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Logic in Poland in the 20th Century

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Abstract:
After Poland gained independence in 1918, logic developed very quickly both as a scientific direction and as a taught discipline. This introduction to the special issue "Logic in Poland in the 20th Century," published in Volume 13:1 (2024) and Volume 13:2 (2024), provides the historical context for the development of logic in the interwar period.
Keywords: Logic, Lviv-Warsaw School, Jan Łukasiewicz, Stanisław Leśniewski, Kazimierz Twardowski.

Zbigniew Semadeni recounts an intriguing moment when, at a mathematics conference towards the end of the 20th century, Benoit Mandelbrot posed the question: "What is the most significant date in 20th-century mathematics?" To the audience's astonishment, he declared it to be the year 1918, marking Poland's resurgence as an independent nation. This period heralded a rapid and remarkable ascent of Polish contributions to mathematical logic on the global stage. Heinrich Scholz, in 1931, had already acknowledged Warsaw as a pivotal center for logic (Scholz, 1931, p. 85), a sentiment echoed by Abraham Fraenkel and Yehoshua Bar-Hillel in 1958, who noted Poland's outsized contributions to mathematical logic and set theory relative to its population (Fraenkel & Bar-Hillel, 1958, p. 185).

The re-establishment of the University of Warsaw in 1916, under German occupation consent, saw Jan Łukasiewicz joining as a philosophy professor, soon followed by Stanisław Leśniewski in 1918, marking their tenure in the Faculty of Mathematics and Physics despite their philosophical backgrounds. This unique positioning was a direct outcome of Zygmunt Janiszewski's vision for Polish mathematics to concentrate on set theory, topology, and their mathematical applications, which naturally extended into the realms of mathematical foundations and logic.

The Warsaw School of Logic (Woleński, 1989) was born from this vision, with notable contributions from mathematicians like Wacław Sierpiński, Stefan Mazurkiewicz, and Kazimierz Kuratowski. Łukasiewicz and Leśniewski's involvement in the editorial team of *Fundamenta Mathematicae* underscored the strategic role of logic within Janiszewski's program. The establishment of the Polish Logic Society in 1936 by Łukasiewicz and the publication of *Collectanea Logica* were pivotal in asserting logic as a distinct discipline, meriting its own academic platforms separate from mathematics and philosophy.

While Warsaw embraced logic as a core component of its mathematical identity, Lviv recognized its value without positioning it at the center. Logic departments flourished across various Polish universities, with logic's prominence further reflected in its advanced level of instruction, even within secondary education, and specialized university curricula in Warsaw that included engagement with contemporary mathematical logic challenges.

This vibrant logic scene was not confined to mathematicians; philosophers, notably Kazimierz Twardowski and his disciples, significantly influenced the field. Twardowski's emphasis on the formal qualities of philosophical discourse fostered a conducive environment for logic's advancement. His students, including Leśniewski and Łukasiewicz, later pioneers of Warsaw logic, and others like Alfred Tarski, further cemented Poland's legacy in mathematical logic. Tarski, reflecting on this heritage, acknowledged the pervasive influence of Twardowski's teaching across the philosophy of exact sciences in Poland, indicating the intertwined evolution of mathematical logic with both philosophy and mathematics in Poland.

The professors of philosophy in Warsaw, including Tadeusz Kotarbiński who significantly contributed to logic's popularization from one of the University of Warsaw's philosophy departments, embarked on vigorous educational initiatives. They nurtured a cadre of mathematical logic specialists, notably Alfred Tarski, Adolf Lindenbaum, Mordechai Wajsberg, Moses Presburger, Bolesław Sobociński, Jerzy Słupecki, Stanisław Jaśkowski, Andrzej Mostowski, Czesław Lejewski, and Henryk Hiż. Others, such as Zygmunt Kobrzyński and Zygmunt Kruszewski, though less central, were also part of this circle. This collective effort gave rise to the Warsaw School of Logic, an unparalleled assembly of logicians during the interwar years, bridging the Polish Mathematical School and the philosophical tradition of the Lviv-Warsaw School.

The story of the Warsaw School of Logic is also woven into the tragic tapestry of Poland's World War II history. Members such as Kobrzyński were killed due to resistance activities, while Lindenbaum, Presburger, and Wajsberg perished in the Holocaust. Others, including Łukasiewicz, Tarski, Sobociński, Lejewski, and Hiż, were compelled to emigrate during or in the aftermath of the war, leaving only Jaśkowski, Mostowski, and Słupecki in Poland. This narrative underscores the school's significant losses, including promising students like Jerachmiel Bryman or Jerzy Billich, whose potential was cut short by the war's devastation.

The logic's evolution in Poland throughout the 20th century extends beyond its mathematical community. Philosophers, notably Kazimierz Twardowski, the progenitor of the Lviv-Warsaw School, and his disciples, played a crucial role. Twardowski's metaphilosophical approach emphasized the formal attributes of philosophical discourse, advocating for clarity, precision, and the eschewing of speculative queries while focusing on the substantiation of arguments. This methodological stance fostered a conducive environment for engaging with logical inquiries.

Many among Twardowski's followers, including Stanisław Leśniewski and Jan Łukasiewicz, who would later pioneer the "Warsaw" approach to logic, dedicated themselves primarily to this field. Other notable students of Twardowski who significantly influenced the development of logic in Poland include Kazimierz Ajdukiewicz in Lviv, Tadeusz Czeżowski in Vilnius, Tadeusz Kotarbiński in Warsaw, and Zygmunt Zawirski in Poznań and then Kraków. Alfred Tarski, swiftly becoming a central figure and the most renowned member of the Warsaw School of Logic, later reflected, "Almost all researchers who pursue the philosophy of exact sciences in Poland are indirectly or directly the disciples of Twardowski, although his own works could hardly be counted within this domain" (Tarski, 1992, p. 20). This statement underscores the profound influence Twardowski had on the field, making mathematical logic in Poland a product of both philosophy and mathematics. Tarski's use of "almost" acknowledges contributions from individuals outside Twardowski's direct lineage, including Chwistek and Sleszyński, indicating the diverse origins of Poland's logical scholarship.

This special issue is a postproceeding of the National Scientific Conference "Logic in Poland in the 20th century" accompanying the publication *Leksykon logików polskich 1900-1939* (*Lexicon of Polish logicians 1900-1939*), (Woleński et al, 2022), which took place on September 22, 2022 in Rzeszów, Poland. It consists of the following recent researches in the field of logic in Poland, published in Volume 13:1 (2024): *From the History of Lesniewski's Mereology* (Andrzej Pietruszczak); *The Warsaw School of Logic: Main Pillars, Ideas, Significance* (Urszula Wybraniec-Skardowska); *100 Years of Logical Investigations at University of Poznań* (Roman Murawski); *Logic and Metalogic: a Historical Sketch* (Jan Woleński); *Proof of the Existence of Hell: An Extension of the Stone Paradox* (Piotr Łukowski). This issue also includes the following three researches, published in Volume 13:2 (2024): *Characterizing Context-Independent Quantifiers and Inferences* (Stanisław Krajewski); *Jan Łukasiewicz and his German Ally. A History of Łukasiewicz-Scholz Cooperation and Friendship* (Anna Brożek); *The Nature of the Anti-Psychologistic Turn in Kazimierz Twardowski's Philosophy* (Krzysztof Nowicki); an interview about the development of logic in Belarus in the 20th century: *The Past and Future of High Technology* (Arkady Zakrevsky), an interview with a Ukrainian logician about the situation of the development of logic in Ukraine in the context of Russian military aggression: *Philosophy and Logic in Time of War* (Yaroslav Shramko), and an interview about the responsibility of Russian philosophers for this terrible war: *Living in Illusion is Dangerous* (Marina F. Bykova). The issue published in Volume 13:2 (2024) also contains the review: *Libertarian Autobiographies: Moving Toward Freedom in Today's World. Edited by Jo Ann Cavallo and Walter E. Block. Palgrave Macmillan, 2023* (Robert W. McGee).

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From the History of Leśniewski's Mereology

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Abstract:
In this paper, we want to present the genesis of Stanisław Leśniewski's mereology. Although 'mereology' comes from the word 'part', mereology arose as a theory of collective classes. That is why we present the differences between the concepts of *being a distributive class* and *being a collective class*. Next, we present Leśniewski's original mereology from 1927, but with a modern approach. Leśniewski was inspired to create his concept of classes and their elements by Russell's antinomy. To face it, Leśniewski had to define the concept of *being an element of* based on the concept of *being part of*. Leśniewski showed that in his theory, there is no equivalent to Russell's antinomy. We will show that his solution has nothing to do with the original approach because, in both cases, we are talking about objects of a different kind. Russell's original antinomy concerned distributive classes, and Leśniewski's considerations concerned collective classes.
Keywords: mereology, Leśniewski, collective class, distributive class, set theory.

1. Introduction

Mereology arose as a theory of collective classes (sets). It was formulated by the Polish logician Stanisław Leśniewski. Collective sets are certain wholes composed of parts. In general, the concept of a *collective set* can be defined with the help of the relation *is a part of* and mereology may therefore be considered as a theory of "the relation of part to the whole" (from the Greek: $\mu\epsilon\rho o\varsigma$, *meros*, 'part').

Leśniewski did not invent the concept of a *collective set*. It is discussed, for example, by Whitehead and Russell in comments in *Principia Mathematica* (1910) concerning the theory of classes developed in that work. Whitehead made use of such sets in his thoughts on the philosophy of space-time (Whitehead, 1929).

In mereology, as in everyday speech, the expression 'part' is usually understood as having the sense of 'fragment', 'bit', and so forth. Thus understood, the relation of part to the whole has two properties *irreflexivity* and *antisymmetry*, i.e.:

(irrP) *No object is its own part.*
(antisP) *There are not two objects such that the first could be a part of the second and the second is a part of the first.*

Thanks to condition (irrP), we have no difficulty interpreting the phrase “two objects” in condition (antisP). One can see that it concerns «two different» objects. Of course, the properties of *irreflexivity* and *antisymmetry* entail the property *asymmetry* which is difficult to express in natural language). Furthermore, *being part of* is assumed to be transitive:

(tP) *Every part of a part of a given object is part of it.*

Colloquially, we treat sets as “wholes consisting of some units” — disregarding the way they are created and whether the units they are composed of have a functional contribution to the whole. We will show that even with this approach, we must divide these wholes into two distinct kinds. Moreover, different wholes of both kinds can be formed from the same units. The so-called *distributive sets* forms the first type. Colloquially, we usually talk about the so-called *collective sets*, although there are also cases of distributive sets here. To understand what Leśniewski meant, we will explain what collective sets are supposed to be and how they differ from distributive sets.

The terminology used is artificial. On the one hand, the word ‘set’ is often a substitute for ‘collection’. On the other hand, the basic meaning of the word ‘distribution’ is division. Thus, the return ‘distributive set’, i.e. «collected and distributed». However, this terminology is beneficial. The combination ‘collective set’ is to remind you that it is about the colloquial meaning of the word ‘set’. The combination ‘distributive set’ suggests that the sets in question have little to do with collecting, accumulating, or combining. If anything, we are to associate them with these activities, understood in the abstract. For example, we unite voivodeships, but at the same time, we do not unite communes, thus not receiving the land territory of Poland. We collect cities, but we do not collect their streets, squares, etc. We understand this abstractly to such an extent that we also admit the existence of the empty set, which is nonsense in common sense. How can assemble «things that are not there»?

Just as the word ‘set’ is ambiguous, so is the term ‘element of a set’. In other words, the word ‘element’ takes on a meaning that depends on the meaning of the word ‘set’. Thus, when we talk about sets (resp. classes) and their elements, some misunderstandings can arise because of the multiplicity of meanings the terms ‘set’ and ‘element of a set’ possess. Let us quote an extensive excerpt from Ludwik Borkowski’s book (1977, p. 146):

> The terms “set” and “element of a set” are used with two meanings. Understood with the first of these meanings, the term “set” signifies objects composed of parts, collections and conglomerations of a different kind. The elements of such type of set are to be understood as arbitrary parts of that set, where the term “part” is understood in its everyday sense, with which, for example, the leg of a table is a part of the table. A pile of stones is in this sense a set of those stones. The elements of that set are both individual stones along with the various parts of those stones, and thus, for example, the molecules or atoms of which those stones are composed. With this meaning, the set of given stones is identical to, for example, the set of all the atoms from which they are composed. Elements of a set so understood, such as the set of all tables, would be not only the individual tables but, for example, the legs of those tables or other of their parts. We shall say that we are using here the term “set” in its *collective* sense, as we are using it with that sense. A theory of sets and the relation *is a part of* understood in line with the above has been constructed by S. Leśniewski, who called it *mereology*.
>
> We use the terms “set” and “element of a set”, with the second meaning in the following example: when talking about the et of European countries, we consider as elements of that set particular European countries, such as Poland, France and Italy, and we do not consider as elements the parts of those countries. With this meaning, the Tatra mountains or the Małopolska Upland are not elements of the set of European countries even though they are parts of certain European countries. We also use these terms with this meaning for example when, talking about the set of Polish towns, we consider as elements of that set towns such as Wrocław and Warsaw whilst not considering as elements of that set particular streets or squares or other parts of those cities. The terms “set” and “element of a set” have long been used with this meaning in logic, when speaking of extensions of names or concepts as certain sets of

> objects. In contrast to the first meaning, it is not possible to identify the concept of an element with the common concept of a part.

The second meaning of the term "set" has come to be called the *distributive* or *set-theoretic meaning*. Let us add further a section from the final paragraph of the book (Słupecki & Borkowski, 1967, p. 279) that makes some philosophical on sets.

> [...] the word "set" has two clearly distinct meanings in everyday speech, of which one is call the collective meaning and the second the distributive. With the collective meaning — a set of a certain objects is a whole composed of those objects in the same way that a chain is composed of links and a pile of a sand of grains of sand. With this meaning, a set of concrete, sensually perceptible objects is also a concrete and perceptually-available object. Using the term "set" with this meaning, we understand "x is an element of the set A" as having the same sense as the expression "x is a part of the set A" (with the word "part" having that meaning such that the leg of a table is a part of the table). A set theory understood in this way was built by S. Leśniewski under the name *mereology*. Using the term "set" with its distributive meaning, we consider the sentence "Mars is an element of the set of planets in our Solar System" as equivalent to the sentence "Mars is a planet in our Solar System". The difference in meaning is attested to by the fact that certain true sentences where "set" is understood with its first meaning are false when it is understood with its second meaning. For example, where the meaning is collective, it is true that a tenth part of Mars is an element of the set of planets in our Solar System, because it is a part of the whole arrangement; whereas that sentence is false if the meaning is distributive, because no tenth part of Mars is a planet in our Solar system.

It is evident from the above texts that the terms 'collective set' and 'distributive set' have different meanings. It would seem indeed that the single common characteristic is that, in both cases, it is possible to say that "a set of certain objects is a whole composed of those objects" (Murawski, 1984, p. 164). To put it another way, there may be a similar "way of creating sets" for both concepts. As Hao Wang (1994, p. 267) writes:

> There are two familiar and natural ways of construing sets [both conceptions of the creation of sets described here obviously concern distributive sets, *A.P.*]. On the one: hand, given a multiplicity of objects, some or all of these objects can be conceived together as forming a set; the process can be iterated indefinitely. This way may be called "the extensional conception of set." On the other hand, a set may be seen as the extension of a concept or a property in the sense that it consists of all and only the objects which have the property. This way may be called "the intensional conception of set." We tend to use both conceptions and expect no conflict between them. Yet in practice it makes a difference whether one takes the one or the other conception as basic.
>
> Roughly speaking, Frege begins with the intensional conception and Cantor begins with the extensional conception.

2. Distributive Sets – the Basic Principle

Used with their distributive senses, the terms 'set' and 'class' are often treated as synonyms. In certain versions of modern set theory a distinction is made between them. In such theories, *each set has to be a class, but not conversely*. Sets are a special kind of class: they are those classes which are elements of other classes.

In the case of distributive classes (sets), the collection — i.e., collecting of objects, regardless of their type — must be understood always in an abstract sense and not a spatio-temporal one. Quine (1981, p. 120) writes :

> The reassuring phrase 'mere aggregates' must be received warily as a description of classes. Aggregates, perhaps; but not in the sense of composite concrete objects or heaps. Continental United States is an extensive physical body (of arbitrary depth) having the several states as parts; at the same time it is

> a physical body having the several counties as parts. It is the same concrete object, regardless of the conceptual dissections imposed; the heap of states and the heap of counties are identical. The class of states, however, cannot be identified with the class of counties; for there is much that we want to affirm of the one class and deny of the other. We want to say e.g. that the one class has exactly 48 members, while the other has 3075. We want to say that Delaware is a member of the first class and not of the second, and that Nantucket is a member of the second class and not of the first. These classes, unlike the single concrete heaps which their members compose, must be accepted as two entities of a non-spatial and abstract kind.

With their distributive meaning, the terms 'class' ('set') and 'element' for any general name S, satisfy the BASIC PRINCIPLE given below in the form of a schema:

($\star$) The elements of the distributive set of Ss are all Ss and only Ss.

So when we talk about the distributive set of Ss, we mean the distributive set of all Ss, and composed only of Ss. For example, the elements of the distributive set of Polish voivodeships are all these and only them (and not communes, towns, villages, etc.).

The following principle of extensionality applies to distributive sets:

if X and Y have the same elements, then $X = Y$.

According to the above, all general names having the same referents determine one distributive set whose elements are their referents and which is their common extension. All empty names designate the same distributive set — null set, denoted by '$\varnothing$'.

The Podkarpackie is one of the Polish voivodships, but it is not a commune. It is the other way around with the Strzyżów commune. Therefore — under the basic principle ($\star$) — distributive sets of voivodships and communes differ in elements because the Podkarpackie voivodship is an element of the first one and not an element of the second one (similarly, the commune of Strzyżów is an element of the second one, but not an element of the first one). Applying the counterposition of the following principle of identity:

if $X = Y$, then X i Y X and Y have the same elements,

we obtain that these sets are different:

the distributive sets of voivodships $\neq$ the distributive sets of communes.

We do not need to apply the principle of extensionality, which is the inverse implication of the principle of identity. What we got proves that attests to the fact that the aforementioned distributive sets may not be identified with any spatiotemporal object. Indeed, the land territory of Poland is the only such object. However, with such an identification, we would get equality instead of inequality. So:

the land territory of Poland $\neq$ the distributive sets of voivodships
$\neq$ the distributive sets of communes.

Similarly, Quine says in the previous passage is that if the class of the states of the USA occupied some 'place' in space, then it would the very same place that the USA occupies. The same would be true of the class of counties in the USA. We should therefore identify these distributive classes, contrary to condition ($\star$).

The presented analyses show that distributive sets are abstract objects. We may paraphrase the preceding considerations: it is possible «to collect abstractly» the communes whilst not collecting voivodeships and vice versa. Quine provides us with another example in support of the theory of the abstractness of distributive classes (sets) in an essay from (Quine, 1953, pp. 114–115):

> The fact that classes are universals, or abstract entities, is sometimes obscured by speaking of classes as mere aggregates or collections, thus likening a class of stones, say, to a heap of stones. The heap is

indeed a concrete object, as concrete as the stones that make it up; but the class of stones in the heap cannot properly be identified ,with the heap. For, if it could, then by the same token another class could be identified with the same heap, namely, the class of molecules of stones in the heap. But actually these classes have to be kept distinct; for we want to say that the one has just, say, a hundred members, while the other has trillions. Classes, therefore, are abstract entities; we may call them aggregates or collections if we like, but they are universals. That is, if there *are* classes.

As in the previous quoted passes, Quine is saying that if a class of stones occupied some «place» in space, then it would be a pile of stones. A similar thing would be said of the molecules in the stones. It is possible to «abstractly take» the stones «without moving» their molecules or vice versa.

Let us remind ourselves that in "the intensional conception of [distributive, *A.P.*] set", "a set may be seen as the extension of a concept or a property in the sense that it consists of all and only the objects which have the property" (Wang, 1994, p. 267). Thus, not as a property, but as its extension. A further excerpt from (Quine, 1981, pp. 120–121) will help clarify what is meant:

> Once classes are freed thus of any deceptive hint of tangibility, there is little reason to distinguish them from properties. It matters little whether we read '$x \in y$' as 'x is a member of the class y' or 'x has the property y'. If there is any difference between classes and properties, it is merely this: classes are the same when their members are the same, whereas it is not universally conceded that properties are the same when possessed by the same objects. The class of all marine mammals living in 1940 is the same as the class of all whales and porpoises living in 1940, whereas the property of being a marine mammal alive in 1940 might be regarded as differing from the property of being a whale or porpoise alive in 1940. But classes may be thought of as properties if the latter notion is so qualified that properties become identical when their instances are identical. Classes may be thought of as properties in abstraction from any differences which are not reflected in differences of instances. For mathematics certainly, and perhaps for discourse generally, there is no need of countenancing properties in any other sense.

It is precisely what the two previously given principles of extensionality and identity providers are. It also shows that the notion of *distributive set* (or of *distributive class*) must be primitive, i.e. undefinable. After all, it is impossible — without falling into a "vicious circle" — to define sets as classes of abstractions in a set of properties.

3. Leśniewski's Views on Distributive Sets

One of the featured quotes from Quine on distributive classes ends with the words: "That is, if there *are* classes." Even such conditional «making the case» irritated Leśniewski, who categorically rejected the existence of distributive sets (classes). This is evidenced by his comments in the first part of his fundamental work "On the foundations of mathematics" (Leśniewski, 1927, 1928, 1929, 1930, 1931). The theory of types created by Russell and Whitehead and the theory of classes as the extensions of concepts created by Frege were both for Leśniewski objectless (1927, pp. 204–205) (the passages from Leśniewski's papers have been translated from the original and not taken from the English edition of this work (Leśniewski, 1991)):

> I do not know what Russell and Whitehead understand in the commentaries on their system by class. The fact that, on their position, "class" is supposed to be the same as "extension" does not help me in the slightest, as I do not know what these authors mean by extension. I do not therefore know either, when they consider the matter of the existence or non-existence of objects as such whether their thoughts on the puzzle of existence and non-existence address those objects which are classes. [...] Not understanding the relevant terminology of Whitehead and Russell, I am not in particular aware where and to what degree their doubts as to the existence of objects, which are classes in their understanding of that term [The authors of the *Principia Mathematica* do in fact introduce as a problem the question of the existence of distributive sets, *A.P.*], may bear on particular positions I take in the theory of classes sketched earlier.

> In "*Principia Mathematica*", I did not find a single paragraph which I felt there was even the weakest presumption of calling into question the existence of classes as I understand them. Sensing in the "classes" of Whitehead and Russell, in a similar fashion as with the "extensions of concepts" of Frege, the scent of mythical paradigms from a rich gallery of "invented" objects, I cannot for my part divest myself of the inclination to sympathise "on credit" with the doubts of the authors on the matter of whether objects that are such "classes" exist in the world. — On the matter of the relation of my conception of class to the views represented in the commentaries of Whitehead and Russell on their system, a certain light may be here thrown by the views of Russell on "heaps". Russell writes in one of his works: "We cannot take classes in the *pure* extensional way as simply heaps or conglomerations. If we were to attempt to do that, we should find it impossible to understand how there can be such a class as the null-class, which has no members at all and cannot be regarded as a "heap"; we should also find it very hard to understand how it comes about that a class which has only one member is not identical with that one member. I do not mean to assert, or to deny, that there are such entities as "heaps". As a mathematical logician, I am not called upon to have an opinion on this point. All that I am maintaining is that, if there are such things as heaps, we cannot identity them with the classes composed of their constituents" [The passage Leśniewski refers to is to be found in (Russell, 1919, p. 183), *A.P.*]. If I understand the cited paragraph correctly, then the fact that a certain object P is a "heap" of some as, composed of all as, would still not be for Russell a sufficient basis on which to affirm that the object P is a "class" of objects a. Russell's terminology would remain most clearly in complete discord with my terminology; in accordance with his use of the expressions "class" and "set", and the use of the expression "heap" in our common, everyday language [...], I could always say of a "heap" of some as, that it is a set of objects a [of as, *A.P.*], but of a "heap" of objects a [of as, *A.P.*] composed of all as, that it is the class of objects a [of as, *A.P.*]. [...] The difficulty is in understanding in what consists the difference a "heap" of objects a [of as, *A.P.*] and "class" of objects a [of as, *A.P.*] from Russell's point of view, if both such things existed and if each of them were *composed* of all as and it is a difficulty which I do not know how to overcome.

As can see, the problem was that Leśniewski understood the word 'class' differently than Russell. Quite simply, Leśniewski categorically rejected the existence of sets (classes) in the distributive sense. Let us add that heaps are the same as corresponding collective classes. Leśniewski is right about this point. He could not, however, understand "on what rests the difference between" a heap of Ss (i.e. a collective class of Ss) and a distributive class of Ss, "if both such things existed and if each of them were composed of all" Ss. This led Leśniewski to claim that Cantor's set theory applies to — just like his mereology — of collective sets (Leśniewski, 1927, p. 190):

> My conception is, in this respect, on the one hand (as far as I have managed to observe) entirely consistent with the way the expressions "class" and "set" are used in the common, everyday language of people who have never held neither any "theory of classes" nor any "theory of multitudes". On the other hand, it is based on a strong academic tradition, running more or less continuously through countless past and present scholars, and in particular through George Cantor.

In Leśniewski's opinion therefore, mereology deserves the title of "The foundation of mathematics" in the same way as in Cantor's theory, since both theories are concerned with the same sets (classes). Leśniewski's main work, in which he presented his mereology, he thus called "On the foundations of mathematics" (1927; 1928; 1929; 1930; 1931). An earlier work pertaining to mereology carried the title "The foundations of the general theory of sets" (Leśniewski, 1916). Nowadays, it is undisputed that set theory deals with sets (classes) in the distributive sense.

However, Cantor's theory differs significantly from mereology. In the first one:

- there is the distributive empty set $\varnothing$, which excludes mereology (see section 4 below);
- the distributive set consisting of one object x is not x, i.e. $x \neq \{x\}$ (for example, $\varnothing \neq \{\varnothing\}$; $1 \neq \{1\}$, but in mereology we have $x = [\![x]\!]$, where $[\![x]\!]$ is the collective set consisting of one object x (see sections 4 and 5 below).

In Cantor's theory, the set consisting of the only object x, i.e. $\{x\}$, has one element. In mereology, the elements of a given set are all its parts and the set itself (see section 7 below). Thus, each part of x is an element of the collective set $[\![x]\!]$ because $x = [\![x]\!]$. It is only a single-element set if x has no parts. Moreover, since each part of an element of a given set is also an element of this set, it is impossible to determine the number of elements of a given collective set. Thus, collective sets have no use in mathematics.

4. A Definition of Collective Classes

The phrase 'is a class' does not appear in Leśniewski as a unary predicate. The term 'class' is always used in the context of 'class Ss'. Leśniewski connects the latter with the conjunction 'is', obtaining the phrase 'is a class of Ss'. We will not go into the details of the syntax of Leśniewski's mereology here, which is based on the syntax of his other theory — ontology. As mentioned, the concept of *being a collective class of Ss* is defined by the concept of *being a part of*. Leśniewski gave various definitions of this concept, equivalent in his mereology.

To get a relatively concise formulation of these definitions, let us use the artificial notion of *being an ingredient* introduced by Leśniewski (1928, p. 264, footnote 1 and definition I):

- an *ingrediens* of a given object is the object itself and each of its parts, where 'part' is understood with its ordinary sense.

By the above definition, the relation *is an ingrediens of* is reflexive, i.e.:

(rI) *Every object is an ingrediens of itself.*

By the above definition and (antisP), we obtain that *being an ingrediens of* is antisymmetric, i.e.:

(antisI) *There are not two objects such that the first could be an ingrediens of the second and the second is an ingrediens of the first.*

Moreover, by the above definition and (tP), we obtain that *being an ingrediens of* is transitive, i.e.:

(tI) *Every ingrediens of an ingrediens of a given object is an ingrediens of it.*

Using the technical notion *is an ingrediens of*, the first definition of a collective class Ss is as follows:

- it that an object x is a collective class of Ss means that the following two conditions hold:
  (a) every S is an ingrediens of x,
  (b) every ingrediens of x has a common ingrediens with some S.

Notice that from the given definition, we get the following three conclusions:

- If a name S is empty, then there is no collective class of Ss.

Assume for a contradiction that x is such a class. Since x is an ingrediens of itself, by condition (b), x has a common ingrediens with some S, but there is no S.

(i) Every object is a collective class of its ingredienses.
(ii) Every object having a part is a collective class of its parts.
(iii) Each object is a collective class of objects identical to it.

For any object x, we take the expression 'ingrediens of x' (resp. 'part of x', 'identical to x') instead of the letter 'S'. Then both conditions in the definition are tautological (we use the fact that each object is its ingredient).

Collective classes preserve the «nature of objects» from which they are built.

5. Axioms of Leśniewski's Mereology

In addition to the previously mentioned properties of the concept of *being a part of* (irreflexivity, antisymmetry, asymmetry, transitivity; see conditions (irrP)–(tP)), Leśniewski adopted two axioms regarding the defined concept of *being a class of Ss*. The first is uncontroversial. Namely, it says there can only be one class of Ss. In other words, for any objects x and y we put:

- *If x and y are collective classes of Ss, then $x = y$.*

Hence, (i)–(iii), we can write:

$$\begin{aligned} x &= \text{collective class of ingredienses of } x \\ &= \text{collective class of parts of } x\text{-a, if } x \text{ has a part} \\ &= [\![x]\!], \end{aligned}$$

where $[\![x]\!]$ is the collective set consisting of one object x.

The second of the axioms adopted by Leśniewski, however, is already controversial. Namely, it says that for any non-empty name S, there is a collective class of all Ss:

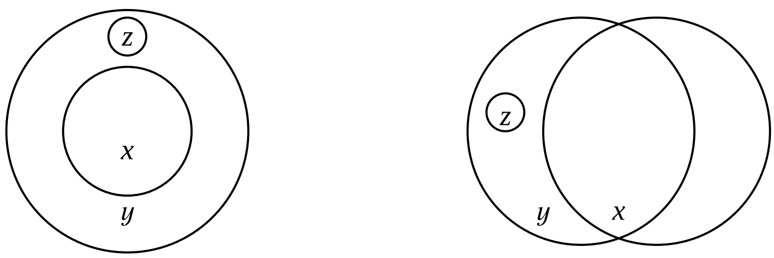
- *If there is an S, then there is a collective class of all Ss.*

This axiom is too strong, which limits its use. In modern terms, it only applies to point-free geometry and point-free topology (for details, see Pietruszczak, 2018).

Note that the given axioms entail the *polarization* of *being of ingrediens of*, also called the Strong Supplementation Principle, which is a fundamental property of the relations *is an ingrediens of* and *is a part of*. For any x i y we have:

(polI) *If y is not ingrediens of x, then there is an ingrediens of y having no common ingrediens with x.*

This principle is illustrated in the figure below:



Notice that from (rI), (tI) and (polI) for any x and y, we get:

- y is an ingrediens of x if and only if every object having a common ingrediens with y also has a common ingrediens with x.

6. Other Definition of Collective Classes

Other of Leśniewski's definition of a concept of *being a class of Ss* is as follows:

- it that an object x is a collective class of Ss means that for object y, the following condition holds:
  (c) y has no common ingrediens with x if and only if y has no common ingrediens with some S.

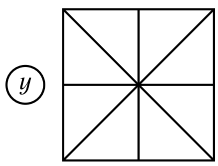
It is obvious that condition (c) is equivalent to the following:

(c′) y has a common ingrediens with x if and only if y has a common ingrediens with some S.

It is also easy to see from (c′) that if S is empty, then there is no collective class of Ss.
First, note that if an object x is a collective class of Ss, i.e., x satisfies conditions (a) and (b), then x satisfies condition (c′) (see Pietruszczak, 2018, p. 81). Indeed, suppose that x is a collective class of Ss. Firstly, assume that y has a common ingrediens, z, with x. Then, by (b), z has a common ingrediens, u, with some S. By transitivity, u is an ingrediens of y. So y has a common ingrediens with some S. Secondly, suppose that y has a common ingrediens, z, with some u, which is an S. Hence, by (a), u is an ingrediens of x. By transitivity, z is also an ingrediens of x. So x and y have some common ingrediens.

Second, in Leśniewski's mereology, both his definitions are equivalent. Namely, by (polI), if an abject x satisfies condition (c′), then x satisfies both conditions (a) and (b) (for details, see Pietruszczak, 2018, p. 142). For (a): By the ⇐-part of (c′), for any y we have: y has a common ingrediens with some S, then y has a common ingrediens with x. Hence for any z being an S and ant y, we have: if y has a common ingrediens with z, then y has a common ingrediens with x. Hence, by (rI), for any z being an S and any y we have: if y is an ingrediens of z, then y has a common ingrediens with x, i.e., every ingrediens of z has a common ingrediens with x. So, by (polI), we get: z is an ingrediens of x. Thus, we obtain that every S is an ingrediens of x. For (b): By the ⇒-part of (c′), for any y we have: if y has a common ingrediens with x, then y has a common ingrediens with some S. Hence, by (rI), if y is an ingrediens of x, then y has a common ingrediens with some S, i.e., every ingrediens of x has a common ingrediens with some S.

Let us see the operation of the given definitions in the figure below:



We see that x = the largest square in this figure satisfies the condition (c) both for S as 'triangle' and as 'square': y has no common ingrediens with x if and only if y has no common ingrediens with some triangle (resp. square) in the figure. So x is the collective class of triangles (resp. squares) in this figure:

the collective classes of triangles	=	the largest square
	=	the collective classes of squares

Similarly, we get:

the collective classes of voivodeships	=	the land territory of Poland
	=	the collective classes of communes

Once again, we see that distributive sets must be considered abstract objects. Namely, the distributive set of triangles in the figure above is different from the distributive set of squares in the figure because these sets have different elements, i.e.:

the distributive classes of triangles ≠ the distributive classes of squares

These classes cannot be drawn because they are abstract objects. Only their elements can be drawn. Notice that for the distributive version, we have the following:

the largest square	≠	the distributive classes of triangles
	≠	the distributive classes of squares

7. Elements of Collective Classes

Leśniewski was «inspired» to create his concept of classes (sets) and their elements by Russell's antinomy, which concerned the distributive class of all distributive classes that are not elements of each other. To face it, Leśniewski had to define the concept of *being an element of*. He meant *being an element of a given class*, but in his theory, all objects are classes and vice versa.

Leśniewski's definition the concept of *being an element of* is similar to the analogous definition adopted by Frege. Namely, Leśniewski assumed that for any objects x and z:

- x is an element of z if and only if for some meaning of 'S': z is a collective class of Ss and x is an S.

This definition and the definition of a collective set of Ss lead in Leśniewski's theory to the following conclusion already mentioned:

($) x is an element of z if and only if x is an ingrediens of z.

Thus, in Leśniewski's mereology, *being an element of* is the same as *being an ingredients of*, and not 15 the same as *being a part of*, as some authors claim (see, e.g., Borkowski, 1977; Słupecki & Borkowski, 1967). Indeed, firstly, suppose that x is an element of z, i.e., for some meaning of 'S': z is a collective class of Ss and x is an S. Then, by (a), we get that x is an ingrediens of z. Secondly, suppose that x is an ingrediens of z. Notice that we have (i): z is the collective class of its ingredienses. So we take S as 'ingrediens of z'. Then z is the collective class of Ss, and x is an S. Hence x is an element of z.

Thus, we see that collective classes and their members do not satisfy the previously mentioned fundamental principle for distributive sets:

(⋆) The elements of the distributive set of Ss are all Ss and only Ss.

Condition (a) from the definition of collective classes and ($) say that we only have:

- All Ss ale elements of the collective class of Ss.

We see that there may be elements of the collective class of Ss which are not Ss.

8. Influence of Russell's Antinomy on the Creation of Mereology

As mentioned, Leśniewski was «inspired» by Russell's antinomy to create his concept of classes and their elements. To solve the problem of a class being composed of classes that are not members of themselves, Leśniewski developed his concept of classes (sets), which is very different from Cantor's. Leśniewski (1927, pp. 185–186) writes:

> Wishing "to conceive of something" and not knowing at the same time how to find any reasonable fault in any of the aforementioned assumptions on which the earlier "antinomy" rests, nor also in the reasoning leading to contradiction on the basis of those assumptions, I began to muse on examples of situations in which in practice I consider or do not consider such and such objects as classes or sets of such and such objects [...] and to submit for critical analysis my faith in the particular assumptions of the "antimony" in hand from that point of view (the puzzle of "empty classes" was not the theme of my considerations on that occasion because I treated the conception of "empty classes" from my first moment of contact with it as a "mythical" conception, taking without any hesitation the position that:
>
> (1) if any object is a class of objects a, then some object is an a.)

In his theory of (distributive) classes, Frege assumed that each concept determines the class of objects that fall under it. Russell noted that this assumption leads to a contradiction. Namely, he showed that the same assumption of the existence of a class of all classes not being their own elements leads to a contradiction. Classes not being their own elements are considered «normal». Thus Russell's antinomy says that the assumption of the existence of a class of all normal classes leads to a contradiction in Frege's theory.

As general name S, we take '(distributive) normal class'. According to Frege's theory, there is a distributive class of all normal distributive classes. Let us denote it by '$\mathcal{N}$'. According to ($\star$):

($\star_{\mathcal{N}}$) The elements of $\mathcal{N}$ are all normal classes and only them.

Hence for any distributive class X we obtain:

($\star_{\mathcal{N}}$) X is an element of $\mathcal{N}$ if and only if X is not an element of itself.

Since X was any distributive class, the above applies to $\mathcal{N}$. Hence we get a contradiction:

- $\mathcal{N}$ is an element of itself if and only if $\mathcal{N}$ is not an element of itself.

However, as Quine points out in (1987), this is based on the tautology of quantifier logic, in which 'R' represents any binary predicate:

($\top$) there is no x such that for any y: yRx if and only if it is not the case that yRy.

Indeed, assume for a contradiction that there is an x such that for any y: yRx if and only if it is not the case that yRy. Then, since y was any object, it applies to x. So we have a contradiction: xRx if and only if it is not the case that xRx.

Taking in ($\top$) R as *is an element of*, we get that there is no class from Russell's antinomy. Similarly, we get the well-known the barber paradox: no one shaves all those and those only who do not shave themselves. It is enough to take the predicate 'is shaved by' as R.

Let us return to the solution of Russell's antinomy in Leśniewski's version. In his theory, no class is normal. Indeed, every object is an ingrediens of itself, and *being an ingrediens of* is the same as *being an element of*, so every object is an element of itself. Hence the concept of *being a normal class* is empty. Furthermore, empty concepts do not designate collective classes. So there is no class of all normal classes.

Does the given solution have anything to do with Russell's original antinomy? Probably not, because in both cases, we are talking about objects of a different kind. Russell's original antinomy was about normal classes in the distributive sense. Leśniewski's considerations concerned normal classes in the collective sense. Finally, let us emphasize once again that for Leśniewski, there were only classes of the second kind. So for him, Russell's original antinomy was about nothing — is talking about «something» that does not exist. So it is a paradox only «apparently». Leśniewski wanted to show that there is no paradox of a class of all normal classes for his classes.

Since Leśniewski wanted to refer to Russell's original antinomy, he used the notion *being an element of*, not the notion *being a part of*. However, the question may arise: what if he used the latter term? Of course, we will not get a contradiction either. Due to the irreflexivity of *of being part of*, the name 'class not being part of itself' applies to all classes, i.e., to all objects in Leśniewski's case. So if there is no object (which Leśniewski did not exclude), we have an empty name, which does not designate any money. If there is an object, then according to the axioms of mereology, the entered name designates an object which is a class of all objects.

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The Warsaw School of Logic: Main Pillars, Ideas, Significance

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Abstract:
The Warsaw School of Logic (WSL) was the famous branch of the Lviv-Warsaw School (LWS) – the most important movement in the history of Polish philosophy. Logic made the most important field in the activities of the WSL. The aim of this work is to highlight the role and significance of the WSL in the history of logic in the 20th century.
Keywords: history of logic, program assumptions and ideas of the WSL, Łukasiewicz, Leśniewski, Tarski, major achievements of representative of the WSL, importance of the WSL in the history of logic in the 20th century.

1. Introduction

The Lviv-Warsaw School (LWS) was the most significant Polish philosophical formation in the 20th century (Woleński, 2015).[1] The interest in logic in its broad scope, understanding of it, was one of the more important attitudes represented by the founders of the LWS – Kazimierz Twardowski and his Lviv disciples. At least several of them undertook to conduct research in the area of logic. Some time later, those disciples, having transferred to Warsaw, formed the Warsaw School of Logic (WSL), shaped the image and contributed to the success of Polish logic as well as to its international recognition.

The work presents the beginnings of logic in Poland (Section 1), outlining the functioning of the domain at the time of establishing the LWS (Section 2). Section 3 focuses on the program assumptions and ideas lying at the heart of the WSL, while Section 4 provides its characteristic. Next, Section 5 discusses the most prominent achievements of logicians forming the WSL.

The aim of the paper is to emphasize the role and the significance of the WSL in the history of logic in the 20th century. They are presented in the last part of this work (Section 6).

2. The Beginnings of Logic in Poland

The history of Polish logic (as a domain of science) dates back to the Middle Ages and is connected with the Cracow Academy (later the Jagiellonian University). Jan of Głogów (1445-1507), called also Głogowczyk or Głogowita, is commonly acknowledged to have been the most outstanding of its representatives. He was one of the teachers of Mikołaj Kopernik (Nicholas Copernicus), himself the author of the coursebook of logic entitled *Exereitium novae logicae*, as well as commentaries on texts by Aristotle and Petrus Hispanus (Peter Spaniard). The end of the Middle Ages was marked by a decline in the interest in logic.

Until the end of the 19th century, in Poland, logic had developed in line with all-European trends and it was the 20th century that saw a revival of interest in logic – primarily owing to the scholars of the Lviv-Warsaw School of Logic (LWS), in particular Warsaw-based logicians forming the Warsaw group of the LWS, i.e. the Warsaw School of Logic (WSL), which was active in the years 1918-1939.

The members of the WSL made Polish logic famous in the world. This would not have been possible if it had not been for the program assumptions and attitude towards logic, which functioned in the LWS.

3. Functioning of Logic at the Beginnings of the LWS

The beginnings of the LWS date back to the end of the 19th century, precisely the year 1895, when Kazimierz Twardowski arrived in Lviv and took the position of professor of philosophy there. His aim was to found a strong philosophical center in Poland, one functioning in the spirit of philosophical and methodological guidelines delineated by his teacher Franz Brentano. Twardowski's philosophical inquiries and his teaching were marked by a clear and concise way of thinking, precision of formulation of the presented theses, as well as their correct justification. All these characteristics were also binding for his disciples.

The following were the features connecting all the members of the School established by Twardowski: the manner of joint practicing philosophy, exchange of thoughts and openness to cooperation with representatives of other disciplines, including Lviv-based mathematicians. The philosophers of the LWS maintained contacts with their counterparts all over the world. Consequently, the LWS became one of the most significant phenomena of European culture.

As regards the program of philosophy of the LWS, logic in its broad sense, i.e. formal logic, semantics and methodology of sciences, made a relevant element. Although Twardowski himself was not an enthusiast of logic, he lectured in logic, in particular algebra of logic, and had an influence on taking interest in logic by some of his disciples.

Therefore, already in the first Lviv period of the LWS, studies in logic were conducted by the following of Twardowski's disciples: Jan Łukasiewicz (principles of contradiction and the excluded middle, logical values, logical entailment, an important monograph on the principle of contradiction); Kazimierz Ajdukiewicz (logical entailment); Tadeusz Czeżowski (theory of antinomy); Tadeusz Kotarbiński (logical values, principles of non-contradiction and the excluded middle); Stanisław Leśniewski (theory of antinomy, principles of contradiction and the excluded middle, logical values); Zygmunt Zawirski (modality theory). All of them exerted a considerable influence on the development of Polish logic. Nevertheless, it is two of them: Jan Łukasiewicz and Stanisław Leśniewski, as well as their disciples, who contributed the most to the success of Polish logic, formal logic, in the international arena.

4. The Genesis of the Warsaw School of Logic: its Program Assumptions and Ideas

The regular scholarly and educational activity of the LWS, suspended in the period of the First World War, was resumed following the reactivation of the Warsaw University in 1916 and developed after Poland's regaining independence in 1918. Twardowski's School ceased to be one based in Lviv and became Nationwide School. Twardowski's former disciples took posts at chairs of philosophy of all Polish universities (with the exception of the Catholic University of Lublin). Many of them transferred from Lviv to Warsaw.

As a result, Lviv and Warsaw were the main centers of the LWS. It is worth noting here that Lviv and Warsaw were also centers of Polish School of Mathematics which was developing parallel to, but independent of, the LWS. Warsaw, in the interwar period (1918-1939) became the chief center of logic, whose development was largely due to Zygmunt Janiszewski and his program of development of mathematics, in which mathematical logic with set theory occupied an important place. (At that time, i.e. in the interwar period, another center of logic in Poland was also Krakow, yet not as important as that in Warsaw, since there was not much acceptance of logic among the mathematicians there.)

The beginnings of the WSL were connected with the establishment of the Chairs of Philosophy at the Faculty of Mathematics and Natural Sciences of Warsaw University. They were especially created for Jan Łukasiewicz (in 1915) and Stanisław Leśniewski (in 1919) who were offered the positions of their heads. Both philosophers met with openness on the part of mathematicians based there: Zygmunt Janiszewski, Wacław Sierpiński, Kazimierz Kuratowski, Stefan Mazurkiewicz and others. In compliance with Janiszewski's program, Warsaw-based mathematicians paid a lot of attention to mathematical logic, set theory and foundations of mathematics. Łukasiewicz and Leśniewski – regarded as the founders of the WSL – in the first period of their activity of their School concentrated on intensive teaching of mathematical logic mainly to mathematicians, still they also taught the subject to philosophers.

Thus, it was the two-stream source: Twardowski's philosophical program and Janiszewski's mathematical one that the manner of practising logic characteristic of the WSL drew on. Mathematical logic was founded here on the assumption that it is an autonomous discipline, self-contained and independent either of philosophy or mathematics.

Obviously, this fact was not deciding for Warsaw-based logicians to abstain from cooperation with either philosophers or mathematicians. Such a cooperation was very fruitful and materialized in maintaining philosophical motivation in logical investigations, and in preserving exactness and precision in logical reasonings, with simultaneous intensive development of formal calculation techniques, similar to those applied in mathematics.

It needs to be added that the founders of the LWS genetically connected with philosophy, yet working in the mathematical environment, as well as their disciples, did not only maintain contacts with the leading Warsaw mathematicians: Sierpiński, Mazurkiewicz and Kuratowski, but also successfully developed close contacts with logicians-philosophers, in particular Tadeusz Kotarbiński who was appointed professor of philosophy at the University of Warsaw in 1919, concentrating logicians of less formal interests (connected rather with semiotics and methodology of sciences, philosophy of science). Kotarbiński attended joint seminars conducted by Leśniewski and Łukasiewicz.

The assumption made by the WSL of the autonomous status of mathematical logic – a discipline not being part of either philosophy or mathematics – was indeed a peculiar one, yet it stimulated the development of theoretical logical studies and working out special techniques to obtain significant scientific results.

The WSL had its own research program and the program of practicing logic in an exact and the most approachable way. Realization of those programs yielded outstanding scientific results and a substantial rise in the significance of Polish logic as well as attractiveness of logical studies.

5. Main Representatives of the WSL and Characteristics of the School

Among the disciples of Jan Łukasiewicz and Stanisław Leśniewski, the founders of the LWS, were: Alfred Tarski – acknowledged to be one of the most outstanding

  

J. Łukasiewicz S. Leśniewski A. Tarski

logicians in the history of logic, Stanisław Jaśkowski, Czesław Lejewski, Adolf Lindenbaum, Andrzej Mostowski, Mojżesz Presburger, Jerzy Słupecki, Bolesław Sobociński, Mordechaj Wajsber. Along with the founders of the School, it is they who formed the core. Most of them were mathematicians, with the exception of Sobociński, a philosopher by education, and Lejewski who studied classical philology.

A few characteristic features of the WSL are listed below:

5.1. The Majority of Problems Dealt with by the School Belonged with Mathematical Logic, Although their Solving was Influenced by the Philosophical Education of its Founders

At the beginning of his scholarly activity, Łukasiewicz dealt with the methodology of empirical sciences. In his monograph entitled *O zasadzie sprzeczności u Arystotelesa* [On the Principle of Contradiction in Aristotle] (Łukasiewicz, 1910) we can find a short lecture on 'algebraic logic.' This work by Łukasiewicz, along with works by Jan Śleszyński, a mathematician and logician representing the Krakow circle, belonged to the first works in Poland dealing with the mathematical logic. Nevertheless, Łukasiewicz never returned to his studies on methodology of empirical sciences.

Leśniewski commenced his scientific activity before the First World War, focusing his interests on problems of semantics of natural language and antinomy; later, however, he distanced himself from those works, stating, among others: "I am very sorry for the fact that they were published at all, and I wish to solemnly 'renounce' them hereby… and to declare the bankruptcy of the "philosophical"-grammatical endeavors of the first period of my activity" (Leśniewski, 1927, pp. 182-183).

5.2. The Founders of the WSL and Their Disciples Were Very Much Concerned with the Intuitiveness of the Value of the Achievements in the Field of Mathematical Logic

Leśniewski wrote about early mathematical logic as follows:

> The matter of far-fetched misunderstandings as regards the sense of basic formulas of this discipline is presently a most current one, the misunderstandings being able to discourage from taking up "logistics" by a considerable number of such research workers who are not satisfied with the very delight in putting down signs and transforming patterns and who […] wish to realize the significance of transformed formulas. (Leśniewski, 1927, pp. 180-181)

5.3. Representatives of the WSL Endeavored to Link Philosophical Questions with the Subject Matter of Mathematical Logic, Solving with its Aid Classical Philosophical Problems

As an example, Łukasiewicz was convinced that the three-valued logic he created (Łukasiewicz, 1920) casts new light on the problem of determinism, while Tarski in his famous work *Pojęcie prawdy w językach nauk dedukcyjnych* [The Concept of Truth in Languages of Deductive Sciences] (published in Polish in 1933 (Tarski, 1933) and translated into many languages) solved one of the fundamental questions of the theory of cognition in such an undisputable way that, probably, no other approach towards the classical problem of truth can undermine Tarski's solution.

One can also note the way in which the founders of the School classified themselves regarding the domains they practiced: Łukasiewicz considered himself to be a philosopher, although he also dealt with purely formal problems, whereas Leśniewski called himself "philosopher-apostate" and built his well-known logical systems: protothetic, ontology and mereology in the most exact and formally perfect manner.

5.4. Another Characteristic of the WSL Was Endeavoring to Find Full, Precise and the Simplest Solutions to Problems

The logicians of the School perfected formally created logical systems or ones presented earlier, repeatedly simplifying axioms, diminishing the number of primitive terms, decreasing the number of axioms themselves. The crowning achievement was reduction of axioms to one only and possibly the shortest. The most significant results in this sphere were achieved by Łukasiewicz (propositional calculus) and Sobociński (Leśniewski's ontology and protothetic).

A consequence of this 'perfectionism', however, was publishing the results with some delay, which brought about a risk of losing the priority as regards the achievement of the results.
The logicians of the School also defined precisely a good number of metalogical terms such as: logical matrix, consequence operation, deduction system, model theory.

5.5. The WSL Made itself Distinctive by Features Connecting All the Members: a Friendly Scholarly Atmosphere and Interpersonal Contacts

The statement below, which was included by Łukasiewicz in his book *Elementy logiki matematycznej* [Elements of Mathematical Logic] (Łukasiewicz, 1929, p. 9), also translated into English, illustrates what cooperation at Warsaw University looked like:

> I owe the most to the scientific atmosphere created at Warsaw University in the field of mathematical logic. It is in discussions with my colleagues, mainly Professor Leśniewski and Assistant Professor Tarski, and often also with students of theirs and mine, that I had a chance to comprehend many a notion, absorb new ways of expressing myself, and learning many a new result, regardless of what their authors were.

That supportive atmosphere and constant discussions, exchanging thoughts between the members of the School, consolidated them greatly and was additionally complemented by their very friendly interpersonal contacts outside the university.

As my teacher Jerzy Słupecki told me, members of the WSL would meet at the café "Lours" located then in Krakowskie Przedmieście in Warsaw and led lively discussions, not only on scientific topics. It is also there where they met with some Warsaw mathematicians. The contacts were void of any prejudice as to the difference in age, social position, political or religious beliefs.

Similarly, as Jan Woleński writes in his books on the Lviv-Warsaw School (Woleński, 1985) and in the article entitled "Tajemnica warszawskiej szkoły logicznej" [The Secret of the Warsaw School of Logic] (Woleński, 1985), we can learn that the School did not recognize any divisions into "the old" and "the young," "beginners" and "advanced". Instead, a strong emphasis was laid on cooperation, regardless of social positions, represented views, characters and personalities of the members. That was a uniting element able to bring closer scientists who were publicly active (Łukasiewicz held the post of Minister for Religious Denominations and Public

Enlightenment in Jan Ignacy Paderewski's Government, the Vice-Rector and twice the Rector of Warsaw University) with modest teachers of comprehensive schools (Wajsberg, Słupecki), the well-to-do (like Lindenbaum) with rather less affluent (Tarski), members who differed regarding their social, political or religious backgrounds: Łukasiewicz and Sobociński were conservatives, Lindenbaum and Presburger inclined towards communism, some who were devout Catholics and others who were followers of Judaism or declared themselves atheists. Apart from Leśniewski, who remained reserved and withdrawn, the majority were persons of kind heart. Irrespective of the natural differences between them, all the members of the WSL were united by a common scholarly idea, charisma of their teachers, the awareness of their exceptionality and the role in the development of logic in the world.

6. The Greatest Achievements of the WSL

The fundamental methods of formal logic attained full maturity in the Warsaw School.

6.1. The Rise and Research Into Propositional Logics

There were many logical systems that were founded and formally investigated in the WSL. It is also here where a series of tools and techniques were created and developed to investigate the properties of the systems.

Łukasiewicz gave a new axiomatics for two-valued propositional logic and the completeness proof of this classical logical system. Inspired by the problem of logical value of sentences describing future contingent events, already formulated by Aristotle, he created the first many-valued systems of logic (1918-1920) (Jadacki, 2009). The discovery of these logics, chronologically preceding slightly that made by American logician Emil Post, should be regarded as one of the greatest achievements of Łukasiewicz. He also introduced a very convenient parenthesis-free notation, later called 'Polish symbolism', which is most useful in theoretical considerations.

The achievements of the WSL include also various technical results connected with seeking the easiest axiomatics of the propositional calculus. In 1935, Wajsberg discovered the criterion of finite axiomatizability of such logics. With regard to axiomatization of many-valued logics, historical results were also achieved by Sobociński (in 1936) and Słupecki (in 1939). The latter gave the criterion of definitional fullness of many-valued systems of propositional logic (in 1939).

The WSL gave rise to the idea of construing prepositional calculi by the matrix method. It is here where the concept of adequate matrix was born and the theory of matrices was developed. The idea-givers behind this were Łukasiewicz, Tarski and Lindenbaum, who proved the theorem of existence of adequate matrices for any closed wrt substitution logics. Jaśkowski gave the criterion of construction of adequate matrix characteristic of intuitionistic logic (in 1936).

6.2. Leśniewski's Systems

Taking into consideration his own nominalistic views, Leśniewski created three original systems: protothetic (1929), ontology (1920) and mereology (1927-31) (Leśniewski, 1929; Leśniewski, 1930), which – in the researcher's intention – were to make a full logical system for the whole science, including mathematics. In the construction of these systems, in the reference to Husserl's idea of semantic category and the theory of logical types, and also in order to increase precision of his research, Leśniewski commenced studies on the theory of semantic/syntactic categories (developed later on by Ajdukiewicz). It made the foundation to formulate the concept of categorial grammar. Leśniewski introduced the distinction between object language and metalanguage, as well.

Significant outcome of the work on simplifying the axiomatics of Leśniewski's systems of ontology and protothetic were obtained by Sobociński. Tarski, under the influence of Leśniewski's mereology, dealt with geometry of solids and jointly with Woodger (in 1937) found an application of mereology in the axiomatic framework of biology. Mereology is still applied in geometry, biology and linguistics, while Leśniewski's systems themselves offer an attractive object of interest on the part of logicians and philosophers not only in Poland.

6.3. Tarski's Achievements in the Field of Semantics and Methodology of Deductive Sciences

Tarski, inspired by Aristotelian tradition in philosophy and non-constructive manner of practicing foundations of mathematics in Poland, published in 1933 (first in Polish) his renown fundamental work on the notion of truth in languages of formalized sciences. This work was translated into German (1936), then English (1956), and next into many other languages (Tarski, 1933; Tarski, 1956). In it, Tarski formulated the semantic theory of truth constituting correct and substantively apt framework of the classical concept of truth.

Tarski proved as well, independently of Gödel, that a set of true expressions of consistent and sufficiently rich mathematical theory cannot be defined in the language of this theory. He introduced the notion of satisfaction by sequence of objects, which made the basic concept of theory of models. By that he laid the foundations of the model theory – one of the central parts of mathematical logic. Making reference to Bernard Bolzano, Tarski gave also the fundamental definition of logical consequence (expression A follows logically from the expressions of the set X if and only if each model of the set X is also a model of the expression A).

As a member of the WSL, Tarski significantly contributed to working out the foundations of syntax and semantics of formalized languages of deductive theories.

In 1930, based on two primitive notions: a well-formed expression and the consequence operation, Tarski built two axiomatic theories of deductive systems (general and a richer one for systems based on classical propositional logic). Within the two theories, he defined the conceptual apparatus relating to basic properties of deductive systems. Lindenbaum also took part in those studies and was the author of the so-called theorem of maximalization, which later in the 20th century became one of the most important tools of research on properties of logical systems.

6.4. Pioneering Studies on Alternative Forms of Formalization of Deductive System

Thanks to S. Jaśkowski's works, independent of G. Genzen, beside the purely axiomatic method of formalization of logical system, a new non-axiomatic method of characterizing such systems was introduced. It is called the natural deduction method and refers to the common natural practice of conducting proofs by mathematicians, but not only mathematicians. The method turns out to be very intuitive and useful in teaching. Later , it became helpful in computer-based testing of correctness of theorems proofs. Jaśkowski applied it not only to classical logic, but also to the intuitionistic one. A version of this method is the well-known method of semantic tables.

The axiomatic method can be complemented by the axiomatic method of rejecting non-acceptable or false expressions of a system. It leads to two-level formalization of the deductive system, on the one hand, as a system due to acceptance, *assertion system*, and, on the other hand, as a system with respect to rejection, *rejection/refutation system*. This dual form of formalization of the deductive system was introduced by Łukasiewicz and Słupecki (1939). It was they who also initiated research dealing with saturation of the logical system, later called by Słupecki its Ł-decidability. The two-level characterization of the logical system became with time a popularly applied method of its formalization and examination of its saturation.

6.5. History of Logic

Łukasiewicz revolutionized the history of logic, establishing the paradigm of studying it from the point of view of mathematical logic. His innovatory logical studies led to the statement that logic of the Stoics was the logic of sentences preceding Aristotle's logic and that it was a logic of "non-provable" forms treated as primitive rules of proving.

Another of Łukasiewicz's findings was the statement that Aristotle's logic was the first axiomatic system (even though a non-formalized one). Łukasiewicz formalized the asertoric logic of Aristotle and, together with Słupecki, characterized it also on the second level as a system wrt rejection/refutation (Łukasiewicz, 1939; Łukasiewicz, 1951). At the same time, he introduced into science the very notion of rejection of expressions itself and a new axiomatic rejection/refutation method.

6.6. A Mention of Other Achievements of The WSL

The above-mentioned outcomes achieved under the auspices of the WSL do not exhaust the repertoire of other significant ones, in particular those relating to relations arising between the very logical calculi and the relation of the calculi to different mathematical, algebraic, geometrical and topological structures.

Earlier, certain results connected with the foundations of mathematics and set theory were not mentioned. Still, some important results connected with elimination of quantifiers, with reference to the problem area of decidability of certain theories were obtained by members of the WSL (Tarski, Presburger, Mostowski). Also, studies were successfully conducted there, among others, on the strength and independence of the choice axiom (Tarski, Lindenbaum, Mostowski).

7. The Significance of the WSL in The History of Logic of the 20th Century

The WSL invested Polish logic of the interwar period with its unique tone. Warsaw was not the only center of logical research in Poland, though. Another important research institute in the field of mathematical logic and history of logic was based in Krakow. Nevertheless, the Krakow center never attained the rank comparable with that of the Warsaw School.

The WSL functioned until the outbreak of the Second World War in 1939. Thus, the development of Polish logic was broken at its peak moment. Shortly before the War, Leśniewski died. Also before the War, Tarski left Poland, emigrating to the United States. A few representatives of the WSL died as a result of wartime actions (Lindenbaum, Wajsberg, Presburger). After the War, Łukasiewicz settled in Dublin, Ireland, Lejewski in England, Sobociński in Notre Dame in the USA. Those who remained in Poland were Mostowski (Warsaw), Słupecki (first in Lublin, then Wrocław) and Jaśkowski (Toruń); they endeavored to cultivate the traditions of the WSL in the post-war Poland and in the new reality, yet were not able to achieve successes similar to those worked out by the WSL or ones which were obtained later in the renown California School founded by Tarski living in exile. Indeed, after the War, there was a new logical center established in Warsaw, managed by Andrzej Mostowski (his results concerning generalization of quantifiers are known especially well (Mostowski, 1957). Continuing Tarski's work, Mostowski's centre studied set theory, model theory, decidability, algebraic and topological methods in logic; however, this research center differed considerably from the School which was founded by Łukasiewicz and Leśniewski and which was also almost as close to philosophy as it was to mathematics. Polish logic never regained the stature that it could boast of in pre-war days.

By 'Polish logic', I meant logic in the narrower sense of the word 'logic', i.e. mathematical logic. The term 'Polish logic' was coined by S. McCall (1967) to underline the great contribution of Polish logicians of the interwar period, in particular members of the WSL, to the development of world logic.

The WSL went down in the history of the 20th century, and its achievements, significance and influence on the world logic were and still are vitally valid.

The achievements of the logicians belonging to the WSL, in the days of the School's development and flourishing, attracted a great number of young people to study logic in Warsaw. It was at the level of master's theses that unprecedented results were achieved there (Wajsberg, Słupecki). Heinrich Scholz of Münster, in his *Geschichte der Logik* (published in 1931; (Scholz, 1931, p. 87) stated long before the outbreak of the War that "Warsaw became the main center of logical studies."

In the lifetime of a generation, Polish logic grew from zero to top of international recognition. The significance of Polish logic of the interwar period was acknowledged already after the war by the authors of the well-known book *Foundations of Set Theory*, published in 1958 by A. Frankel, Y. Bar-Hillel and A. Levy (Fraenkel, Bar-Hillel, & Levy, 1958, p. 200). We can read in it, among others, that "Probably no other country, taking into account the size of its population, has contributed so greatly to the development of mathematical logic and foundations of mathematics as Poland" and that "this curious fact should be explained sociologically."

The achievements of the WSL logicians have survived. They were specifically mentioned in the 400-page volume entitled *Polish Logic 1920-1939*, published by S. McCall (1967) It contains 17 articles by Polish logicians and with the exception of two of them, all were written by authors connected with the WSL. At more or less the same time, there came out Łukasiewicz's *Selected Letters* (Łukasiewicz, 1961), edited and compiled by J. Słupecki, in the Polish language (in 1961), and then in English (in 1970) – Łukasiewicz's *Selected Works* (1970), edited by L. Borkowski. Half of them were Łukasiewicz's letters from the interwar period.

In 1962, the well-known monograph *The Logical Systems of Leśniewski* was published by E.C. Luschei (Luschei, 1962). All the works by Leśniewski were translated into English and collected in (Leśniewski, 1992). It is worth noting here that although all the original works written by Leśniewski (their reprints) were collected and edited in two volumes by J.J. Jadacki (2015).

An extensive selection of Tarski's works from the period of his activity in the WSL (the years 1923-1938) appeared in English under the title *Logic, Semantics, Metamathematics* (translated by J.H. Woodger) already in 1956; the year 1983 saw the second edition prepared by J. Corcoran enter the market (Tarski, 1956). Let us add that Tarski's logical-philosophical letters "Prawda" [The Truth] and "Metalogika" [Metalogic] were published by J. Zygmunt in Polish in 1955 (Volume 1) and 2001 (Volume 2) (Tarski, 1995).

Thus, works by the creators of the WSL: J. Łukasiewicz and S. Leśniewski and their outstanding disciple A. Tarski, as well as those by other disciples of the founders of the School, are now widely available. Owing to this, reaching for the works of representatives of the WSL, one can discover and appreciate anew the significance of the School of Polish logic in the 20th century, as well as find stimulation not only to solidify this significance, but also to work out solutions to new logical problems that arise.

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Notes

1. For the needs of this the study I made use of materials dealing with the Lviv-Warsaw School of Logic, the Warsaw School of Logic and Polish logic found in the following: (Słupecki, 1972), (Tkaczyk, Wybraniec-Skardowska, 2011), (Woleński, 1985), (Woleński,1985), (Woleński, 2015), (Woleński, 2009), (Jadacki, 2009), (Jadacki, Paśniczek, 2006), (Burdman Feferman, Feferman, 2004), (Wybraniec-Skardowska, 2009), (Wybraniec-Skardowska, 2018), (Zymunt, 1998).

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100 Years of Logical Investigations at the University of Poznań

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Abstract:
The aim of this paper is to describe the history of logical investigations at the University of Poznań. The organisational structures within the discipline as well as the outstanding logicians and their achievements are presented. Connections with the Lviv–Warsaw School are indicated.
Keywords: logic, University of Poznan, Lviv–Warsaw School.

1. Introduction

Logic was present at the university in Poznań since its very beginning. Though the academic tradition was present here already in the 16th century, the establishment of a university was possible only after the First World War in 1919 — the first inauguration took place on 7 May 1919. At the beginning, the university was called Piast University [*Wszechnica Piastowska*], but in 1920 it was renamed to the University of Poznań [*Uniwersytet Poznański*]. During the Second World War, staff and students of the university (expelled by the Nazis) founded an underground Polish University of the Western Lands [*Uniwersytet Ziem Zachodnich*]. The university was reopened in Poznań after the war. In 1955 the name was changed to Adam Mickiewicz University. In what follows, I will abstract from the changes of the name and say simply "University of Poznań".

The first chair of logic at the University of Poznań was the Chair of Theory and Methodology of Sciences [*Katedra Teorii i Metodologii Nauk*] at the Philosophical Faculty, founded in 1920. Its head was Władysław Mieczysław Kozłowski (1858–1935). The next year, the Philosophical Faculty was split into the Faculty of Humanities and the Faculty of Mathematics and Natural Sciences. In the former faculty, the Chair of Theory and Methodology of Natural Sciences and Humanities [*Katedra Teorii i Metodologii Nauk Przyrodniczych i Humanistycznych*] was established, with Władysław Mieczysław Kozłowski as its head.

Who was Kozłowski? Born in Kyiv, he studied medicine at St Vladimir University in Kyiv. In 1890, he received a Candidate of Sciences in botany (equivalent to a doctorate) at the Faculty of Natural Sciences of the University of Dorpat (today's Tartu in Estonia). After having moved to Warsaw, he was involved in editorial work for several years. In 1896–1898, he worked as a teacher of Polish among Polish immigrants in North America. In 1899, he obtained his doctoral degree in philosophy at the Jagiellonian University. In 1900, he presented his habilitation thesis to the Jan Kazimierz University of Lviv; however, he did not get *veniam legendi*, since, under political pressure, the ministry refused to accept his habilitation. Beginning in 1901, Kozłowski lectured at Université Libre in Brussels, and beginning in 1902, he was a docent of the University of Geneva. In 1905, he settled in Warsaw, where he taught philosophy in the Society of Science Courses [*Towarzystwo Kursów Naukowych*]. Beginning in 1919, he was a professor at the University of Poznań. His scientific interests included philosophy, sociology, history, and botany. However, he was known mainly as a philosopher and logician — he regarded problems from logic and the methodology and philosophy of various disciplines as fundamental. He proposed an original and interesting classification of disciplines. His lectures in logic and methodology given in Poznań enjoyed great interest; they were published as *Logika przyrodoznawstwa: Wykłady na Uniwersytecie Poznańskim* [Logic of natural sciences: Lectures at University of Poznań] in 1922.

Though Kozłowski knew quite well the new achievements of contemporary logic, his approach to logic was in fact traditional. He characterised logic as "the science about the activities of the mind which seeks truth" (Kozłowski, 1916, p. 8). The first chapter of his book *Podstawy logiki* was entitled "Thinking as object of logic" (Kozłowski, 1916, p. 22). He repeated this thought in *Krótki zarys logiki* [Short outline of logic], claiming that logic is a normative science whose task is "to examine the ways leading the mind to truth" (Kozłowski, 1918, p. 1). However, he stressed that logic:

> analyses mental operations conducted to reach the truth in a form that is so general that could be apply to any content. It investigates its form, separating it completely from the content. Logic shares this property with mathematics […]. […] This formal character, common to logic and mathematics, made these sciences close in their attempts, which were less or more developed, and led to the creation of mathematical logic. (Kozłowski, 1918, p. 8–9)

Finally, Kozłowski stated that logic can be defined as "the science about the forms of every ordered field of real or imaginary objects" (Kozłowski, 1918, p. 9). So he treated logic rather as a tool of science rather than an independent and autonomous discipline. His publications did not influence Polish logicians; however, his books were the first in Polish in which the theory of Boolean algebras and the theory of relations were presented.

In 1928, Kozłowski retired, and his chair at the Faculty of Humanities was cancelled. In the following academic year of 1929/1930, a new chair, the Chair of Theory and Methodology of Sciences [*Katedra Teorii i Metodologii Nauk*], was established at the Faculty of Mathematics and Natural Sciences. Its head became Zygmunt Zawirski (1882–1948).

Zawirski brought a new spirit and new ideas to Poznań. He was educated mainly in Lviv, where from 1901 to 1906 he studied mathematics, physics, and philosophy at Jan Kazimierz University. He completed his studies in Berlin (1909) and Paris (1910). Zawirski earned a doctorate in 1910 in Lviv under the supervision of Kazimierz Twardowski, the founder of the Lviv–Warsaw Philosophical School (Woleński, 1989). Then he taught mathematics and the propaedeutics to philosophy in various Lviv gymnasiums. He was habilitated in 1924 at the Jagiellonian University in Cracow on the basis of his thesis on the axiomatic method in the natural sciences. From 1924 to 1928, he lectured on philosophy at the Faculty of General Studies of the Lviv Polytechnic. And in 1928 he was appointed to the University of Poznań.

Zawirski, being a student of Twardowski, is treated as a member of the Lviv–Warsaw School. However, his scientific interests were not directly connected with the main trends of investigations of this school. He concentrated mainly on the methodology of sciences as well as the theory of cognition and ontology, especially on problems related to the development of physics — here he was interested in relativity theory and quantum theory. He was then the most outstanding Polish specialist in problems concerning the borderline of physics and philosophy. He was also interested in mathematical logic, especially in its applications. His Poznań period was the most creative in his scientific career.

Problems of logic were not at the centre of Zawirski's investigations. However, one should say two things here. Zawirski was interested in problems on the borderline between logic and mathematics, in particular in connections between them as well as in the problem of meaning of non-classical logics, first of all of many-valued logics and intuitionistic logic. He treated logic in a broad sense; hence, logic for him was not only a formal system (or collection of such systems), as he included here also studies on reasoning. This was in fact a reflection of contemporary tendencies in Poland (and not only there) in both investigations and didactics. In *Logika teoretyczna* [Theoretical logic], he wrote that "logic is a general science and it indicates a structure common to all disciplines, ways in which in particular domains their statements are justified" (Zawirski, 1938, p. 2). He also wrote:

> The name of the science, which is now called logic, comes from the Greek *logos*, i.e. 'word,' 'speech' and 'reason' as well as 'reasonable thinking'; the name of the science is associated exactly with the last meaning. Since it is not a science about reason but rather about forms of reasoning that we use in all deductions or argumentations. (Zawirski, 1938, p. 1)

Considering Zawirski's views connected with the problem of relations between mathematics and logic, one should mention first of all his paper "Stosunek logiki do matematyki w świetle badań współczesnych" [The relation between logic and mathematics from the point of view of contemporary investigations] (Zawirski, 1927). In it, he claimed that "Mathematics, as an exact science, was created much earlier than logic; the Greek had known how to construct proper mathematical proofs before systematic investigations on the essence of all logical deduction and argumentation began" (Zawirski, 1927, p. 171).

Emphasising the importance of the Stoics' logic, Zawirski claimed that it was more important to mathematics than Aristotle's logic. He appreciated the works of Leibniz, Peano, and Frege, whereas he refuted Kant's conception. Analysing Whitehead and Russel's work *Principia mathematica*, Zawirski stressed that it is of no greater importance whether the judgements of logic and mathematics are regarded as analytic or synthetic — what is important is the problem of the consistency and independence of axioms.

Zawirski stressed that mathematics and logic do influence our cognition of the world. Therefore, logic and mathematics are of significance for the natural sciences. He dedicated much attention to the problem of the axiomatisability of theories in physics.

As mentioned above, Zawirski was interested in intuitionistic logic. He devoted to it a paper "Geneza i rozwój logiki intuicjonistycznej" [The origin and development of intuitionistic logic] (Zawirski, 1946). It has rather an informational character, as the author limited himself to discussing in a very competent way the effects of other people's investigations, not mentioning his own sympathies or antipathies towards intuitionistic logic. He wrote about the basic ideas of Luitzen Egbertus Jan Brouwer, discussed Arend Heyting's attempts to construct a system of intuitionistic logic, and presented results of Kurt Gödel and Stanisław Jaśkowski on matrices adequate for this logic.

Zawirski greatly appreciated Jan Łukasiewicz's idea of many-valued logics. He was of the opinion that the new logic was the only way to understand the phenomena of the micro-world. Combining the ideas of Łukasiewicz and Emil Leon Post, he tried to construct a system of logic that would be proper to interpret both certain problems of contemporary physics and probability calculus. He presented his ideas in various papers (Murawski, 2011; Murawski, 2014). as well as during various conferences. At the International Congress of Scientific Philosophy in Paris in 1935, he met Hans Reichenbach, who had also worked on similar problems. It turned out that their approaches to probability calculus and non-classical logics were different. Reichenbach interpreted some expressions of probability calculus as a kind of generalised logic, whereas Zawirski outlined the parallelism between the expressions of probability calculus and formulas of the many-valued logics. In Zawirski's opinion, probability calculus and many-valued logic should be treated as two separate systems. He was convinced that such compatibility of many-valued logics, in particular three-valued logic, with probability calculus would allow its application in quantum mechanics. Let us add that further studies of this problem, in particular the investigations of Patrick Suppes and Paulette Destouches-Fevrier, followed just this direction. Therefore, Zawirski can be seen as a forerunner of quantum logic.

Zawirski directed the Chair of Theory and Methodology of Science till the end of 1936, and in 1937 he left the University of Poznań for Jagiellonian University in Cracow. He also left his students in Poznań. Among them were Franciszek Zeidler (1907–1972) and Zbigniew Jordan (1911–1977). Zeidler continued Zawirski's investigations of problems on the borderline of physics and philosophy. Jordan studied philosophy from 1930 to 1934 at the University of Poznań (Konstańczak, 2010). Under the influence of Zawirski, he got interested in the axiomatic method in philosophy. In 1936 he was granted a PhD on the basis of his dissertation *O matematycznych podstawach systemu Platona* [Mathematical foundations of Plato's system] (Jordan, 1937) — his supervisor was Zawirski. Jordan continued his studies in Bonn and Paris and prepared his *Habilitationsschrift* devoted to the problem of infinity. Unfortunately, the outbreak of the Second World War prevented the habilitation (the manuscript went missing). After the war, he worked in England and Canada, where his works were devoted mainly to the history of logic and philosophy.

After Zawirski left Poznań, there appeared a vacancy at the Chair of Theory and Methodology of Sciences. What happened then is explained by Anita Burdman-Feferman and Solomon Feferman in their book *Alfred Tarski: Life and logic*[1]:

> The Ministry of Education asked all the relevant professors in Poland to suggest a candidate to fill the vacancy, and Tarski was unanimously recommended. However, Poznan, always a stronghold of right-wing conservatism and dominated by the Catholic church, had, since Piłsudki's death in 1935, moved even farther to the right and become outright fascistic and anti-Semitic. Unanimous recommendations notwithstanding, Poznan University did not appoint Tarski, and since there would have been no way to appoint anyone else without making the reasons for denying him the professorship patently clear, the position was eliminated. (Feferman & Feferman, 2004, pp. 102–103)

Woleński explains this in the following way: "According to Hiż[2], the people in Poznań were afraid that Tarski would apply and win the competition. Poznań was perhaps the most anti-Semitic region in Poland. This would explain the situation" (Woleński, 1995, p. 400).

In fact, the position left by Zawirski was not filled in the period 1937–1939, and due to the outbreak of the Second World War, it remained unfilled until 1945. In 1945, the head of the chair became Kazimierz Ajdukiewicz (1890–1963), who rejected offers from universities in Warsaw and Cracow and decided to take the position in Poznań.

Ajdukiewicz studied philosophy, physics, and mathematics at the University of Lviv (Murawski, 2014). In 1912, he obtained there his doctoral degree under the supervision of Kazimierz Twardowski. He continued his studies (1913–1914) at the University of Göttingen, where he listened to lectures by Edmund Husserl, Leonard Nelson, and David Hilbert. The views of the latter had a considerable influence on Ajdukiewicz (cf. his *Habilationsschrift*). In the years 1919–1922, he worked as a teacher in a gymnasium in Lviv while conducting scientific research. In 1921, he completed his habilitation at the Philosophical Faculty of the University of Warsaw. From 1922 to 1925, he lectured as a private docent at the University of Lviv and taught in secondary schools in Lviv. In 1925, he became a professor at the University of Warsaw, and in 1928 he became a professor at the University of Lviv. In 1940–1941, he lectured on psychology at the Lviv State Medical Institute. During the Nazi occupation, he was active as an accountant while at the same time involved in underground education. In 1944–1945, he held the Chair of Physics at the Ivan Franko University in Lviv.

Ajdukiewicz is one of the outstanding representatives of the Lviv–Warsaw School. He had a significant influence on the development of logic and philosophy not only in Poland. When coming to Poznań, he was already widely known around the world. His scientific interests included mainly semiotics, epistemology, logic, and the general methodology of sciences.

Among Ajdukiewicz's main achievements is the conception of meaning, which formed the logical base of his radical conventionalism. Later he moved towards empiricism, stressing the role of experience and measurement in science. Formal logic was treated by him as a tool of philosophy making possible precise and strict considerations. Among his main achievements in formal logic, one should mention his proposal of the definition of a consequence (in a certain sense, it prepared the way for Tarski's definition), formulation of the deduction theorem, and considerations on the rule of infinite induction and the calculus of syntactic types. In methodology, he was interested in problems connected with practical logic (classification of reasoning or the problem of rationality of inferences). He proposed a new definition and classification of reasoning and considered non-deductive reasoning. Ajdukiewicz always represented anti-irrationalism, and in works published in his Poznań period, he criticised severely and explicitly various idealistic tendencies in philosophy. He referred in those critiques to logical analysis of discussed conceptions and attempted to indicate logical mistakes and errors. He also engaged in discussions with Marxist philosophy (prevailing at that time in Poland) and Marxist philosophers, defending his own philosophical views against attacks from opponents and suggesting some solutions in favour of their ideas.

Among Ajdukiewicz's main works published in the Poznań period, one finds the following papers: "Logika i doświadczenie" [Logic and experience] (Ajdukiewicz, 1947), "Zmiana i sprzeczność" [Change and contradition] (Ajdukiewicz, 1948), "Epistemiologia i semiotyka" [Epistemology and semiotics] (Ajdukiewicz, 1948), "Metodologia i metanauka" [Methodology and metascience] (Ajdukiewicz, 1948), "On the notion of existence" (Ajdukiewicz, 1951), "W sprawie artykułu prof. A. Schaffa o moich poglądach filozoficznych" [Concerning the paper by Professor A. Schaff on my philosophical views] (Ajdukiewicz, 1953), and "Klasyfikacja rozumowań" [Classification of reasonings] (Ajdukiewicz, 1955). Ajdukiewicz paid great attention to the problem of teaching logic. He wrote some excellent textbooks of logic and philosophy,[3] took part in discussions concerning the didactics of logic,[4] and organised meetings devoted to the teaching of logic and philosophy.

Ajdukiewicz was the head of the Chair of Theory and Methodology until 1955. In the meantime, the Faculty of Mathematics and Natural Sciences was transformed in 1951 into the Faculty of Mathematics, Physics, and Chemistry, and Ajdukiewicz's chair was renamed as the Chair of Logic [*Katedra Logiki*]. Ajdukiewicz created here a significant scientific centre in logic and philosophy. Many scholars from various Polish universities took part in logico-methodological seminars directed by him (Murawski & Pogonowski, 2008). Numerous papers in logic, methodology, and philosophy representing the highest scientific level were written here.

Ajdukiewicz's activity in editing scientific journals must be mentioned as well. During his Poznań period, the journal *Studia Logica* was founded. Ajdukiewicz was editor-in-chief, and Roman Suszko the first secretary of the editorial board. The journal *Studia Philosophica* was also published in Poznań and was co-edited by Ajdukiewicz in the period 1935–1951. One can certainly say that Ajdukiewicz really instilled in Poznań the spirit of the Lviv–Warsaw School.

As said above, Ajdukiewicz left the University of Poznań and moved to Warsaw University in 1955. However, he left behind some of his collaborators and students who continued his tradition. Among them were Seweryna Łuszczewska-Romahnowa (who became the head of the Chair), Roman Suszko, Zbigniew Czerwiński, and Andrzej Malewski.

Roman Suszko (1919–1979) studied physics, mathematics, and chemistry from 1952 to 1956at the University of Poznań and during the war at underground schools in Cracow. In 1945 he obtained a master's degree in philosophy at Jagiellonian University under the supervision of Zawirski, and in 1946 he started work in Ajdukiewicz's Chair of Theory and Methodology of Sciences at the University of Poznań. Here in 1948 he obtained a doctoral degree under the supervision of Ajdukiewicz and in 1951 the habilitation. He was also, as mentioned above, the secretary of the editorial board of *Studia Logica*. In 1952, Suszko left Poznań and moved to Warsaw (to the Chair of Logic at the Philosophical Faculty of the University of Warsaw). His papers written during his Poznań period were devoted to logical rules of reasoning and their relations with laws of logic, the theory of mathematical definitions, as well as some problems connected with the theory of axiomatic systems. In particular, he considered systems of logic without axioms but with appropriate finitistic inference rules. His *Habilitationsschrift* "Canonic axiomatic systems" (Suszko, 1951) was devoted to the explication of the Skolem paradox and contained general metatheoretical considerations regarding models of axiomatic theories, in particular models of set theory. During his work in Poznań, Suszko also published a few other minor papers, in particular a critical discussion of logical positivism (Suszko, 1952), and began his work on diachronic logic.

Zbigniew Czerwiński (1927–2010) studied law and economy from 1945 to 1949 and logic from 1950 to 1952 in Poznań, and in 1952 he became assistant to Ajdukiewicz. In his works devoted to logic, Czerwiński was interested mainly in the theory of induction and its connections with the statistics and theory of games. He wrote also about the paradox of implication and about deductive reasonings. Later, his scientific interests moved towards problems of economy and econometrics (starting in 1961 he was at the Higher School of Economics in Poznań) — he was mainly interested in applications of mathematics and statistics in economy.

Andrzej Malewski (1929–1963) was assistant in Ajdukiewicz's chair, and in 1956 he moved to the Institute of Philosophy and Sociology of the Polish Academy of Sciences in Warsaw. However, he collaborated with Jerzy Topolski, a historian from the University of Poznań; they were interested in the methodology of historical sciences. Malewski also wrote an interesting and popular handbook of logic: *ABC porządnego myślenia* [ABC of a proper thinking] (Malewski, 1957).

As said above, the successor of Ajdukiewicz as the head of the Chair of Logic became Seweryna Łuszczewska-Romahnowa (1904–1978), his student from Lviv (Murawski & Pogonowskim, 2018). There she studied philosophy and mathematics under Twardowski, Ajdukiewicz, and Roman Ingarden (philosophy), as well as Hugo Steinhaus and Stefan Banach (mathematics). In 1932 she obtained a doctoral degree. Her real supervisor was Ajdukiewicz; however, for formal reasons, the official supervisor was Kazimierz Twardowski. She then started to work at the Chair of Philosophy of the University in Lviv, whose head was Ajdukiewicz. In 1943 she was arrested by the Gestapo and sent to concentration camps in Majdanek, Ravensbrück, and Buchenwald. In December 1946, she came to Poznań, and in 1947 she started work at the Chair of Theory and Methodology of Sciences of the University of Poznań, directed by Ajdukiewicz. In 1970 the chair was incorporated into the newly founded Institute of Mathematics and renamed the Department of Mathematical Logic [*Zakład Logiki Matematycznej*].

Łuszczewska-Romahnowa worked mainly in mathematical logic, methodology, and the history of logic. Due to her dramatic experiences during the war, she published relatively few papers. However, one can recognise in her works the influence of her studies in Lviv. This can be seen in particular in the synthesis of analytical philosophy and logic characteristic of her style of writing.

As her main works, one can mention "Wieloznaczność a język nauki" [Polysemy and the language of science] (1948), devoted to the problem of the ambiguity of concepts used in the language of science (Łuszczewska-Romahnowa, 1948); "Analiza i uogólnienie metody sprawdzania formuł logicznych przy pomocy diagramów Venna" [An analysis and generalisation of Venn's diagrammatic decision procedure] (Łuszczewska-Romahnowa, 1953), in which she proposed a method of checking the decidability of the first-order monadic predicate calculus; and papers dealing with argumentation theory (Łuszczewska-Romahnowa, 1962; Łuszczewska-Romahnowa, 1964) and with the problem of induction (Łuszczewska-Romahnowa, 1957). She also wrote papers on multi-level classifications and on the distancefunctions connected with such classifications (Łuszczewska-Romahnowa, 1961; Łuszczewska-Romahnowa & Batóg, 1965; Łuszczewska-Romahnowa & Batóg, 1965).

Łuszczewska-Romahnowa retired in 1974. Her successor as the head of the Department of Mathematical Logic was Tadeusz Batóg (born 1934). He studied Polish philology at the University of Poznań. In 1956–1957 he was an assistant at the Chair of Logic at the Philosophico-Historical Faculty, and in 1957 he moved to the Chair of Logic at the Faculty of Mathematics, Physics, and Chemistry. Here he was awarded the doctor degree in 1962 and habilitation in 1968. His scientific interests belong to applications of mathematical logic and set theory to theoretical linguistics (in particular to phonology), methodology, the history of logic, the philosophy of mathematics, and the history of philosophy. He is also the author of works on the history of literature. Concerning his logical achievements, one should mention his monograph *The axiomatic method in phonology* (Batóg, 1967) in which an axiomatic-deductive system of theoretical phonology was presented and developed. This system was based on type theory and an extended mereology. Batóg also published (together with his wife Maria Steffen-Batogowa) an extensive *Słownik homofonów polskich* [Dictionary of Polish homophons] (Steffen-Batogowa & Batóg, 2010). As examples of his analyses devoted to the methodology and philosophy of mathematics, one should mention at least two studies: *Dwa paradygmaty matematyki* [Two paradigms of mathematics] (Batóg, 1996) and "Kantowska filozofia matematyki a paradygmat Euklidesa" [Kant's philosophy of mathematics versus Euclidean paradigm] (Batóg, 2015). Batóg also published a handbook of logic, *Podstawy logiki* [Foundations of logic] (Batóg, 1986), which enjoyed and still enjoys great interest — it was and still is used in courses of logic for students of mathematics, as well as students of philosophy and the humanities in general.

In the 1970s there were several students of mathematics who were interested in logic and the foundations of mathematics. They became assistants in the Department of Mathematical Logic, directed then by Batóg. In this way, the number of members of this department grew, and the scope of interests and the spectrum of scientific investigations were significantly extended. Among those members were (in chronological order) Roman Murawski, Wojciech Zielonka, Wojciech Buszkowski, Zygmunt Vetulani, Wojciech Nowakowski, and Jerzy Pogonowski. Later, they were joined by Maciej Kandulski, Izabela Bondecka-Krzykowska, and Kazimierz Świrydowicz (the latter moved here from the Departmet of Legal Applications of Logic — see below). Their fields of scientific interests were broad. Buszkowski, Zielonka, and Kandulski worked mainly in the logical theory of categorical grammars and Lambek calculus. Buszkowski's and Kandulski's interests included also substructural logics, algebra of logic and its applications in computer science, and mathematical linguistics. Murawski worked in mathematical logic and the foundations of mathematics, in particular in the theory of models of arithmetic. Nowadays, he deals mainly with the philosophy and history of logic and mathematics. Vetulani started from the foundations of mathematics and later moved towards problems of computer linguistics. Pogonowski was interested mainly in applications of logic in linguistics. Świrydowicz dealt at the beginning with legal applications of logic and then moved to non-classical logics and algebraic methods in logic. Bondecka-Krzykowska's interests include the history and philosophy of computer science and of mathematics, as well as didactics of computer science.

From the Department of Mathematical Logic evolved in 1993 the Department of Theory of Computation [*Zakład Teorii Obliczeń*] (head: W. Buszkowski) and the Department of Computer Linguistics and Artificial Intelligence [*Zakład Lingwistyki Informatycznej i Sztucznej Inteligencji*] (head: Z. Vetulani). Pogonowski moved to the Institute of Linguistics and founded there the Department of Applied Logic [*Zakład Logiki Stosowanej*]. Nowadays, he is in the Department of Logic and Cognitive Science [*Zakład Logiki i Kognitywistyki*] at the Faculty of Psychology and Cognitive Science. Batóg was the head of the Department of Mathematical Logic till 1996; his successor was R. Murawski.

The Department of Mathematical Logic at the Faculty of Mathematics and Natural Sciences and the Faculty of Mathematics, Physics, and Chemistry was not the only centre of logical investigations at the University of Poznań. Since 1952, there is also the Chair of Logic [*Katedra Logiki*] at the Philosophico-Historical Faculty. It refers to the tradition of the Chair of Theory and Methodology of Natural Sciences and Humanities, which existed in the 1920s and whose head was Kozłowski (see above). The first head of the Chair of Logic was Adam Wiegner (1889–1967). He studied philosophy, mathematics, and psychology from 1909 to 1914 at Jagiellonian University, where in 1923 he was awarded a doctoral degree in philosophy. Beginning in 1928 he was at the University of Poznań, where in 1934 he was given the habilitation. After the Second World War, he became the head of the Chair of Philosophy (reactivated in 1945 at the Faculty for Humanities and renamed in 1951 as the Chair of History of Philosophy). From 1952 till his retirement in 1960, he directed the Chair of Logic.

The scientific interests of Wiegner were broad and included the history of philosophy, epistemology, ontology, psychology, philosophical foundations of physics, and formal logic. Most important were his achievements in epistemology. His logical works were devoted to the modern treatment of so-called traditional logic; however, they were far from current investigations in logic. He defended the principle of reciprocity between the contents and the extension of a notion. He also claimed that sources of some critics of this principle can be seen in terminological mistakes and in unsound assumptions concerning the concept of richness of the contents. He attempted to axiomatise the traditional logic — he developed and improved the result of Ajdukiewicz by extending his system by an axiom ensuring the non-universality of all considered names. Wiegner also carried out an analysis of important concepts of philosophical logic, such as abstraction, generalisation, idealisation, and concretisation. His analyses significantly influenced the methodological reflection undertaken later in Poznań.

Wiegner was an author of two handbooks of logic: *Elementy logiki formalnej* [Elements of formal logic] (Wiegner, 1948) and *Zarys logiki formalnej* [An outline of formal logic] (Wiegner, 1952). They were written in a very accurate way. He proposed axiomatics for propositional calculus, which turned out to be of didactic value (it has two primitive notions, namely conjunction and negation, and is based on four axioms).[5]

Wiegner retired in 1960, and the head of the Chair of Logic at the Philosophico-Historical Faculty became Jerzy Giedymin (1925–1993). He studied English philology from 1945 to 1950 in Cracow and in Poznań as well as economics in Poznań. In 1953 he became assistant in Wiegner's chair. In 1951 he earned a doctorate under the supervision of Wiegner, and in 1960 he was awarded the habilitation. He considered himself a pupil of Kazimierz Ajdukiewicz and Karl R. Popper; he took part in Ajdukiewicz's seminars in Poznań and attended Popper's seminar at the London School of Economics during his stays as a scholar in London in the late 1950s. In 1968, Giedymin left Poznań and moved to Great Britain. Beginning in 1971 he was professor at the School of Mathematical and Physical Sciences at the University of Sussex. His works from his Poznań period were devoted to various problems of the methodology of empirical sciences as well as to some methodological problems of social sciences. In his main work from that period, *Problemy, założenia, rozstrzygnięcia* [Problems, assumptions, decidability] (Giedymin, 1964), he dealt with the general theory of questions and with the empirical methodology, in particular of social disciplines referring to it. He also wrote (together with Jerzy Kmita) a handbook of logic: *Wykłady z logiki formalnej, teorii komunikacji i metodologii* [Lectures on formal logic, theory of communication, and methodology] (Giedymin & Kmita, 1966).

After Giedymin left Poznań, the head of the Chair of Logic became Jerzy Kmita (1931–2012). The chair was incorporated as the Department of Logic and Methodology [*Zakład Logiki i Metodologii*] into the Institute of Philosophy, founded in 1970. Its head was Kmita, and after him Włodzimierz Ławniczak and later on Paweł Zeidler.

Under the direction of Kmita, this centre of logic became a vivid and creative centre of investigations in the methodology, especially the methodology of humanities. One should mention here the collaboration with philosophers interested in methodology (Jan Such, Leszek Nowak) and with the historian Topolski. They founded the so-called Poznań Methodological School. Main problems considered by members of this group included analysis of fundamental concepts of the theory of literature by applying the conceptual apparatus of logical semantics, analysis of methods of explanation of facts and phenomena as well as of justification of theses in humanties, analysis of methodological assumptions of Karl Marx's *Kapital*, and investigations concerning theory and methodology of the history of art.

Members of this group were also authors of important handbooks. One should mention here in particular *Wykłady z logiki i metodologii nauk dla studentów wydziałow humanistycznych* [Lectures in logic and methodology of science for students of faculties of humantities] by Kmita (Kmita, 1975) and *Wstęp do metodologii ogólnej nauk* [Introduction to general methodology of science] by Such (Such, 1973). From that group came also Andrzej Wiśniewski, who was interested mainly in erotetic logic and epistemic logic (he is now in the Department of Logic and Cognitive Science at the Faculty of Psychology and Cognitive Science).

Besides groups of logicians at the University of Poznań described above, still one more should be mentioned here: the Department of Legal Applications of Logic [*Zakład Prawniczych Zastosowań Logiki*] at the Faculty of Law. Its head was outstanding legal theorist and logician Zygmunt Ziembiński (1920–1996). He dealt with logical problems of jurisprudence, applications of deontic logic in legal reasoning, and logic of norms. He also wrote a famous handbook of logic for students of law: *Logika praktyczna* [Practical logic] (published for the first time in 1956; since then the book has had many editions).

The above panorama of logical investigations at the University of Poznań shows that logic was present there from the very beginning and that it was intensively developed and kept pace with other centres in the world. The broad spectrum of concepts and problems studied there should be stressed. One developed there not only mathematical and formal logic but also logic connected with specific problems of humanities, natural sciences, or jurisprudence. Logical investigations were connected with methodological considerations and studies. The latter concern both formal (mathematical) disciplines as well as natural sciences and humanities. Specific philosophical problems of particular disciplines were also studied. Note that Poznań logicians were usually educated in several disciplines, which made such studies easier. In the interwar period, the scope of logical investigations and the obtained results had a rather local character. The situation changed radically after 1945: logic developed in Poznań was situated in the main trend of its development in the world, and results obtained here were known and quoted in the literature by specialists abroad.

It can be said that, in a certain sense, the Poznań centre of logic was in fact a continuation or a part of the Lviv–Warsaw School and in particular of the Warsaw School of Logic (which formed a part of the former) (Woleński, 1989). Some members of the school (students of Twardowski or students of students of his) were active at the University of Poznań, in particular Zawirski before the war and Ajdukieiwcz and Łuszczewska-Romahnowa after the war. They brought here the spirit of this school and instilled its tradition. It can be seen in the supervisor–doctoral student relations and, above all, in directions and tendencies of investigations undertaken here and in promoted methods, in the understanding of logic as a discipline and of its meaning for other disciplines, as well as in the importance attached to didactics of logic. The spirit of the Lviv–Warsaw School influenced the next generations of scholars active in the field of logic in Poznań.

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Notes

1. No documents concerning this problem survived in the archives of the University of Poznań (cf. Michalski, 2012, pp. 131–132).
2. Henryk Hiż (1917–2006), logician and philosopher. He studied at the University of Warsaw, where he was a student of Tadeusz Kotarbiński. In 1950, Hiż left Poland. He lectured at various universities, in particular at the University of Pennsylvania in Philadephia. He had strong connections with Tarski, first as a pupil at the gymnasium in Warsaw and later as *protégé* in the USA [my remark – R.M.].
3. Let us mention here *Zagadnienia i kierunki filozofii* [Problems and trends in the philosophy] (Ajdukiewicz, 1949) and *Zarys logiki* [The outline of logic] (Ajdukiewicz, 1957).
4. Let us mention here the discussion which took place in the journal *Myśl Filozoficzna* [Philosophical thought] in the 1950s. Among its participants were leading Polish logicians (Ajdukiewicz, Andrzej Grzegorczyk, Klemens Szaniawski, Roman Suszko), as well as Marxist philosophers (e.g., Adam Schaff). This discussion was important not only from the point of view of teaching logic but also for ideological reasons.
5. For details, see, for example, (Murawski & Pogonowski, 2008).

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Logic and Metalogic: a Historical Sketch

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Abstract:
This paper briefly discusses the relations between logic and metalogic in history. Metalogic is understood as a reflection on logic in its various senses, particularly *sensu stricto* (formal, mathematical) and *sensu largo* (formal logic plus semantic plus methodology of science). It is shown that metalogic in its contemporary understanding arose after mathematical logic had become a mature discipline. Special passage is devoted to metalogic in Poland. The last part of the paper discussed so-called logocentric predicament.
Keywords: metaphysics, metamathematics, logic *sensu largo*, logic *sensu stricto*, philosophy.

1. Introduction

Six words with "meta" at the front have a philosophical significance as related to some specific fields of research. They are: metaphysics, metaphilosophy, metaethics, metascience, metalogic and metamathematics. Doubtless, the first is the most popular – it refers to study of being qua being, one of the most respectable philosophical problems; roughly speaking (but I omit various conceptual and historical issues) the subject of metaphysics can be identified with the scope of ontological investigations. It was probably Andronicos of Rhodes, the scholar of the Peripatetic philosophical school (the Lyceum) in the second half of the 1st B. C., who introduced the phrase *Τὰ μετὰ τὰ φυσικά* (*ta meta ta physika*; what comes after physics – the book called *Physika* was the first item in Andronicos' catalogue of Aritotle's books) as the title for Aristotle's books devoted to the first philosophy (other characterizations include theology, wisdom or the science investigating the first causes of things). Thus, the Greek counterpart of the word "metaphysics" arose after some simplification of the classificatory Andronicos' locution, that is, in a rather accidental way. Medieval philosophers translated *Metaphysika* as *Metaphysica* – this word was employed as the title of Aristotle's book on *prote filosofia* (just the first philosophy). However, the word *metaphysica as* well as its various counterparts in other languages, like German *Metaphysik* or Polish *metafizyka*, lost its meaning as related to ordering the Stagirite writings and began to denote the part of philosophy investigating being, its kinds and properties.

The above mentioned terminological circumstances blocked using the word "metaphysics" as referring to a theory of physics, directed to one of particular sciences. According to the most common view, metaphysics studies the world and its objectual furniture. Other above listed meta-nouns refer to considerations on something (a domain) to which the word standing just after the prefix "meta" refers. Consequently, metaphilosophy is about philosophy, metaethics about ethics, metascience about science, metalogic about logic and metamathematics about mathematics. We can eventually distinguish first-order disciplines (FOD, for brevity) and second-order ones (SOD, for brevity) – the latter are about of the former. Consequently, philosophy, ethics, science, logic and mathematics are first-order, but metaphilosophy, metaethics, metascience, metalogic and metamathematics – second-order. If we consider FOD as theoretical (I abstract here from entering into a definition of the word "theory), we can distinguish between the theoretical level (that is FOD) and the metatheoretical level as identified with SOD – theories are about the world, but metatheories about theories. Still another aspect appears in saying FOD are expressed in the object language, but metatheories in the metalanguage.

There appear various problems concerning relations between FOD and SOD. In particular, one can ask whether the methodological status is meta-disciplines is the same as disciplines related to them. Is metaphilosophy, a part of philosophy, metaethics – of ethics, metascience – of science, metalogic – of logic and metamathematics – of mathematics. Clearly, it requires further conceptual elaborations concerning all mentioned fields. For instance, logical empiricist defined science as a set of sentences satisfying some epistemic constraints, like the principle of testability, but metascience investigate syntactic and (in the later account of this movement) semantic properties of scientific locutions. However, other approach considers science as constituted by activities of scientists in the academic sense, and metascience as logical, sociological and psychological studies on science – the former belong to FOD, but the latter to SOD. Furthermore, so-called normative ethics aims to formulate ethical norms and evaluations – some authors consider ethical scientific theories to be possible, others deny such a possibility. However, both sides agree that metaethics is fairly legitimate in which concepts employed in normative ethics are analyzed. Perhaps the most dramatic situation occurs in philosophy. Metaphilosophy is certainly considered as a part of philosophy. According to Wittgenstein (Wittgenstein, 1922, 4.111).

Philosophy is not one of the natural sciences. (The word "philosophy" must mean something which above or below, but not beside the natural sciences).

Husserl and his followers treat philosophy as a super-science staying above natural science, but thinkers, like logical empiricists, consider philosophy as located below mathematics, physics or sociology. Yet philosophical reflection on philosophy itself is accounted as a part of the latter by both parties, identified by Wittgenstein as seeing philosophy as being (staying) above or below natural sciences. Logical empiricists, directly inspired by Wittgenstein's quoted metaphilosophical remarks, accused the traditional philosophy as mostly meaningless (=unscientific) metaphysics. This pejorative qualification of metaphysics has ancestors in Hume and Kant, although the borderline between science and metaphysics was (and still is) drawn differently in each case. Anyway, the problem of how FOD is related to that denoted by the acronym SOD is important in each specific case.

Aristotle's syllogistic was the first fully developed logical theory. Various metalogical rules, for instance, that a correct syllogism must have at least general premise, supplemented theorems of this system. The Stagirite also offered a theory of non-deductive inferences and commented on the question of their value in accommodating truths about the world. He elaborated various philosophical problems, for instance, a general definition of truth and its application to future-contingents. Although Aristotle did not speak about logic *sensu stricto* (in the narrow sense; formal logic) and logic *sensu largo* (in the wide sense; semantics plus formal logic plus methodology of science), this distinction is present in his writings, similarly to the Stoics. Medieval logicians worked in all domains of logic *sensu largo.* John of Salisbury prepared the book *Metalogicon*, but it was rather a textbook of practical logic and its role in human thinking; consequently, he cannot be considered as an anticipant of metalogic in the contemporary sense. Petrus Hispanus attributed the universality property to logic saying that *dialectica* (that is, logic) *est art atrium et scientia scientiarum ad omnium aliarum scientiarum methodorum principia viam habent* (logic is science of sciences, which provides the methodological principles for all other sciences). Theory of *consequentiae* and *suppositiones* or Occham's nominalism illustrate metalogical themes in the Middle Ages, Leibniz and his ideas of *calculus rationaticator* and *characteristica universalis* can be considered as further examples of logico-metalogical considerations, being an anticipation of so-called *logica magna* (grand logic), a system covering at least the entire mathematics, if not knowledge at all. Kant's account of logic (I abstract from his later idea of transcendental logic) as analytic was a methodological view with an explicit philosophical flavour. Fichte and Bolzano tried to develop *Wissenschaftslehre* (theory of science) with formal logic as its part. The word "metalogic" as referring to logical systems, their nature, their properties, relations to other fields, etc. began to be used in the 19th century, mostly by philosophers from the Neo-Kantian School (Rentsch, 1980); many historical facts are noted in Boos, 2018). Some strange uses occurred as well, for example, Ernest Troeltsch, a distinguished German historian, referred this word to methods of concrete historical investigations as metalogical, and Walter Harburger, a German composer and musicologist, the author of the book *Die Metalogik* (1919), was speaking about metalogic as the logic of music. Yet such usages became forgotten in the course of time.

Mathematics appeared as a separate science in ancient Greece even before logic, namely not later in the Pythagorean School, and very soon achieved a remarkable stage of development, culminating in the antiquity in works of Euclid, Archimedes and Claudius Ptolemy. Such considerations as treating numbers as the *ache* of the reality, seeing geometry as the basic mathematical theory, the invention of deductive method, Plato's account of mathematical objects as abstracts or investigating the relations of the fifth axioms of Euclid to other postulates can be taken as examples of ancient metamathematics (Schütte, 1980) as a brief presenting more facts, also from the subsequent history). Medieval metamathemical reflection did not develope very much, due to very poor development of mathematics itself. The situation changes in the 16th century and later, of course, after the new mathematical discoveries of Descartes (analytic geometry), Newton (calculus) and Leibniz (also calculus). Berkeley's critique of the concept of infinisitemals was a philosophical-metamathematical analysis. Kant's view that mathematics is, contrary to logic, synthetic a priori, Attempts to prove the parallel axiom from other geometrical assumptions motivated meta-geometry as the first systematic metamathematical theory. However, some mathematicians, even very eminent, like Gauss, strongly protested against the word "metamathematics" as suggesting metaphysical speculations to be avoided by the real sciences. The construction of models for Non-Euclidean geometry (Beltrami, Riemann) convinced mathematicians that metamathematical reflection on mathematics is fruitful and should be continued. This stage was concluded by the rise of set theory (Cantor), program of arithmetization of analysis (Dedekind, Weiestrass) and the axiomatization of geometry (Hilbers) as well as arithmetic of natural numbers (Dedekind, Peano). Mathematical logic developed concurrently to the mentioned novelties in mathematics, firstly as algebra of logic (Boole, Schröder) and secondly as consisting of propositional calculus and quantification (predicate) logic (Frege, Russell) as axiomatic systems.

Since formal properties, like axiomatization, completeness, consistency or independence of axioms, appeared to be essential, this immediately directed logicians' attention to metalogical issues. Three other circumstances strengthened interests in metalogic and metamathematics in 1900-1939. Firstly, logical antinomies had to be solved, which needed various subtle logical investigations, for instance, concerning logical types. Secondly, three leading programs in the foundations of mathematics, namely logicism (a reduction of mathematics to logic), formalism and intuitionism, required a logical elaboration. In the case of logicism (Frege, Russell), the relation of logic and set theory was crucial (resulting systems might be considered as *logica magna* – Leśniewski's logic belongs to this group as well), in the case of formalism (more precisely in the version of this project as presented by Hilbert), the explanation of the scope of finitary methods, and in the case of intuitionism – the logic of constructive methods in mathematics. Hilbert's program inspired metamathematics (metalogic was understood as a part of metamathematical research) much more than other mentioned views, because it claimed that mathematical systems should be investigated by explicitly formulated formal means. Consequently, metalogic became a part of metamathematics (Gödel and Tarski worked within these frameworks). For logicism, the former was still a mixture of mathematics and philosophy. Brouwer was not interested in logic very much – his foundational project was based on some very speculative philosophical ideas about time-intuition. Intuitionistic logic achieved a mature shape in Heyting's hands in 1930s. Thirdly, various alternative formal logical systems (logic as formalized by Frege and Russell was identified as classical) were proposed in the period in question. C. I. Lewis offered systems of modal logic (or based on strict implication), Łukasiewicz and Post constructed many-valued logic and (see above) intuitionistic logic arose as a device of formalization of intuitionistic mathematics. The plurality of logics generated several metalogical problems, like comparisons of proposed schemes or the question, if any of them is correct in a sense. Łukasiewicz argued that, at least in the case of many-valued logic, the difference between it and two-valued (classical) logic does not concern this or that theorem, in particular, the law of excluded middle, but the principle of bivalence, that is, a fundamental metalogical principle.

Metalogic flourished in Poland. We read (Łukasiewicz, Tarski, 1930, pp. 38, 59):

> In the course of the years 1926-1930 investigations were carried out in Warsaw belonging to that part of metamathematics – or better metalogic – which has as its field of study the simplest deductive discipline, namely the sentential calculus. These investigations were initiated by Łukasiewicz; the first results originated both with him and with Tarski. In the seminar for mathematical logic, which was conducted by Łukasiewicz in the University of Warsaw beginning in 1926, most of the results stated below of Lindenbaum, Sobociński, and Wajsberg were found and discussed. The systematization of all the research and the clarification of concepts concerned was the work of Tarski. […]. In conclusion we would like to add that, as the simplest deductive discipline, the sentential calculus is particularly suitable metamathematical investigation. It is to be regarded as a laboratory in which metamathematical methods can be discovered and metamathematical concepts constructed which can then be carried over to more complicated mathematical systems.

Simultaneously, Tarski papers on metamathematics (Tarski, 1930; Tarski, 1930) appeared. One can find in these writings investigations on various metamathematical concepts and problems, like deductive system, consequence operation or logical matrix. These results established to a great respect the position of metamathematics and metalogic in mathematical community. It is perhaps worth noting that Polish logicians did not assume any particular system of the foundations of mathematics. Following the tradition of Polish Mathematical School they admitted any accepted mathematical method in order to carry out investigations on logical and mathematical systems – a free use of a controversial axiom of choice is a good example of this attitude. In other words, "Polish" metamathematics was not logicist, formalist or intuitionistc as well as not bounded by any general philosophical view as it occurred in the Vienna Circle syntactic approach (Carnap, 1934).

In the last part of the present paper I would like to make some remarks about a problem from the borderline of metalogic and philosophy. Sheffer observed the following situation (Carnap, 1934, p. 218) "In order to give an account of logic, we must presuppose and employ logic." He called this dependence "the logocentric predicament." Clearly, it concerns the relation between FOD and SOD as restricted to logic itself. The difficulty consists in the fact that we can either suffer from an regressum ad infinitium or vicious circle. Assume that **L** is logic, which is analysed in metalogic, that is, $\mathbf{L}^{\mathbf{M}}$. In order to explain the validity of **L** (more exactly, its theorem), we need to use logical rule in $\mathbf{L}^{\mathbf{M}}$. However, in order to do that, we must either go to $\mathbf{L}^{\mathbf{MM}}$ (the third level) or to fall into vicious circle. Since the latter outcome is not good, we return to the former, but in consequence, we need to step into $\mathbf{L}^{\mathbf{MMM}}$ and so on. How to resolve this dilemma? Ajdukiewicz (see 1.) proposed the following solution of a dilemma stated by the Sceptics. According to this philosophical truth, any correct truth-criterion **C** is problematic, because in order to use **C**, one needs a criterion, let denote it by $\mathbf{C}^{\mathbf{C}}$, by that **C** is good. Yet $\mathbf{C}^{\mathbf{C}}$ either leads to the infinite regress or is to be blamed for circularity. According to Ajdukiewicz, it is enough to use, but it is unnecessary to know that **C** is good. This idea as applied to the logocentric predicament suggests that it is enough to apply logical rules without knowledge that they are logically valid. This solution can be supplemented by the following observation. We can (see [19]) define logic as universally valid, that is, true (correct) in all possible models (world). Consequently, **L** is also valid in $\mathbf{L}^{\mathbf{M}}$. If one observes that the universality property is defined in $\mathbf{L}^{\mathbf{M}}$ and uses **L,** which is problematic without its grounding in $\mathbf{L}^{\mathbf{M}}$, we can observe that logic is used before being grounded, but this situation does not generate any theoretical objection. For instance, we can say that logic is genetically inborn in our mental capacities. This example shows a connection between metalogic and fundamental epistemological problems.

2. Conclusion

This paper does not claim to be, even approximately, an exhausted treatment of metalogic and its history. See, for instance (Bedürftig & Murawski, 2018), (Beth, 1968), (Irvine, 2009), (Jacquette, 2007) for detailed treatments.

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Proof of the Existence of Hell:
An Extension of the Stone Paradox

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Abstract:
As shown in (Łukowski, 2013), the paradox of the stone is a failed attempt to show that "omnipotence" is a contradictory concept. An element of the argument presented there is that God, while unable to lift the stone, can nevertheless annihilate it. This work considers the amplification of the paradox of the stone to the form generated by the question: can God create a stone which He will not be able to lift, nor, once created, will He be able to destroy.
Keywords: stone paradox, omnipotence of God, mercy of God, free will, apokatastasis, empty hell, emptiness of hell.

1. Introduction

Although, according to the teaching of the Catholic Church, God is merciful and just, quite often two alternative visions of God clash in the minds of many Christians. According to the first, God is just: He damns unrepentant sinners to hell for their sins. In this vision, the mercy of God plays no major role, practically speaking: it exists, but it is without consequence. According to the second vision, God is merciful. He forgives sinners, because He must forgive: He loves human beings so much. In this vision, divine justice does not play a major role – it, too, exists, but without consequence. It is also easy to see that more and more often the first vision loses to the second in Christians' minds – to the point, supposedly, that the greatest authorities of the Catholic Church have proclaimed the validity of the thesis that hell does not exist, or, even worse, that it is empty. And this is so despite the fact that, in the discussions that theologians have about universal salvation, the dominant view seems to link God's justice to His mercy; this is only strengthened by the hope of universal salvation.[1]

This paper proposes a logical analysis of this theological problem based on the Catechism of the Catholic Church in the sense that premises based on the Catechism will be adopted and then conclusions will be drawn from them. This means that conclusions should be accepted by anyone who accepts the rationale. In this sense, belief in the thesis that hell exists is conditional; it depends on belief in the underlying assumptions. The essay is written in the convention of a logical analysis of a theological problem, similar to the publications of Jan Woleński (2005) or Ireneusz Ziemiński (2013).

The purpose of this article is not to resolve the question of whether God's justice is actually doomed to the status of a fiction and in this sense loses out to His mercy. Our sole purpose is to ascertain whether the thesis about the emptiness or non-existence of hell can be true in light of the Catechism of the Catholic Church. Our considerations will be based on an analysis of the well-known stone paradox – more precisely, its stronger version. Let us start by recalling the paradox in its basic form.

The point of the stone paradox is to try to prove the non-existence of an omnipotent being. The effect of the simple question of whether God can create a stone that He cannot bear is to show that the concept of omnipotence leads to a contradiction. If God can create such a stone, it means that the stone will effectively limit God's power and God will no longer be almighty. If God cannot create such a stone, it means that He is not omnipotent. There is no other way out; therefore, an omnipotent being cannot exist. Perhaps the simplest solution to this dilemma is to assume that God is bound neither by the laws of logic nor by the principle of non-contradiction. Such a solution would be consistent with the views of Pietro Damiani (n.d.) and Descartes (n.d., 1960, 1988). It turns out, however, that this solution is unnecessary, because the argumentation of the stone paradox is itself not logical. Does God's ability to create a known stone mean that the stone already exists? Of course not. In *Topics* (Book IV, 136a) Aristotle (n.d.) cautions against making the mistake of equating the ability to do something with doing it. Therefore, proof of the non-existence of an omnipotent being does not lead to a contradiction.

However, the problem identified in the question about the possibility of making a stone is quite serious. The stone is here a representative of potential objects, each of which would limit God's omnipotence in a particular way. There are many potential objects of this type. The question about creating the known stone can be replaced by other analogous ones: *Can God formulate a mathematical problem so difficult that He cannot solve it Himself? Can God create two flowers that differ in fragrance so subtly that He Himself will not recognise the difference?* And so on. All such questions have a common essence, a problematic one: can God in some specific way, and therefore to a certain extent, limit His omnipotence? A number of issues arise here: among them are the issue of how we understand divine omnipotence, how we understand the scope and temporality of its limitation, and how we think about the possibility of God's return to omnipotence. An extensive discussion that takes into account these and other issues can be found in Łukowski (2013).[2]

As previously noted, the possibility of creating a known stone does not contradict God's omnipotence. However, an inability to create this or any other stone already contradicts it. Therefore, of these two options, we choose the first: we assume that God, if He wishes, can create a stone that will limit His omnipotence to a certain extent. This limitation is expressed in the fact that God will no longer be able to pick up this one stone, though He will still be able to pick up all the other stones among which they will certainly be heavier than the special one. After all, the problem does not lie in the weight of the stone, but in God's self-limitation. However, adopting this assumption raises another important question: can God regain His omnipotence? A return to omnipotence would mean that God could pick that stone up as well. Thus, the stone would lose its essence. It would no longer be the stone that God cannot lift. It should be noted that God's return to full omnipotence does not change the fact that to the question of whether God can create a stone that He cannot lift, an affirmative answer still remains the true answer. After all, depriving the stone of its unique property makes it no longer a stone that God cannot lift, so it is no wonder that God can lift it. God's regaining of omnipotence can be achieved either by depriving the stone of its unique property or by annihilating the stone. In both cases, the stone that God cannot lift ceases to exist. In the first, the stone ceases to exist as the one that God cannot lift, and in the second, it ceases to exist altogether.

It is not difficult to see that the question that constitutes the essence of the stone paradox in its traditional form can thus be reinforced: *Can God create a stone that He cannot lift and after creation* *cannot destroy?* Of course, the word 'destroy' can take one of two meanings: stripping the stone of this unique property, that is, to make it an ordinary stone, or annihilating it literally. This means that we are dealing with two questions:

1. Can God create a stone that He cannot lift and after creation cannot transform into an ordinary stone?

2. Can God create a stone that He cannot lift and after creation cannot annihilate?

Naturally, the stone still represents any logically possible object for which God would deprive Himself of some of His power, but this time He is doing it irreversibly. The answer to each of these two questions is the same: Yes. Apparently, God can create such an object now that He has already created it. It is a human being endowed with free will – that is, the freedom to make decisions.

We adopt the understanding of human freedom following the Catechism of the Catholic Church (n.d.):

Part 3, Section 1, 1730: 'God created man a rational being, conferring on him the dignity of a person who can initiate and control his own actions.'

Part 3, Section 1, 1731: 'Freedom is the power, rooted in reason and will, to act or not to act, to do this or that, and so to perform deliberate actions on one's own responsibility. By free will one shapes one's own life.'

Part 3, Section 1, 1732: 'As long as freedom has not bound itself definitively to its ultimate good which is God, there is the possibility of choosing between good and evil, and thus of growing in perfection or of failing and sinning.'

Part 3, Section 1, 1733: 'The more one does what is good, the freer one becomes. There is no true freedom except in the service of what is good and just. The choice to disobey and do evil is an abuse of freedom and leads to "the slavery of sin."'

Part 3, Section 1, 1734: 'Freedom makes man responsible for his acts to the extent that they are voluntary.'

According to the Catechism, as long as the human being's freedom is not established in God, the human being can choose good, but she or he is also able to choose evil, thus opposing God. God's limitation in being able to lift a stone has its counterpart here in God's limitation in being able to prevent human beings from doing evil. On the other hand, God's limitation in the possibility of annihilating the stone (totally or only essentially) has its counterpart in God's limitation in the (total or essential) annihilation of the human being, which means the immortality of the soul. A person decides for her- or himself who she/he is and who she/he will eventually become. However, regardless of whether the human soul will strive towards God or towards satan, God will keep it for all eternity. The immortality of the soul is a guarantee for the human being's freedom. Therefore, on the one hand, the human being has the right to resist God's will by sinning, and therefore to act against Him. Undoubtedly, this limits the omnipotence of God, which ceases to be complete in the presence of the human being. The human being can therefore limit God's omnipotence with her/his actions that are not in accordance with the will of God. On the other hand, God cannot annihilate the human being since He has given her/him freedom. The annihilation of a person acting against God's will would be tantamount to denying a person her/his free will: she/he would have freedom as long as she/he acts in accordance with God's will, but as soon as she/he opposes Him, she/he is annihilated. In such a situation, human freedom would only be an illusion. Human freedom, then, will not be reduced to a mere fiction, but only if God actually allows human being to lapse without the threat of annihilation.

Human beings will have freedom only when it can follow their own decisions without being destroyed for doing so. Human beings may even turn away from God, and God has to accept that, because this was a decision God made when He created them. God can grant human freedom only by endowing human beings with immortality. What can God do with a human being who consciously and consistently refuses to obey Him, who consciously turns away from Him, who does not want to be with God, who chooses satan? God's gift to this person is to respect this decision, that is, to allow this human being to choose eternity with satan and eternal isolation from God. This is the consequence of human freedom. Hell is nothing more than isolation from God and eternal communion with satan.

The Catechism teaches nothing else:

Part 1, Section 2, 1033: 'To die in mortal sin without repenting and accepting God's merciful love means remaining separated from him forever by our own free choice. This state of definitive self-exclusion from communion with God and the blessed is called "hell."'

Part 1, Section 2, 1035: 'The teaching of the Church affirms the existence of hell and its eternity. Immediately after death the souls of those who die in a state of mortal sin descend into hell, where they suffer the punishments of hell, "eternal fire." The chief punishment of hell is eternal separation from God, in whom alone man can possess the life and happiness for which he was created and for which he longs.'

Part 1, Section 2, 1037. 'God predestines no one to go to hell; for this, a willful turning away from God (a mortal sin) is necessary, and persistence in it until the end.'

Hell is necessary for God to respect human freedom. Since human being does not want to be with God, but with satan, God must respect human being's decision, otherwise He would deny human freedom. From this perspective, hell is not a punishment for human sins, but a necessity resulting from human freedom; it is the guarantee that a human being will be able to achieve a goal sovereignly chosen by her- or himself. It is not God who condemns humans to hell; human beings condemn themselves to hell with their sovereign choice. With the existence of hell, God completes His promise to make humans free. Moreover, it is clear from 'Part 1, Section 2, 1033' that human being who chooses eternity with satan blocks the working of God's mercy - by rejecting God she/he rejects Him along with His mercy. Thus, not only is there no contradiction between God's mercy and God's justice, but the two attributes require each other.

Therefore, the answers to the two questions asked above are in the affirmative:

1. *Yes, God can create a stone that He cannot lift and not make it an ordinary stone after creation.*
2. *Yes, God can create a stone that He cannot lift and after creation annihilate.*

2. Conclusion

Acceptance of specific assumptions should result in the acceptance of the resulting conclusions. This should also be the case when accepting the thesis that God created the human being and gave her or him the freedom to choose between good and evil. If we accept these assumptions, we should in consequence also accept the thesis about the existence of hell and reject the thesis assuming its emptiness.

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Notes

1. E.g. Edith Stein (1962), Hans Urs von Balthasar (2007-2008), (1998), (2004), (2005-2009), (2001), (2004-2005), Karl Rahner (1987), Michael Schmaus (1989), Gisbert Greshake (2010), Henri de Lubac (2008), Wacław Hryniewicz (1982), (1989), (1987), (1991), (2009), Romano Guardini (2004), Walter Kasper (1983).
2. It contains detailed analyses of the positions of various philosophers and their works, such as Johannes Duns Scotus (2020), Pier Damiani (n.d.), Thomas de Aquino (n.d.), René Descartes (n.d., 1960, 1988), Bertrand Russell (1959), Richard Swinburne (2016), Ralph McInerny (1986), John L. Mackie (1955), G. B. Keene (1960), Bernard Mayo (1961), C. Wade Savage (1967), G. I. Mavrodes (1963), David E. Schrader (1979).