **Contradictoriness**: two norms are contradictory when it is the case that they cannot both be true, and they cannot both be false.
- **Subcontrariety**: two norms are subcontrary when it is the case that they can both be true, but they cannot both be false.
- **Subalternation**: two norms are subalterns when it is the case that the subaltern norm (which is at the bottom of the square) always is true if the superaltern norm (which is at the top of the square) is true and if the subaltern norm is false, then the superaltern norm will be false either.

For Bobbio, two juridical norms are said to be *inconsistent* if, and only if, there are two regulations for φ and those regulations are either contraries or contradictories:

Contrary juridical norms
$O\varphi \wedge F\varphi$

Contradictory juridical norms
$O\varphi \wedge P\neg\varphi$ $F\varphi \wedge P\varphi$

What of legal gaps? In theory, a legal gap would represent a situation of *empty legal space*, that is, there would be an unregulated φ whenever there is no juridical norm that says whether φ is associated with O (obligation), F (prohibition), P (positive permission) or P¬ (negative permission). We could represent this situation as:

Legal Gap
$\neg(O\varphi \vee F\varphi \vee P\varphi \vee P\neg\varphi)$

However, there are reasons sufficiently developed by various philosophers of law to accept that empty legal space does not exist (Bobbio, 1999). Law, as a *legal order*, has general norms with a very high degree of abstraction that can be applied to any behavior φ. We will analyze these general norms in the following section.

3. Bobbio's Insight

If there is no empty legal space, then there is only *full legal space*, and we can only think of a full legal space if the legal order has an infinity of specific norms for each possible behavior and situation or a few general norms that can be applied to a vast number of cases, if not all. Bobbio believes that the second case is the correct one to ground the legal order from a logical general theory of law.

The first general juridical norm presented by Bobbio is the *Exclusive General Norm*. This norm always accompanies a *specific norm*, much like a shadow. Consider, for example, an elevator with a sign stating that cigarette smoking, *s*, is prohibited, F*s*. What Exclusive General Norm does is dictate that, supposing that the sign constitutes the only juridical norm in place, then it is permitted to do anything else that does not involve cigarette smoking. Thus, Exclusive General Rule eliminates from the field of incidence of the specific legal norm, F*s*, any other action that is not cigarette smoking. This ensures that the legal order within the elevator example (which possesses only a single specific rule) is complete (i.e., it has no gaps) since, for every action, if the action is cigarette smoking then it is prohibited (by the specific rule), and if the action is anything other than cigarette smoking then it is permitted (by the Exclusive General Rule).

In addition to Exclusive General Norm, Bobbio identifies a second general legal norm that he calls Inclusive General Norm. The function of this norm is to ensure that actions or situations similar to those regulated by the specific norm are treated identically. To continue with our elevator example, the existence of the specific norm prohibiting cigarette smoking includes in its prohibition, acts that are similar to cigarette smoking, for example, the use of electronic cigarettes or vaporizers, hookahs or pipes, actions involving inhalation of smoke, etc. So, for every action, if the action is cigarette smoking then it is prohibited (by the specific rule), if the action is similar to cigarette smoking then it is prohibited (by Inclusive General Norm), and if the action is anything other than cigarette smoking or any action similar to it then it is permitted (by the Exclusive General Rule).

But, if the legal order is complete, once again, where are legal gaps? For Bobbio, the issue of legal gaps is not a problem of a lack of legal norms because, for every action, it will either be excluded

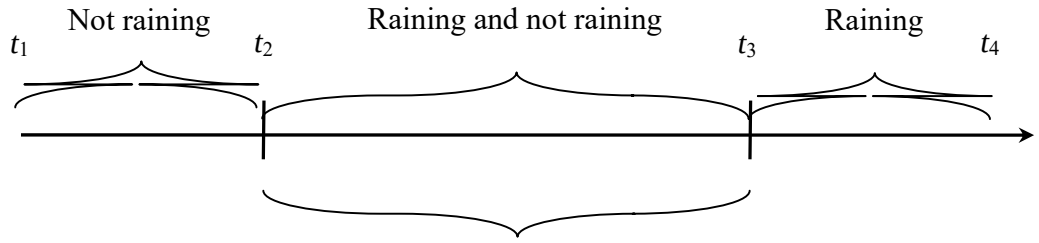
from the scope of a specific norm by Exclusive General Norm or included into the scope of a specific norm, if it is a similar action, by Inclusive General Norm.

The legal gap, therefore, constitutes a problem of a more profound nature. It is evident that, in our elevator example, some situations will obviously be excluded from the scope of the cigarette smoking prohibition, like the act of whistling, while others will also quite clearly be included within the scope of prohibition due to their similarity, like smoking a pipe. The problem arises when one encounters a borderline situation that generates quite reasonable doubts about which general norm, whether exclusive or inclusive, would be applicable.

The most problematic issue related to Bobbio's reasoning is that he appears to transform the problem of legal completeness into a problem of legal consistency. It is not clear if Bobbio is aware of how much his rationale brings the legal gap and legal antinomy closer together, since the problem of completeness becomes a problem, not of lack or absence of legal norms, but of the existence of two legal norms and a doubt about which of these two legal norms should be applied. In summary, at least from the point of view of Bobbio's philosophy of law, the problem of consistency is also a problem about two legally inconsistent norms associated with a doubt about which one should be applied.

In this sense, perhaps von Wright's reference to different types of truth may help. Suppose there is a norm that provides that "If it rains, then it is prohibited to drive a car". Since rain is a natural continuous process that extends over time, there is a moment that will obviously be understood as rain, as well as another moment that will also be obviously considered as non-rain. But what about the intermediate moments, i.e. when only a few drops are falling from the sky (von Wright, 1986)?

This question was formulated by von Wright to expose two different views of what would be a true proposition in these cases. One view understands that at this intermediate moment it is neither raining nor not raining, and another that it is both raining and not raining at the same time. Wright called the first form (*neither ... nor ...*) *strict truth*, and the second form (... and ...) *liberal truth*.



The Finnish author's thinking seems useful for understanding what Bobbio seems to mean. For Bobbio, the *intermediate zone* is the one that generates the completeness problem because, for him, this would be a situation where one cannot, from a legal point of view, choose any of the general norms for application. In this way, Bobbio seems to assume, according to von Wright's categorization, a strict view of truth. And if the judge cannot apply either of the two general rules then, indeed, in this case we would have as fact that the legal order is incomplete.

Bobbio's strict view of truth implies that, in these intermediate zone situations, a judge can ultimately resort to what is called *judiciary law*, which is a method of legal resolution that admits the judge's creative power to create the law beyond what is already established, as if the judge were the legislator for that situation. Bobbio's view is endorsed independently by H. L. A. Hart, who asserts that legal order is always partially incomplete in possessing an open texture through which the judge, in these hard cases contained within an area of imprecision, creates the Law to be applied to the case – as if he were a legislator, albeit with greater limitations than the Legislative Power itself (Hart, 2021). Bobbio understands that the application of judiciary law is a very serious matter as it violates the separation of powers, thereby generating uncertainty and disorder in democratic orders. Hart, on the other hand, views the issue in a less harmful way, understanding that, even in this creative activity, the judge acts in a much more restrained way than a real Legislative Power could act.

Bobbio's original insight is a starting point for formulating a deeper view of legal gap. However, the general norms used by Bobbio do not correspond to the closure rules most commonly worked on in current philosophy of law. In addition, his notion of Inclusive General Norm resembles the technique of using *analogy*, something admitted by the author himself, which is nothing new or revolutionary.

More problematic, however, is the fact that Bobbio's solution generates more problems than one might perceive at first reading. For instance, the application of the General Exclusive Norm on specific permissive norms seems particularly problematic, due to its ability to generate infinite prohibitions, something that will be better visualized in the example of the next section. In addition, the use of the analogy technique presents its own controversial issues, as the problem of modes of truth proposed by von Wright can affect its very use: how similar must one conduct be to another to be included in the field of incidence of the specific norm by General Inclusive Norm?

In the following section, we will seek to advance this debate and continue our deepening of legal gaps by working with the idea of closure rules.

4. Woleński's Closure Rules and the Insufficiency of Standard Deontic Logic

Like Bobbio, Woleński understands that any normative system has *closure rules*. He suggests that the two normative systems are British legalism and German legalism,[22] each of which is differentiated by its own unique closure rule, as follows (Woleński, 2006):

(I) if something is not prohibited then it is permitted (Common Law)
(II) if something is not permitted then it is prohibited (Civil Law)

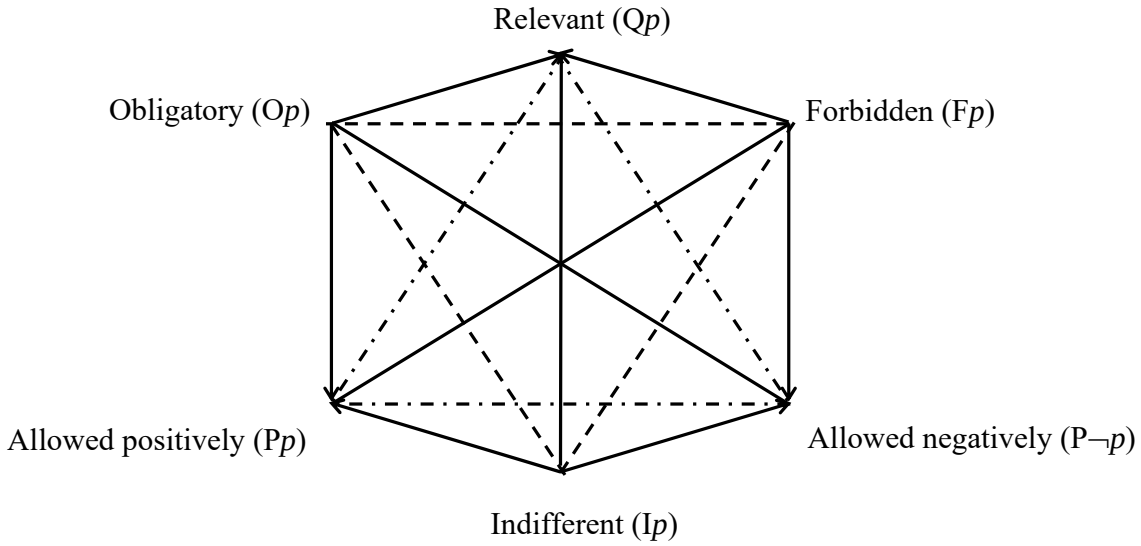
It is easy to see the similarity between these two closure rules and Bobbio's General Exclusive Norm. There may be a tendency, due to a hasty reading of Bobbio's general norms, to associate Common Law with the General Exclusive Norm, while Civil Law would be associated with General Inclusive Norm. This is a mistake, however. Closure rules (I) and (II) are actually versions of General Exclusive Norm, which excludes from the norm's field of incidence everything that is *contradictory to* what norms stipulate. Furthermore, we previously talked about the prohibition of smoking cigarettes; but consider, for example, an elevator with a sign stating that cigarette smoking, *s*, is allowed, Ps. What Exclusive General Norm does is dictate that, supposing that the sign constitutes the only juridical norm in force, then it is prohibited to do anything else that does not involve cigarette smoking. Thus, Exclusive General Rule eliminates from the field of incidence of the permissive specific legal norm, Ps, any other action that is not cigarette smoking. The role of Inclusive General Norm, unlike the closure rule of Common Law, is not to serve as a general permission but to authorize the use of analogy to include

into the field of incidence of the specific norm, Ps, everything that is similar to action *s*, for example, smoking pipes or cigars.

Actually, Bobbio's view contemplates only one closure rule, Exclusive General Norm, whereas Inclusive General Norm represents only the traditional method of applying analogy to fill legal gaps. Woleński's reasoning represents an advancement, because it contemplates not one but two closure rules, both representative of the two consecrated ways of viewing the principle of legality in the history of legal reasoning. However, as we will see next, this view also has its own difficulties.

For instance, Woleński uses deontic logic to assess the applicability of (I) and (II) in legal reasoning. Deontic logic is the branch of philosophy of logic that is concerned with the study of the logical consequences of using terms like Prohibition (F), Permission (P), and Obligatory (O). There is no single language of deontic logic that is accepted unanimously, and it is a field still in frank evolution and development. However, one could say that the so-called 'Standard Deontic Logic' (SDL) is the most cited and studied system of deontic logic (Shapiro & Kouri Kissel, 2022).

Suppose a proposition φ that represents some given behavior. By associating a deontic expression of prohibition with the proposition φ it is possible to infer Fφ, which is read as 'It is forbidden to φ'. The relationships between the deontic operators in SDL can be visualized in the below hexagon of oppositions (Kalinowski, 1996 and Woleński, 2006):



In accordance with the relations of opposition, the deontic hexagon helps to derive the following:

- **Subalternation Relations**

A1. $O\varphi \rightarrow P\varphi$
A2. $F\varphi \rightarrow P\neg\varphi$
A3. $\neg P\varphi \rightarrow \neg O\varphi$
A4. $\neg P\neg\varphi \rightarrow \neg F\varphi$

- **Subcontrariety Relations**

A5. $P\varphi \wedge P\neg\varphi$
A6. $\neg(\neg P\varphi \wedge \neg P\neg\varphi)$

- **Contrariety Relations**

A7. $\neg(O\varphi \wedge F\varphi)$

- **Contradictory Relations**

A8. $F\varphi \leftrightarrow \neg P\varphi$
A9. $O\varphi \leftrightarrow \neg P\neg\varphi$

- Other important relations include:

A10. $I\varphi \rightarrow (P\varphi \vee P\neg\varphi)$
A11. $O\varphi \rightarrow Q\varphi$
A12. $F\varphi \rightarrow Q\varphi$

As can be observed, a legal system operating in accordance to a deontic logic described by this hexagon would be consistent and complete. For any φ, there would always be one unique solution.
However, the Polish philosopher noticed that for SDL (I) and (II) mean the same thing, as the below proof shows:

[1]	$\neg F\varphi \rightarrow P\varphi$	(Common Law)
[2]	$\neg P\varphi$	(Hypothesis)
[3]	$\neg P\varphi \rightarrow F\varphi$	(1, 2, MT) (Civil Law)
[4]	$(\neg F\varphi \rightarrow P\varphi) \leftrightarrow (\neg P\varphi \rightarrow F\varphi)$	

Common Law is known to validate (3), but not (1). This means that both kinds of juridical systems do not share the same definitions of the aforementioned norms. Moreover, this entails that the normal hexagon of deontic oppositions is unable to represent the separate logics of Common Law and Civil Law.

For this reason, despite its theoretical, conceptual, and pedagogical importance, the hexagon is not sufficient to translate Bobbio's intuition that there is a problem of completeness in legal systems or that there are not one, but two different closure rules. It cannot capture the legal intuition that common law systems (primacy of permission, of broad freedom) are substantially different from civil law systems (primacy of prohibition, of restricted freedom). Therefore, we will need to expand our tools beyond clas- sical logic if we want to describe the problem of gaps in law in a formally adequate manner.

Woleński tried to understand the distinction between these two types of legalisms from the study of the specific case of competence norms, i.e., those norms that assign competences to institutions to perform acts based on what he called 'strong permissions', which would be those permissions (not generic) that are associated with specific 'conditional obligations'. The author recognized that to include this new concept he would have to extend the domains of SDL, which led him to opt for a semi-formal approach (in his own words) in order to proceed with his analysis.

In this sense, an institution would only possess those competencies that were explicitly determined (strong or explicit permissions, $P^s\varphi$) and an action of an institution that was not strongly permitted would be prohibited. Woleński defined this new primitive norm as:

$$(III)\ P^s\varphi \leftrightarrow (\varphi \rightarrow O^x\varphi)$$

(III) means that, if φ is done, then obligations *x* come into being. The example given by the author would be that of a committee that has permission in law to dissolve itself before the end of its members' term. In this case, this strong permission would be read as 'If the committee dissolves itself, then its decision must be respected by all, it must have the majority of votes, etc.', i.e., a permission associated with conditional obligations that result from the realization of the permitted action. However, Woleński's solution, besides being only a semi-formal solution, is also a solution that can only differentiate the common law system from the civil law system in specific cases of competence norms, which makes his approach quite restricted.

Bobbio's analysis of legal gaps is analytical, but not formal. Woleński's, on the other hand, is analytical and semi-formal. In the next section we will evaluate a formal analysis of legal gaps from an interpretation of one of the most respected philosophers of law today, Joseph Raz.

5. Raz's Legacy

In Chapter 4 of the second edition of the book *The Authority of Law*, entitled 'Legal Reasons, Sources and Gaps', Joseph Raz will express his view of what would be legal gaps, with a both analytical and formal approach (Raz, 2009).

One of the great problems of the analysis of gaps in versions like those of Bobbio and Woleński is that legal philosophers, when adopting the sources thesis,[33] viz. that every legal statement has a legal source that underpins it, are forced to adopt a reductive perspective of legal statements, which becomes synonymous with statements about what 'someone' orders. That is to say, if there is a legal statement, *p*, there is a source of law that underpins this legal statement, S*p*:

$$\text{(IV)} \quad \vdash p \leftrightarrow \mathrm{S}p$$

For Raz, this entails both:

$$\text{(V)} \quad \vdash \neg p \leftrightarrow \neg \mathrm{S}p$$

$$\text{(VI)} \vdash \neg p \leftrightarrow \mathrm{S}\neg p$$

And therefore, (IV) entails:

$$\text{(VII)} \vdash \neg \mathrm{S}p \leftrightarrow \mathrm{S}\neg p$$

The problem is that (VII) is false, according to practical experience of law (Raz, 2009). Raz's solution goes through the formalization of what he believes to be a 'conclusive reason for φ'. This statement has the form 'There is a conclusive reason for a person *x* to φ', and Raz uses a special logical form to represent it:

$$\text{(VIII)} \ (\mathrm{R}_c x, \varphi)$$

Conclusive permission is formalized as:

$$\text{(IX)} \ (\mathrm{PER}_c x, \neg\varphi)$$

$$\text{(IX′)} \ (\neg \mathrm{R}_c x, \varphi)$$

From this point, Raz philosophically assumes some conditions for this system he is developing, which are:

‘There cannot be conflicting conclusive reasons’:

$$(X) \vdash \neg(R_c x,\varphi \wedge R c_x,\neg\varphi)$$

‘It is not assumed that in every case to which reason applies there is a conclusive reason either for the action or against it’:

$$(XI) \vdash \neg(R_c x,\varphi \vee R_c x,\neg\varphi)$$

‘A conclusive permission to act is the contradictory of a conclusive reason for refraining from that act’:

$$(XII) \vdash (R_c x,\varphi) \leftrightarrow \neg(PER_c x,\neg\varphi)$$

In our understanding of what Raz proposes, there are conclusive reasons for someone to do something and there are also, let’s say, partial reasons. One type of partial reason for someone to do or not to do something, and which interests us more than any other, is *legal reasons*. If the account of sources is admitted, then every legal reason has a source, ‘*so*’. In this way, the legal statement ‘*so* is a legal reason for *x* to φ’ can be written

$$(XIII)\ so\mathrm{LR}x,\varphi$$

And concerning explicit permissions that, for Raz, in the same way as Woleński, constitutes a permission to cancel an existing legal reason or to forestall possible reasons by cancelling them in advance:

$$(XIV)\ so\mathrm{LPER}_c x,\neg\varphi$$

It is this type of legal statement that, for Raz, allows a deep understanding of the source thesis. The source thesis implies a truth criterion for evaluating the values of legal statements. Statements of the form $so\mathrm{LR}x,\varphi$ are true if, and only if, *so* can be substituted by a source of law (a social fact), without recourse to moral arguments. Statements of the form $so\mathrm{LPER}_c x,\neg\varphi$ are true if, and only if, *so* can be substituted by a source of law (a social fact) that cancels a legal reason, without recourse to moral arguments. In this sense, it follows that negative legal statements ($\neg so\mathrm{LR}x,\varphi$) do not have sources, while explicit permissions ($so\mathrm{LPER}_c x,\neg\varphi$) always have sources.

And what does it mean to say that a legal statement does not have a source? For Raz, it means that *so* cannot be substituted by a source in a “complete” manner and, therefore, the answer to the question “What is the source (social fact) that legally substantiates this legal statement?” is non-existent or incomplete. In this way, Raz defines *legal completeness* and *legal gap* as: “A legal system is legally complete if there is a complete answer to all the legal questions over which the courts have jurisdiction. It contains a legal gap if some legal questions subject to jurisdiction have no complete answer” (Raz, 2009).

In other words, when the question ‘What decision does the law require in this case?’ is met with the answer ‘No decision is required by law’, then there is a legal gap. There are only two possible legal complete answers.

The law conclusively requires that action:

(XV) $LR_c x,\varphi$

The law conclusively permits the omission of that action:

(XVI) $LPER_c x,\neg\varphi$

Conclusive permission can be written as follows:

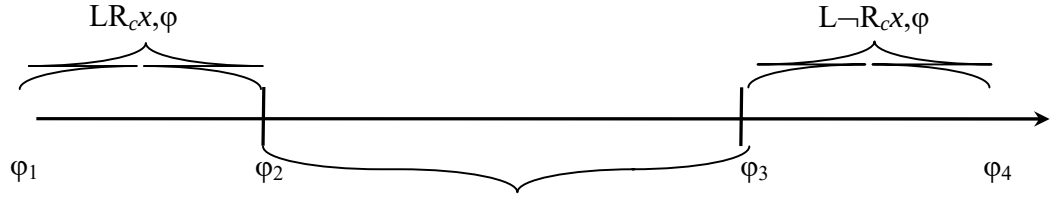
(XVI′) $L\neg R_c x,\varphi$

Therefore, if there are two types of conclusive legal responses then there are two types of gap:

(XVII) $LR_c x,\varphi$ is neither true or false and $L\neg R_c x,\varphi$ is neither true or false

(XVIII) $\neg LR_c x,\varphi \wedge \neg L\neg R_c x,\varphi$ is true

Note that (XVII) exactly reflects Bobbio's insight from a strict perspective of truth, now formalized:



Neither $LR_c x,\varphi$ nor $L\neg R_c x,\varphi$
nor Inclusive General Norm

The gap (XVII) is not exactly a result of a problem with legal language but, rather, with the nature of natural language as a whole. It shows that, in borderline cases, the interpreter of the law simply does not have a legal criterion to determine whether to include conduct in the scope of a conclusive norm that requires φ or a permissive norm that allows ¬φ.

This is particularly problematic in cases where the laws themselves use adjectives or moral terminology to define actions (Raz, 2009). For example, saying that a murder committed with cruelty will have a corresponding increased sentence that will certainly bring the judge to the impossibility of deciding by exclusively legal means. In these cases, it is likely that case law, or the judge's discretion, will determine the solution (of the definition of cruelty), with a low degree of legal certainty.

Raz's advancement over Bobbio is that his formalization allows for a clearer distinction between antinomy and legal gaps. While (XVII) and (XVIII) represent types of legal gaps, the antinomy would be represented as (Raz, 2009):

(XIX) $LR_c x,\varphi \wedge LR_c x,\neg\varphi$

According to Raz, the legal gap presented by (XVIII) would be solved with a closure rule, more precisely, Raz's version of Common Law legalism:

$$(XX)\ \neg LR_c x,\varphi \rightarrow L\neg R_c x,\varphi$$

$$(XX')\ \neg LR_c x,\varphi \rightarrow LPER_c x,\neg\varphi$$

If (XX) is used to replace $\neg LR_c x,\varphi$ in (XVIII), we get:

$$(XXI)\ L\neg R_c x,\varphi \wedge \neg L\neg R_c x,\ \varphi \text{ is true}$$

$$(XXI')\ LPER_c x,\neg\varphi \wedge \neg LPER_c x,\neg\varphi \text{ is true}$$

(XXI) and (XXI′) are contradictions and, because of that, Raz rejects them. Note that these expressions do not represent antinomies, since the problem is not the conflict between contrary or contradictory legal reasons equally valid ; rather, the conflict falls on the existence or not of legal reasons grounded by sources of law.

However, Raz's solution is also unable to resolve the distinction between the legalisms of Common Law and Civil Law. If we used Raz's formalism to describe (I) and (II), we would get:

[1]	$\neg LR_c x,\varphi \rightarrow L\neg R_c x,\varphi$	[(XX)]
[2]	$\neg LR_c x,\varphi \rightarrow LPER_c x,\neg\varphi$	[(XX′)], (Civil Law)
[3]	$\neg LPER_c x,\neg\varphi \rightarrow LR_c x,\varphi$	[2, MT], (Common Law)

[2] could be read as 'That which is not legally conclusively required is legally conclusively permitted not to do', while [3] could be read as 'That which is not legally conclusively permitted not to do is conclusively required to do'. Thus, in Raz's system [2] is equivalent to (I) and [3] is equivalent to (II), and both [2] and [3] are equivalent in Raz's formalization, as [I] and [II] are equivalent in SDL.

For this reason, in the following sections, we will seek to construct a legal logic that is capable of making sense of the two closure rules warranting, in theory, the completeness of legal systems. If it is not clear by now, we will discuss, from now on, just the legal gap in the sense expressed by (XVIII).

6. Logic and Norms

Let φ be an arbitrary sentence. It is taken for granted that not every kind of sentence is entitled to be a legal statement, that is, an information whose content is relevant with respect to the social facts that ground the law (sources). If φ stands for the sentence 'I am watching TV', it seems clear that φ is not entitled to become a legal statement; whereas, if φ is 'My neighbour killed his dog', then the sentential content of φ is relevant and should be of concern for the law. It is also taken for granted that killing an animal is either permitted or forbidden, given the special circumstance at which the fact occurred (the neighbour may have killed in the name of legitimate self-defence because his dog contracted rabies, for example).

Whilst it is not the job of logic to discriminate between ordinary sentences and legal statements, i.e., those sentences that are juridically relevant or are not, the aim of a proper legal logic is to explain what follows from such legal statements.

7. Legal Logic

In order to know whether φ is a legal statement or not, it suffices to see whether φ is made explicit or *stated* or not by an arbitrary source, that is, any source that states φ and thereby makes φ a legal statement. In other words:

Definition 1. A sentence φ is a *legal statement* if, and only if, there is a source (henceforth: S) that states φ as a norm.

Let us call this a *promulgation*, i.e. whenever a source states a sentence in its set of explicit norms. In symbols:

$$S\varphi =_{df} \exists S\ \varphi \in S$$

This also means that a sentence is not a legal statement whenever no source states it. It is said indifferent (Woleński, 2006), such that:

Definition 2. A sentence φ is said *indifferent* if, and only if, there is no source that states φ as a norm.

This means that an indifferent sentence is whatever is not a legal statement. In symbols:

$$I\varphi =_{df} \neg\exists S\ \varphi \in S$$

However, a difference is to be made between two ways of denying the legal status of an arbitrary sentence φ. It may be so because φ does not belong to the set of relevant data that have to do with the sources, as e.g. "I am watching TV". If so, then it is the case that Iφ. It may also be because the sentential content is condemned by a given source, so the sentential content is promulgated by that source. If so, then we have a situation where S¬φ holds. The logical relation between legal statements is such that, for any given source S embedding φ, three main properties may be established about the behavior of logical negation.

Definition 3. For any legal statement Sφ, the negative promulgation of a negative sentence amounts to promulgating an affirmative sentence. In symbols:

$$S\neg(\neg\varphi) \leftrightarrow S\varphi$$

The lack of not promulgating a legal statement amounts to promulgating it. In symbols:

$$\neg(\neg S\varphi) \leftrightarrow S\varphi$$

Promulgating a negative sentence φ entails not promulgating the corresponding affirmative sentence φ. In symbols:

$$S\neg\varphi \rightarrow \neg S\varphi$$

A set of norms can then be defined accordingly, once legal statements are identified by means of the above definition. These norms are: obligation, O, forbiddance, F, and permission, P. An usual logical analysis of these is exemplified by the square of deontic oppositions, as presented by e.g. (Woleński, 2006). Thus, for any legal statement φ:

$O\varphi \leftrightarrow \neg P\neg\varphi$
$F\varphi \leftrightarrow O\neg\varphi$

The normal square of deontic oppositions lists a set of ensuing logical relations between legal statements, accordingly:

(i) $O\varphi \rightarrow P\varphi$
(ii) $O\varphi \rightarrow \neg F\varphi$
(iii) $\neg O\varphi \rightarrow P\neg\varphi$
(iv) $F\varphi \rightarrow P\neg\varphi$
(v) $F\varphi \rightarrow \neg O\varphi$
(vi) $\neg F\varphi \rightarrow P\varphi$
(vii) $P\varphi \rightarrow \neg F\varphi$
(viii) $\neg P\varphi \rightarrow F\varphi$

Now there is a distinction between two kinds of legal systems, namely: *Common Law* and *Civil Law*, that do not validate any of the above theorems (i)-(viii). Common Law is known to validate (vi), but not (8); and Civil Law is known to validate (viii), but not (vi). This means that both kinds of law do not share the same definitions of the aforementioned norms. Moreover, this entails, as demonstrated throughout the present paper, that the normal square (or hexagon) of deontic oppositions is unable to represent the logic of Common Law and Civil Law: the normal square stands for a 'classical' set of norms that are both complete and consistent with respect to one unique given juridical system, S.
We still need to define what a juridical system is, in order to account for Common Law, Civil Law, and non-normal situations such as legal gap. This can be account by the fact that a system is a set of several distinctive laws, and that the latter may be at odds about the range of legal statements.

Definition 3. A *juridical system* S is a finite set of sources $S_1,...,S_n$ establishing logical conditions for assigning the norms P and F to a legal statement φ.

The set of logical relations (i)-(viii) depicts a special kind of juridical system, namely: legally consistent and complete. The logical properties of legal consistency and completeness can be defined in terms of basic legal statements. Thus:

Definition 4. A juridical system S is said S-*consistent* only if, for any legal statement φ, φ cannot be both stated and not stated by any source. In symbols:

$$\vDash_{\mathsf{S}} \varphi \text{ only if } \nvDash_{\mathsf{S}} \neg\varphi$$

Definition 5. A juridical system S is said S-*complete* only if, for any legal statement φ, φ cannot be neither stated nor not stated by any source. In symbols:

$$\nvDash_{\mathsf{S}} \varphi \text{ only if } \vDash_{\mathsf{S}} \neg\varphi$$

The following wants to show that the complete and consistent juridical system is just one among other ones. The way to construct and compare such juridical systems is the aim of the following non-classical juridical logic.

8. Non-Classical Juridical Logic

We are especially interested in a special case of normative drawback: legal *gap*. A legal gap is a situation in which the juridical system is not S-complete, that is:

$$\nvDash_S \varphi \text{ and } \nvDash_S \neg\varphi$$

Whilst such a gappy situation is made impossible by S-consistent and S-complete juridical systems, it can make sense in juridical systems that infringe any of these "classical" properties.

8.1. Common Civil and Civil Law

The central problem is about how to define one and the same norm, i.e., permission. In Common Law, any legal statement that is not forbidden is thereby permitted. But that is not the case in Civil Law, whereby a legal statement is permitted only if the source has promulgated that it is not forbidden. In order to show that asymmetry between both juridical systems, we want to afford a list of distinct definitions of juridical norms that parallel other works about the logic of truth (von Wright , 1986) and epistemic criteria of justification (Schang, 2017).

Here is a set of four juridical systems, S_1-S_4, each being a specific way of dealing logically with a set of sources S. Thus, for any legal statement φ, the conditions of permission and forbiddance for φ in a given juridical system differ as follows:

Definition 7. The juridical system S_1 is such that, for any legal statement φ that belongs to S_1:

- φ is permitted in S_1 if, and only if, either its sentential content *p* is promulgated or its negation ¬*p* is not promulgated. In symbols:
  $\vDash_{S_1} P\varphi$ iff $\vDash_{S_1} Sp$ or $\vDash_{S_1} \neg S\neg p$
- φ is forbidden in S_1 if, and only, both its sentential content *p* is not promulgated and its negation ¬*p* is promulgated. In symbols:
  $\vDash_{S_1} F\varphi$ iff $\vDash_{S_1} S\neg p$ and $\vDash_{S_1} \neg Sp$

Definition 8. The juridical system S_2 is such that, for any legal statement φ that belongs to S_2:

- φ is permitted in S_2 if, and only, both its sentential content *p* is promulgated and its negation ¬*p* is not promulgated. In symbols: $\vDash_{S_2} P\varphi$ iff $\vDash_{S_2} Sp$ and $\vDash_{S_2} \neg S\neg p$
- φ is forbidden in S_2 if, and only, either its sentential content *p* is not promulgated or its negation ¬*p* is promulgated. In symbols: $\vDash_{S_2} F\varphi$ iff $\vDash_{S_2} S\neg p$ or $\vDash_{S_2} \neg Sp$

Definition 9. The juridical system S_3 is such that, for any legal statement φ that belongs to S_3:

- φ is permitted in S_3 if, and only, both its sentential content *p* is promulgated and its negation ¬*p* is not promulgated. In symbols: $\vDash_{S_3} P\varphi$ iff $\vDash_{S_3} Sp$ and $\vDash_{S_3} \neg S\neg p$
- φ is forbidden in S_3 if, and only, both its sentential content *p* is not promulgated and its negation ¬*p* is promulgated. In symbols: $\vDash_{S_3} F\varphi$ iff $\vDash_{S_3} S\neg p$ or $\vDash_{S_3} \neg Sp$

Definition 10. The juridical system S_4 is such that, for any legal statement φ that belongs to S_4:

- φ is permitted in S_4 if, and only, either its sentential content *p* is promulgated or its negation ¬*p* is not promulgated. In symbols: $\vDash_{S_4} P\varphi$ iff $\vDash_{S_4} Sp$ or $\vDash_{S_4} \neg S\neg p$
- φ is forbidden in S_4 if, and only, either its sentential content *p* is not promulgated or its negation ¬*p* is promulgated. In symbols: $\vDash_{S_4} F\varphi$ iff $\vDash_{S_4} S\neg p$ or $\vDash_{S_4} \neg Sp$

Now it can be easily proved that none of the juridical systems S_1-S_4 validates only one of the characteristic formulas of Common Law or Civil Law, however: both are valid in S_1, S_2 and S_4, whereas none is valid in S_3. The latter is an incomplete or 'gappy' juridical system, in accordance with our expected case of legal gap. But, we still want juridical systems providing with different criteria for permission and forbiddance. A characteristic semantics for Common Civil and Civil Law is still in order, accordingly.

8.2. Semantics for Common Law and Civil Law

Definition 11. Each legal statement φ is interpreted as an ordered pair of states about whether φ or its negation ¬φ is promulgated, such that its characteristic value is of the form:

$$v(\varphi) = \langle S\varphi, S\neg\varphi \rangle$$

Definition 12. Juridical bivalence: Any legal statement or its negation is to be promulgated, and no legal statement can be promulgated and not be promulgated at once. That is, for any φ:

Either Sφ or S¬φ
Not Sφ and S¬φ

Φ	¬φ
11	11
10	01
01	10
00	00

Juridical system of Common Law: S_{CoL}

- φ is permitted in S_{CoL} if, and only if, either its sentential content φ is promulgated or its negation ¬φ is not promulgated. In symbols:
  $\vDash_{S_{CoL}} P\varphi$ iff $\vDash_{S_{CoL}} S\varphi$ or $\vDash_{S_{CoL}} \neg S\neg\varphi$
- φ is not permitted in S_{CoL} if, and only if, either its sentential content φ is not promulgated or its negation ¬φ is promulgated. In symbols:
  $\vDash_{S_{CoL}} \neg P\varphi$ iff $\vDash_{S_{CoL}} \neg S\varphi$ or $\vDash_{S_{CoL}} S\neg\varphi$
- φ is forbidden in S_{CoL} if, and only if, its sentential content φ is not promulgated and its negation ¬φ is promulgated. In symbols:
  $\vDash_{S_{CoL}} F\varphi$ iff $\vDash_{S_{CoL}} \neg S\varphi$ and $\vDash_{S_{CoL}} S\neg\varphi$
- φ is not forbidden in S_{CoL} if, and only if, its sentential content φ is promulgated and its negation ¬φ is not promulgated. In symbols:
  $\vDash_{S_{CoL}} \neg F\varphi$ iff $\vDash_{S_{CoL}} S\varphi$ and $\vDash_{S_{CoL}} \neg S\neg\varphi$

φ	Pφ	¬Pφ	Fφ	¬Fφ
11	1	1	0	0
10	1	0	0	1
01	0	1	1	0
00	1	1	0	0

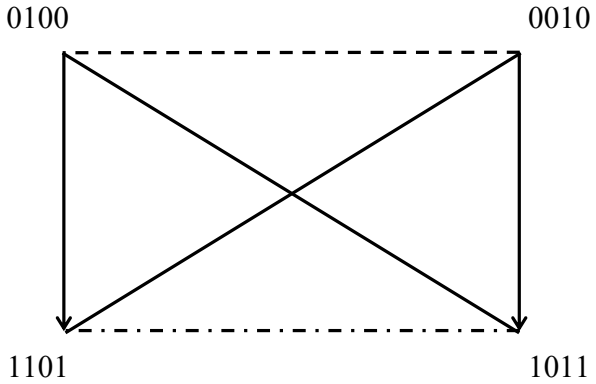
Juridical system of Civil Law: S_{CiL}

- φ is permitted in $\mathsf{S}_{\mathrm{CiL}}$ if, and only if, either its sentential content φ is promulgated and its negation ¬φ is not promulgated. In symbols:
  $\vDash_{\mathrm{SCiL}} P\varphi$ iff $\vDash_{\mathrm{SCiL}} S\varphi$ or $\vDash_{\mathrm{SCiL}} \neg S\neg\varphi$
- φ is not permitted in $\mathsf{S}_{\mathrm{CiL}}$ if, and only if, either its sentential content φ is not promulgated and its negation ¬φ is promulgated. In symbols:
  $\vDash_{\mathrm{SCiL}} \neg P\varphi$ iff $\vDash_{\mathrm{SCiL}} \neg S\varphi$ or $\vDash_{\mathrm{SCiL}} S\neg\varphi$
- φ is forbidden in $\mathsf{S}_{\mathrm{CiL}}$ if, and only if, its sentential content φ is not promulgated or its negation ¬φ is promulgated. In symbols:
  $\vDash_{\mathrm{SCiL}} F\varphi$ iff $\vDash_{\mathrm{SCiL}} \neg S\varphi$ and $\vDash_{\mathrm{SCiL}} S\neg\varphi$
- φ is not forbidden in $\mathsf{S}_{\mathrm{CiL}}$ if, and only if, its sentential content φ is promulgated or its negation ¬φ is not promulgated. In symbols:
  $\vDash_{\mathrm{SCiL}} \neg F\varphi$ iff $\vDash_{\mathrm{SCiL}} S\varphi$ and $\vDash_{\mathrm{SCiL}} \neg S\neg\varphi$

φ	Pφ	¬Pφ	Fφ	¬Fφ
11	0	0	1	1
10	1	0	0	1
01	0	1	1	0
00	0	0	1	1

9. A Non-Classical Square of Legal Oppositions

From the aforementioned study, we can represent the relationships of $\mathsf{S}_{\mathrm{CoL}}$ and $\mathsf{S}_{\mathrm{CiL}}$ in a non-classical square of legal oppositions that includes the legal statements and their logical relationships.



Description of the nodes in the non-classical square of legal oppositions

0100: $P_{\mathrm{SCiL}}\varphi$, $\neg P_{\mathrm{SCiL}}\neg\varphi$, $F_{\mathrm{SCoL}}\neg\varphi$, $\neg F_{\mathrm{SCoL}}\varphi$
1101: $P_{\mathrm{SCoL}}\varphi$, $\neg P_{\mathrm{SCoL}}\neg\varphi$, $F_{\mathrm{SCiL}}\neg\varphi$, $\neg F_{\mathrm{SCiL}}\varphi$
0010: $P_{\mathrm{SCiL}}\neg\varphi$, $\neg P_{\mathrm{SCiL}}\varphi$, $F_{\mathrm{SCoL}}\varphi$, $\neg F_{\mathrm{SCoL}}\neg\varphi$
1011: $P_{\mathrm{SCoL}}\neg\varphi$, $\neg P_{\mathrm{SCoL}}\varphi$, $F_{\mathrm{SCiL}}\varphi$, $\neg F_{\mathrm{SCiL}}\neg\varphi$

The 'non-classical' import of the above square is obviously due to the previous definitions of permission and forbiddance. More precisely, it is related to the non-dual relationship between the

corresponding operators P and F. It has been recalled in the following tables that forbiddance does not amount to the lack of permission in Common Law and Civil Law: non-permission implies forbiddance in the latter, whereas non-forbiddance implies permission in the former; but the converse never holds in both juridical systems, unlike the square (and its hexagonal extension) of deontic oppositions wherein norms behave like normal modal operators.

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Notes

1. The author does not use the term “normative proposition”, but the term “norm”, not discussing the distinction between them, i.e., the descriptive and prescriptive senses of norms (von Wright, 1991).
2. From this point onward, we will call British legalism ‘Common Law‘ and German legalism ‘Civil Law.’
3. For Raz, sources are basically the origin of the existence and content of Law, and any legitimate legal decision refers to these objective social facts (the sources) and not to moral considerations.

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Neural Networks in Legal Theory

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Abstract:
This article explores the domain of legal analysis and its methodologies, emphasising the significance of generalisation in legal systems. It discusses the process of generalisation in relation to legal concepts and the development of ideal concepts that form the foundation of law. The article examines the role of logical induction and its similarities with semantic generalisation, highlighting their importance in legal decision-making. It also critiques the formal-deductive approach in legal practice and advocates for more adaptable models, incorporating fuzzy logic, non-monotonic defeasible reasoning, and artificial intelligence. The potential application of neural networks, specifically deep learning algorithms, in legal theory is also discussed**.** The article discusses how neural networks encode legal knowledge in their synaptic connections, while the syllogistic model condenses legal information into axioms. The article also highlights how neural networks assimilate novel experiences and exhibit evolutionary progression, unlike the deductive model of law. Additionally, the article examines the historical and theoretical foundations of jurisprudence that align with the basic principles of neural networks. It delves into the statistical analysis of legal phenomena and theories that view legal development as an evolutionary process. The article then explores Friedrich Hayek's theory of law as an autonomous self-organising system and its compatibility with neural network models. It concludes by discussing the implications of Hayek's theory on the role of a lawyer and the precision of neural networks.
Keywords: legal analysis, generalisation, legal concepts, logical induction, semantic generalisation, formal-deductive approach, fuzzy logic, non-monotonic defeasible reasoning, artificial intelligence, machine learning, neural networks, deep learning algorithms, legal theory**.**

1. Introduction: Problems and Limitations of Formal, Logical and Mathematical Methods in Legal Analysis

The domain of legal analysis encompasses the generation, scrutiny, and application of law in relation to particular cases. Thus, legal analysis entails employing diverse analytical methodologies, such as categorising factual information into specific legal frameworks, employing legal reasoning,

and engaging in the decision-making process. These methodologies are fundamental to the systematic study of law and, furthermore, hold great significance from the vantage points of artificial intelligence and machine learning. The utilisation of formal methods not only facilitates the processes of reasoning and decision-making, but also allows for the conceptualisation of legal analysis as a comprehensive *holistic* undertaking. Indeed, in essence, the analysis of legal systems encounters similar fundamental inquiries. These inquiries pertain to the collection, interpretation, retention, and identification of patterns, and the response to incoming, legally relevant information. Each of these questions holds significant importance and is inherently interconnected with the others.

Every interaction with the domain of legal regulations or the realm of actual juridical relations results in the accumulation of experiential legal knowledge. If discernible regularities exist within this wealth of real legal experience, then they can be identified, scrutinised and subsequently utilised. The identification of such regularities implies the presence of "shared recursive patterns in legal forms and judicial opinions, which enables the use of process and technology to routinize and scale very cheap and very high quality solutions to the myriad of legal needs" (Henderson, 2013, p. 479). Consequently, the isolation of these shared entities is commonly referred to as *generalisation*. Generalisation represents a pivotal task across all disciplines associated with data analysis (mathematical statistics, machine learning, etc.). Naturally, the human brain does not remain exempt from this cognitive pursuit, as evidenced by our occasional observations of its adeptness in generalisation.

The classical position holds that legal norms are written in reference to those cognitively shared patterns (shared, uniform and inflexible properties with clear distinct boundaries) which define *juridical categories*. According to Lakoff:

> The classical view that categories are based on shared properties is not entirely wrong [w]e often do categorise things on that basis[…] it has become clear that categorisation is far more complex than that […] new theory of categorisation, called prototype theory, has emerged. [..] shows that human categorisation is based on principles that extend far beyond those envisioned in the classical theory (Lakoff, 1987, p. 5).

More recent studies in the field of cognitive science and brain neurophysiology have demonstrated that categorisation, the study of generalisable representations, is a type of decision making and that categorisation learning research would benefit from approaches developed to study the neuroscience of decision making and generalisation (Seger & Peterson 2013). The multitude of approaches to generalisation implies that the generalisation procedure lacks a universal framework. Despite the ubiquity of generalisation, the task itself, when considered in its broadest form, remains somewhat ambiguous. The formulation of the generalisation problem can vary extensively, depending on the specific context in which it is required. Different problem formulations engender diverse and sometimes disparate methods of solution.

Legal analysis encompasses the examination of semantic constructions of specific legal concepts, wherein ideas are expressed and documented through natural language expressions in legal texts.[1] The philosophical-semantic approach to generalisation can be outlined as follows: when there are interconnected concepts sharing a common generic attribute, it becomes necessary to transition towards a new concept that offers a broader, albeit less specific, interpretation by eliminating the generic attribute.

Philosophy is concerned with the examination and interpretation of semantic constructions. To illustrate this point, we can consider the paradigmatic instance of a rule, namely, "no vehicles in the park." Here, the term "a vehicle" can be defined as a mechanical device typically equipped with wheels and an engine, utilised for the conveyance of individuals or goods, particularly on land. As colloquially, "a vehicle" is understood to refer specifically to an automobile, the underlying core meaning of this rule can be construed as "no automobiles in the park." However, if we eliminate the

specific attributes of “wheels” and “an engine,” we arrive at the more generalised concept of “a vehicle,” encompassing any machine employed for transportation purposes. In the example, the very term “a vehicle” itself indicates the process of generalisation. By simply discarding the extraneous terms, we attain a broader and consequently more ambiguous “penumbral” concept, thereby enabling the application of the rule to bicycles or chariots (Hart, 1958).

The process of generalisation yields the development of legal concepts that are employed in constructing further descriptions. Different viewpoints exist regarding the fundamental principle governing the identification of specific concepts. All the items enumerated in this inventory are directly relevant to this matter. During the process of generalisation, we acquire concepts that encompass numerous phenomena encountered in some manner beforehand. By isolating commonalities among these phenomena, we are able to describe the properties of ideal concepts, which are detached from the specific details of individual occurrences. Through the process of generalisation, it becomes possible to convey the outcome of such generalisation using a systematic framework of concepts. In this scenario, the generalised concepts are not merely a collection of unrelated elements, but rather assume an inherent structure composed of interrelationships.

It is these ideal concepts that form the foundation of law. All legal concepts, like “liability”, “contract”, “tort”, “crime”, etc., are idealisations of objects derived from our everyday experiences. Law introduces a formal system of rules for these concepts, enabling the construction, interpretation, and application of these rules. However, while these concepts are primary for law itself, they are connected to human experiences of their application. Consequently, legal scholars can employ a more targeted legal inquiry based on the experiential background associated with the ideal concepts, using logical devices such as induction.

Logical induction involves deriving general laws from a collection of specific cases. In the case of complete induction, set A comprises the elements $A_1, A_2, A_3, ..., A_n$.. If A_1 possesses attribute B and A_2 possesses attribute B, then all elements from A_3 to A_n also possess attribute B. Consequently, all elements of set A possess attribute B. In a case of incomplete induction, set A comprises elements $A_1, A_2, A_3, ..., A_n$. If A_1 possesses attribute B and A_2 possesses attribute B, then all elements from A_3 to A_k also possess attribute B. Consequently, it is likely that A_{k+1} and the remaining elements of set A possess attribute B (incomplete induction pertains to probability and can be fallible). Induction addresses generalisation in two ways. First, when referring to a set of objects, it implies that something has previously served as the basis for combining these objects into a unified set. In other words, a mechanism has been identified that facilitated the preceding generalisation. Second, through induction, if we discover a characteristic peculiar to the elements of a particular group that describes a specific concept, we can employ this characteristic as a criterion for categorising it within that group. Logical induction shares several similarities with the semantic generalisation of concepts. However, the semantic approach places a slightly different emphasis, focusing on the features comprising the description of a concept and the possibility of discarding certain features to obtain a more general formulation. Nonetheless, the question remains open, regarding the source of such concept definitions that enable the process of generalisation through the act of discarding. Incomplete logical induction elucidates the way that descriptive features are formed.

On other hand, it is obvious that the logical (either formal-deductive or inductive) approach to legal decision-making comes close to what Perelman and Olbrechts-Tyteca describe as:

> a strategy which may be called logical, is that in which the primary concern is to resolve beforehand all the difficulties and problems which can arise in the most varied situations, which one tries to imagine, by applying the rules, laws, and norms one is accepting [...] The logical approach assumes that one can clarify sufficiently the ideas one uses, make sufficiently clear the rules one invokes, so that practical problems can be resolved without difficulty by the simple process of deduction. This implies moreover that the unforeseen has been eliminated, that the future has been mastered, that all

problems have become technically soluble (Perelman & Olbrechts-Tyteca, 1971, pp. 197-198).

In the initial stages of formalist advancements, a notable inclination toward strong symbolism emerged, promoting the acquisition of legal knowledge through the quasi-algebraic manipulation of symbols. These symbols served as representations that conveyed precise legal meanings or events, enabling the deduction of rules based on their manipulation. Within this context, reasoning was understood to be the systematic manipulation of existing legal knowledge, employing algebraic techniques, with the aim of deciding whether established legal knowledge (a norm or a norm's interpretation) applies in a particular case. Such manipulation encompasses exploring an algebraic space encompassing various potential solutions.

This approach has proved to be a significant source of inspiration for the development of legal applications, encompassing traditional computer programs (e.g. payroll systems or social security payments) as well as legal expert systems (e.g. databases of legal norms). The operation of such expert systems draws predictable conclusions from a predetermined initial set of norms, concepts, and facts, predominantly according to the so-called 'syllogistic model of adjudication'. There is much to be said for this approach in a number of legal contexts. It is an obvious, and even recommended, choice when standardisation and efficiency are paramount, when individual cases do not merit special adaptation or when the relevant legal rules can be easily defined, formalised, and updated. Even theoretical issues that can be addressed within the deductive model of law (such as the formalisation of *deontic* and *normative* conditions) should not be underestimated.

Nevertheless, this formal-deductive approach fails to capture a number of central, socially significant, and theoretically interesting phenomena of legal practice, which include the ability of jurisprudence to use insufficient or contradictory information, draw analogies, learn from examples and experiences in applying vague and imprecise rules, etc. From a perspective of classical legal positivism, it is usually asserted that empirical statements, concepts or terms within norms are not inherently vague but, rather, exhibit open-textured characteristics. Open-textured concepts of language refer to those concepts whose extension are not predetermined for all instances before their application, and yet they possess a certain procedure (that is of judicial decision-making) for determining their applicability in specific cases and extending those concepts to 'undecided cases' outside their standard domain of application (Hart, 1994, pp. 123, 128-136). Within the realm of law, this procedure is known as judicial decision-making, through which the courts render a ruling on a case.

Henceforth, a multitude of legal theorists have advocated for the adoption of a more adaptable formal model, underpinned by fuzzy logic and non-monotonic defeasible reasoning (Hage, 2005). In addition, another faction of scholars has pushed the boundaries further by augmenting this model with the distinct characteristics of artificial intelligence and machine learning.[2] These learning mechanisms enable the legal practice to serve as an effective problem-solving process in contentious cases, wherein the application of legal rules paradoxically necessitates the alteration and transformation of the rules themselves. The following section aims to demonstrate the potential utilisation of neural networks, a prominent deep learning algorithm, in addressing analytical tasks within the realm of legal theory.

2. Neural Networks: Formal Description

Neural networks can be described as neurally inspired computational tools for modelling neurological and cognitive processes. The capacity for an artificial neural network to effectively process and generalise information from previously unseen data is commonly referred to as generalisation. In Frank Rosenblatt's formulation, the concept of pure generalisation encompasses the following scenario: "In a learning experiment, a perceptron is typically exposed to a sequence of patterns containing representatives of each type or class which is to be distinguished, and the

appropriate choice of response is ‘reinforced’ according to some rule for memory modification. The perceptron is then presented with a test stimulus, and the probability of giving the appropriate response for the class of the stimulus is ascertained. Different results will be obtained, depending on whether or not the test stimulus is chosen to correspond identically to one of the patterns which were used in the training sequence. If the test stimulus is not identical to any of the training stimuli, the experiment is not testing ‘pure discrimination’, but involves generalisation as well. If the test stimulus activates a set of sensory elements which are entirely distinct from those which were activated in previous exposures to stimuli of the same class, the experiment is a test of ‘pure generalisation’. The simplest of perceptrons, which will be considered initially, “have no capability for pure generalisation, but can be shown to perform quite respectably in discrimination experiments, particularly if the test stimulus is nearly identical to one of the patterns previously experienced” (Rosenblatt, 1962, p. 68).

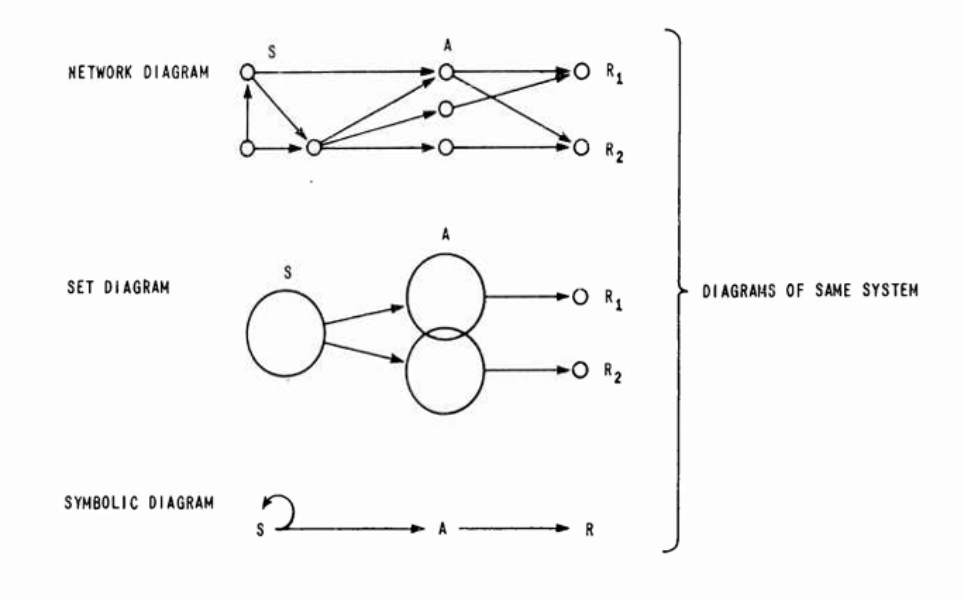

Figure 1. Perceptron diagrams. Source: Rosenblatt, F. *Principles of Neurodynamics: Perceptrons and the Theory of Brain Mechanisms*. Washington Spartan Books (Rosenblatt, 1962, p. 86).

A neural network can be described as a collection, characterised by a specific arrangement, of interconnected neurons (Haykin, 2006, p. 32). In this context, neurons are regarded as individual entities responsible for the reception and transmission of information. In isolation, neurons do not possess significant individual significance; their relevance lies solely within the interconnected network they form. Upon receiving incoming signals, a neuron assigns a specific weight to each of them. Subsequently, the signal is multiplied by its corresponding weight, the resulting values are aggregated, and a singular numerical value is generated. This resultant value is then passed on to the activation function, which determines whether the signal should be propagated further along the neural pathway (Haykin, 2006, pp. 42-44).

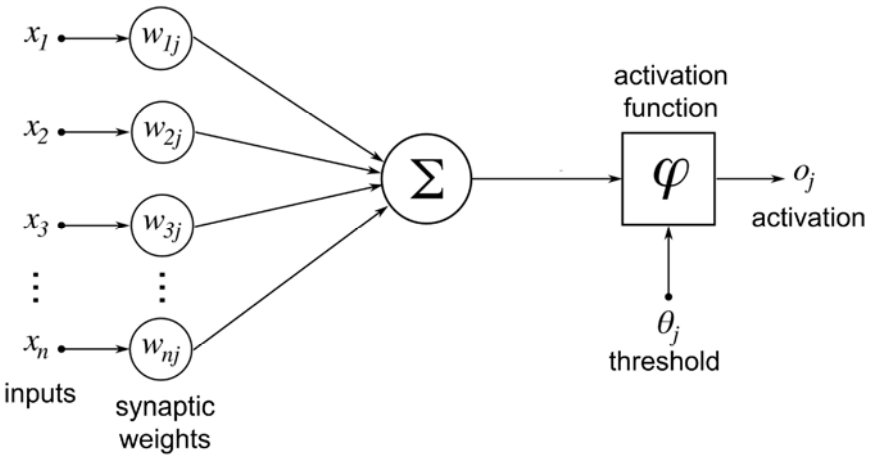

Figure 2. Diagram of an artificial neuron with n inputs and their corresponding synaptic weights. All weighted inputs are added and an activation function controls the generation of the output signal. Source: (Camuñas-Mesa et al., 2019).

An elementary neural network comprises three layers and facilitates unidirectional data transmission. The network encompasses input neurons, a concealed intermediary layer of neurons that remains imperceptible to external observation, and an output neuron.

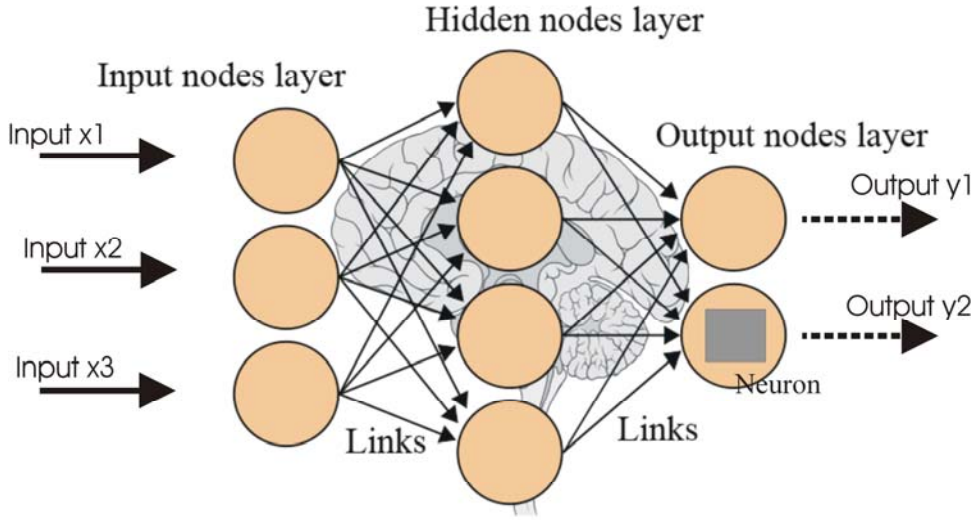

Figure 3. The architecture under consideration is a fully-connected direct propagation neural network featuring a single hidden layer and a single output layer. Source: (EE 260, 2020).

Henceforth, it becomes evident that a neural network can be regarded as a form of mathematical function, essentially operating as a program. Instead of explicit programming, the neural network necessitates a process known as 'training' or adjustment. The process of training a neural network appears to be straightforward: by presenting a set of well-understood examples, we modify the coefficients of the underlying mathematical function, constituting the neural network, in adherence to specific rules, contingent upon the network's responses, whether they are deemed to be correct or incorrect.

3. Neural Networks and Legal Theory

Neural networks inherently challenge fundamental principles of the syllogistic model, a widely employed formal approach in jurisprudence. This contradiction arises due to several key distinctions between these two paradigms:

1) In contrast to the syllogistic model, which condenses legal information into a collection of axioms, neural networks encode legal knowledge within the synaptic connections of their computational units.
2) The implementation of neural networks relies on reactive dispositions, whereas the syllogistic model operates on the basis of the logical validity of conclusions.
3) Neural networks possess the capability to assimilate novel experiences, whereas the deductive model of law can only be altered by the inclusion of overarching axioms.
4) Unlike intermittent updates commonly employed by the syllogistic model, neural networks exhibit evolutionary progression, enabling gradual refinement over time.

Obviously, these contradictions affect the two main trends in theoretical jurisprudence, namely natural law and legal positivism, which are united by the idea of law as a system of axioms (although with disagreements about the source and content of these axioms). Meanwhile, turning to the history of jurisprudence, we can find theories that are very close to the basic ideas underlying neural networks.

Statistical analysis of legal phenomena, developed within the framework of sociological and realistic approaches to the study of law, is the earliest and most obvious precedent for the use of ideas underlying the modern use of neural networks. Nevertheless, ideas close to the concept of neural networks are also found in other areas of legal research. In particular, we can recall a number of theories that, based on customary law, consider legal development as an evolutionary process, i.e. as a process of the selection and development of individual normative provisions, e.g. the historical school of Friedrich Carl von Savigny and Georg Friedrich Puchta in Germany, and the social philosophy of David Hume and Adam Smith in Scotland.

More recently, this approach to law has found its fullest formulation in the legal theory of the great economist Friedrich Hayek. First, Hayek offers us an unusual conception of the rule, which contrasts with the usual assumption that the rule is a linguistic entity, i.e. a statement or proposition. He views a rule as a special kind of disposition that "causes an organism to respond to stimuli of a certain class... with a response of a certain kind" (Hayek, 1977, p. 40). The imposition of numerous regulations (dispositions) upon a specific situation governs both our cognitive and practical behaviour. In order for these regulations to structure our experiences, it is not imperative that we possess a conscious awareness of them. Our unconscious cognitive tendencies are even more overarching and conceptual than our linguistic expressions. In fact, our language frequently proves inadequate in conveying the full extent of the mind's capacity for considering the nature of required actions, and we often struggle to articulate, in words, what we inherently understand through practical knowledge. The intricate rules that govern our behaviour can only be acquired through emulation, whereby individuals learn to act in accordance with the same principles by imitating specific actions, although they can never assert those principles themselves.

Furthermore, this rule-based perspective of the human mind, as espoused by Hayek, is applicable to our sense of justice. Our ability to perceive the actions of others as meaningful and to evaluate our own or others' actions as just or unjust must be grounded in the possession of highly abstract rules governing our behaviour, even if we remain oblivious to their existence and lack the means to articulate them verbally. The practical duty of a judge extends beyond adhering to these rules (thus safeguarding the expectations derived from them) to verbalising them in a manner accessible to the general public. This is a formidable task akin to the challenge of formulating scientific laws.

Lastly, Hayek presents us with an evolutionary model of the development of these rules (dispositions) in which they originate "from human action but not from human design." According to Hayek, fundamental (and exceedingly complex) moral and legal principles do not arise from deliberate human choices but, rather, emerge unpredictably and spontaneously through social and cultural evolution (Hayek, 1976, p. 165). Evolution leads to the spread of behavioural dispositions that are best adapted through the persistence, expansion, and imitation of those groups that adopt them. Thus, the 'best-adapted' rules situation does not require anyone to know the reasons for the

success of those rules but depends only on the 'greater success' of the social order based on them. These reasons are implicitly included in the system of rules handed down by tradition, even when they are beyond the comprehension of individuals.

This leads us to Hayek's central thesis, which posits that social order emerges as a result of an autonomous process of self-organisation governed by selective evolution. Consequently, the scope for legislator-initiated reforms within the realm of social order is relatively limited. While Hayek acknowledges the need for occasional improvements in established rules, he contends that the human mind is only capable of immanent criticism, which represents a constrained and partial attempt to enhance the internal coherence of the existing order. In other words, this form of criticism evaluates specific rules within a particular system by considering their consistency and compatibility with other recognised rules that shape a specific order of action. Since any established system of rules of conduct is founded on a partially understood body of experiential knowledge, and serves to guide actions in a manner that is only partially comprehended, it is impractical to aspire to its improvement (Hayek, 1976, p. 165). In this context, Hayek posits that the concept of consistency should not be misconstrued as logical consistency and it is unnecessary to reframe existing rules into a coherent set of axioms. Rather, achieving such consistency can be better accomplished by assigning priority to conflicting rules and establishing criteria for resolving and eliminating conflicts.

To encapsulate the main theses of Hayek's theory in relation to the fundamental components of the neural network model, it is worth noting that Hayek's proposition that norms are derived from "learning by example" aligns with the learning strategy commonly observed in neural networks. Moreover, the notion of the legal system as an autonomous self-organising system corresponds to the phenomenon of self-organisation exhibited by neural networks. Lastly, the task of a lawyer, as Hayek sees it (the arrangement and harmonisation of normative material), corresponds to the process of enhancing the precision of neural networks by generating and refining a precise formal representation of input data.

4. The Problem and the Critique

It is imperative to enumerate several foundational, unresolved inquiries in legal theory, which manifest as technological predicaments in the realm of neural networks. **The primary concern** pertains to the correlation between the 'context of discovery' and the 'context of justification', i.e. the manner in which a legal solution is attained and subsequently substantiated through appropriate reasoning. In neural networks, solutions are not derived through the formulation of reasons but through the application of analogies to previous instances of successful problem-solving (the notion of neural reasoning as an analogy is challenged in Dan Hunter's (1994) article). This conveys the predicament of whether and how such solutions ought to be justified. Should some form of justification for the outcomes produced by neural networks not exist? Should it be a retrospective opportunistic rationalisation? (This notion finds support in the works of Andrew Stranieri and John Zeleznikow (2005), and Mark Gawler and Bryn Lewis (Stranieri et al., 1999), who acknowledge the realist perspective of perceiving decision motivation as a form of rationalisation.) If there is a genuine attempt to derive a coherent set of principles from the problem-solving patterns of the network, what are the principles that can influence problem-solving behaviour and even modify its 'unconscious' patterns? How does legal reasoning integrate symbolic and sub-symbolic processes and how can the deliberate utilisation of refined conceptual structures interact with the unconscious activation of parallel connections?

In 1994, Dan Hunter emphasised that one of the primary challenges with neural networks is their inability to comprehend the rationales underlying their decisions (Hunter 1994). Consequently, in the event of conflicts, the system will be incapable of providing a logical justification for its conclusions. Although it is presently feasible to partially comprehend certain conclusions through

reverse engineering and the algorithmic reporting process, understanding the mechanisms behind machine-generated results necessitates the development of diverse approaches.

Neural networks possess an inherent proficiency in pattern classification, rendering them seemingly promising candidates for emulating analogical reasoning processes. Hobson and Slee explored the utilisation of artificial neural networks to emulate this facet of reasoning in their work. They constructed a neural network 'index' of the 1968 Theft Act (England) (Hobson & Slee, 1994). In this index, the researchers analysed the factual circumstances to determine the presence or absence of various concepts, as defined by the wording of the Theft Act. The presence or absence of each concept was represented in the form of a state matrix, which subsequently served as input for their neural network. The ultimate verdict on whether a given situation constituted theft within the confines of the Act was employed as the desired outcome for the neural network. Based on this material, Hobson and Slee argued that a neural network could be trained to classify cases falling under the purview of the law. During the training process, the neural network autonomously groups the cases utilised for training into shared categories. Following training, new cases can be presented to the neural network. The network then classifies the cases into the general groups established during training to reach a verdict. Through this classification process, the neural network seemingly emulates similar reasoning, as comparable cases yield congruent verdicts.

The second theoretical inquiry pertains to the acquisition of legal knowledge. It has been observed that neural networks employ an example-based learning paradigm, rather than one based on legal theory. This approach to legal learning is not novel to legal professionals, as cases have always played a pivotal role in the instruction of common law. Even lawyers practicing within continental legal systems are progressively recognising the significance of case precedents (as some have historically done, exemplified by Friedrich Carl von Savigny's recognition that mastery of Roman casuistry is essential for comprehending law). A fundamental question arises: must the selected learning patterns be pristine prototypes? Easily discoverable learning patterns that address specific problems can be found in textbooks. However, an alternative approach involves utilising all available court cases to train the neural network. Nevertheless, this approach carries the risk of incorporating examples shaped by secondary problems that may 'contaminate' the main problem.

An experiment conducted by Filipe Borges, Raoul Borges and Daniele Bourcier (2003) sought to model certain aspects of the French penal code using a neural network. A cursory examination of the hidden neurons' activity revealed the emergence of distinct 'tendencies' or preferences in data processing. Notably, some neurons focused specifically on 'murders', while others focused on 'sexual offences', and others displayed either stoical or hyperactive behaviours. Despite such a latent specialisation of individual neurons, the overall decisions made by the neural network remained entirely pertinent. The development of specialised functions within the hidden neurons suggested the possibility of refining the representation of the legal dispute model within the neural network.

The **third theoretical concern** pertains to the concept of 'self-organisation', which holds significance, not only in the realm of neural networks, but also in diverse disciplines, such as biology, systems theory, economics, and computer science. Within the field of contemporary sociology of law, this concept is frequently invoked to construct abstract and comprehensive frameworks for understanding the development of legal systems, drawing upon ideas of evolution and self-organisation (as exemplified by Luhmann's legal sociology). However, legal research has thus far encountered challenges in offering precise and, potentially, controlled explanations of specific facets of law based on self-organisation theories.

An argument has been posited suggesting that neural networks are incapable of modelling the process of legal decision-making due to their inability to apply norms. However, this claim is subject to debate. If the assertion implies that neural networks are unable to apply norms because of their normative content, it is inaccurate. If norms can be expressed in the form of cases or rules, a neural network can be employed to model them. In this context, the normative content within these cases or rules becomes irrelevant. In fact, the fundamental functioning of neural networks can be

viewed as the application of a norm, mandating that similar cases be resolved in a consistent manner. If the implication is that norms cannot be expressed in terms of cases but necessitate representation as rules, it is still premature to assume that neural networks cannot model legal decisions. There remains a possibility that neural networks can be utilised to model norms.

Moreover, the contention that neural networks are unable to apply norms because they lack normative content within the neural network itself is also a matter of debate. This question is intertwined with the broader inquiry of whether neural networks and computers can engage in thinking, which, though beyond the scope of this article, remains an open question. Nonetheless, it may be valid to assert that the outcomes generated by a neural network cannot determine value-based decisions. Simultaneously, when examining the classification of legal systems into continental and Anglo-Saxon (which constitute the two fundamental models today), the potential for employing neural networks in law enforcement practice can be assessed as follows.

In the continental legal system, the approach is apparent. Since the continental criminal process can be formally categorised into pre-trial and trial proceedings, the utilisation of a neural network to make decisions regarding the termination of criminal proceedings or sentencing presents certain challenges. This is due to the fact that the entire process is characterised by police (investigative) procedures and the introduction of neural network-derived sources as evidentiary materials may hinder the adherence to cornerstone principles. The Anglo-Saxon legal system is not entirely straightforward either. On one hand, it is a system grounded in fundamental principles concerning the safeguarding of human rights (e.g. the Human Rights Act of 1998), which precludes the possibility of employing a neural network. But, on the other hand, it is a precedent-based system that, from the perspective of the Borges theory mentioned earlier, can accommodate the utilisation of various electronic systems in practical applications. For instance, one could consider a form of collective program capable of incorporating precedent norms (thus serving as a custodian of legal information) that could be employed by demonstrating competence in reactive dispositions and leaving room for evolutionary development. Overall, these considerations shed light on the potential applications of neural networks in legal contexts, taking into account the distinctive features and requirements of different legal systems.

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Notes

1.This semantic construction of concept should not be confused with statutory construction, i.e. the activity of translating the semantics of a legal text into legal rules.

2. Technically, one can consider machine learning to be a type of artificial intelligence, working by identifying patterns in data and then applying a learned model to new data.

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Logic Matters – Gender and Diversity, Too

Abstract:
This interview features Andrea Reichenberger. Currently she holds a substitute professorship for history of technology at TUM Technical University of Munich.

She is junior research group leader at the Department of Mathematics, University of Siegen, Germany, and leads the research project "Rethinking the History of Mathematics and Physics: Women in Focus." Reichenberger has held several postdoctoral positions, e.g., at the Center for the History of Women Philosophers and Scientists (HWPS) at Paderborn University (Germany) and in the DFG research project "Thought Experiment, Metaphor, Model" at the Institute for Philosophy I at the Ruhr University Bochum. Between 2019 and 2021, she was a fellow at the University of Paderborn and principal investigator of the research project "Foundational research in mathematical logic – relativity – quantum physics. Case studies on the integration of women philosophers." Reichenberger has written a book on Émilie du Châtelet (Springer, 2016) and has published many articles in journals, collected editions, and encyclopedias.

Keywords: logic, feminist philosophy, ethics, war, Ukraine.

Andrew Schumann: The shameful euphemism "female logic" denotes unpredictability and a lack of consistency in reasoning. It is erroneous because neither men nor women use logic in everyday life. Logic is not natural. It is not an innate ability but a technique to be mastered only through learning and training. To what extent can we equate gender and logic?

Andrea Reichenberger: Language is a powerful tool we can use or misuse, just like any other technique. It plays a crucial role in how we perceive the world, including ourselves and others. A big philosophical question is whether we can think and communicate without language. By using language, we not only differentiate and categorise; we also discriminate, violate, and hurt. History teaches us how the use of language is interwoven with atrocities, genocide, and war. This applies not least to political euphemisms. One might ask: What does logic have to do with euphemisms such as "female logic"? Professional logic, as learned and practiced at universities today, is a highly specialised field of research and teaching. In this context, the use of the term "female logic" would seem inappropriate and irritating. According to my opinion, combining the adjective "female" with the noun "logic" has a similar effect as combining the adjective "artificial" with the noun "intelligence." Such word combinations invite us to ask the question of what logic is repeatedly anew. What do we mean when we talk about logic, and why does it irritate us when we speak of

female logic? One might reply that logic is unitary, independent of concrete human relations, transcends historical circumstances, and is pure thinking regardless of sex and gender.

The history of logic teaches us that the concept of logic is deeply gendered. One might object to the idea that logic is gender-neutral if we understand logic as the theory and practice of correct reasoning in terms of inferences or arguments. In this sense, the validity of an argument just states that it is not possible that the conclusion is false when the premises are true. This is just a conditional claim; it has nothing to do with the content, the circumstances of the utterance, etc. According to the widespread narrative, the validity of an argument is determined by its structure, not its content. In fact, it is a matter of dispute what "structure," "form," and "content" mean. Although the formalisation of logic achieved a certain clarity and precision, it had its price and limits. Many questions remained open as to whether reasoning can be reduced to a theory of inference or whether reasoning involves rationality. In the 19th century, the logician Christine Ladd-Franklin (1847–1930) protested against the philosophical doctrine that reason was masculine and intuition was feminine. The German philosopher Margherita von Brentano (1922–1995), the first woman to hold the office of vice president at the Freie Universität Berlin, once remarked that statements about the nature of those who are discriminated against are statements about the nature of discrimination. If that is true, and if you are correct that logic is a technique that can be mastered only through learning and training, then it is disputable whether logic in research and teaching practice is gender-neutral and free of bias. From this perspective, the idea that the mind has no sex (which we find in Augustine and the work of the Cartesian Francois Poulain de la Barre) functions as an ideal. It should not be confused with real-world practice.

When one reflects on this topic today, it seems to be forgotten that the distinction between *res cogitans* (mind) and *res extensa* (body) was theologically motivated. For Descartes, God was an eternal, omnipotent, omniscient, and omnipresent mind that could be understood through reasoning, but the theological context disappeared with the Enlightenment. The idea that logic was pure thinking survived, and this doctrine was often used in order to defend the "autonomous nature" of logic. Historical inquiry helps us to reflect critically on clichés and prejudices regarding what logic is and should be. Current philosophical reflections on contemporary mathematical logic have radically transformed such narratives and stereotypes. Logic is practiced and investigated as a social phenomenon in its rich diversity and multimodality.

Now, let's come to the project "Gender & Logic," which I developed with PD Dr Jens Lemanski at FernUniversity in Hagen. The project did not concern what logic is, can, and should be; nor was it about "female logic," whatever that means. First, the project focused on women's contributions to the field of logic in the late 19th and early 20th centuries (e.g., Rózsa Péter's impact on the development of recursion theory, Johanna Piesch's work on switching algebra, and Christine Ladd-Franklin's work on the algebra of logic); secondly, the project aimed to explore the constitution of knowledge in light of the development of cultural techniques, notation systems, and standardisation in the history of logic (e.g., the process of standardisation of logical notations had a price, namely, the exclusion of visually impaired people from learning logic because of a lack of Braille provision).

One lesson we learned: In principle, everyone should have an equal opportunity to learn logic. Doing logic should be open for everyone as a fundamental value and human right. Once again, that is the ideal, not the practice. The value-ladenness of scientific knowledge is also evident in logic and its history.

Andrew Schumann: There have been many outstanding female mathematicians and female physicists. How does the history of mathematics and physics change when we focus on women?

Andrea Reichenberger: History is not the past. History is a story about the past told in the present, and it is supposed to be useful in constructing the future. This also applies to the history of logic, science, and technology, which is constantly being rewritten and re-evaluated in light of current developments. Logic and its history are essential parts of scientific inquiry. In this context, feminist

studies of women and gender play a crucial role. They help to correct standard narratives, they uncover multiple dimensions of gender stereotypes in scientific research, they integrate women's contributions into our picture of logic's rich and diverse history, and, in doing so, they promote gender equality and epistemic justice in current research and teaching practices.

Again, one might object that, let's say, for example, the validity of the Pythagorean theorem $a^2 + b^2 = c^2$ eludes historicity. However, history teaches us that it is a legend that Pythagoras discovered the theorem. In fact, the theorem is far older. Maybe it is more important to mention in this context that among the Pythagoreans, *women* played an important role and participated actively in the philosophical life. And this is not just a legend.

Andrew Schumann: What is your position on the war in Ukraine? Is it of interest to German philosophers?

Andrea Reichenberger: I can't speak for all German philosophers. For me, the only alternative to war is peace, which is unenforceable; both sides must be willing to find a way to peace.

Andrew Schumann: How can this terrible war be stopped, then?

Andrea Reichenberger: I wish there were a simple answer to your question. According to my opinion, the way to peace is not just a matter of specific decision-making processes. It is often a painful process that requires goodwill and honesty on both sides. The intricate problem here seems to me not to be that simple if we try to recapitulate the situation in Ukraine. We all know that many countries are involved in his terrible war with their own socio-economic interests, but not all countries are affected by the war to the same extent. I'm not a political expert, but it seems to me that this is one of the reasons why there is no simple answer to the question of how this war can be stopped.