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Is the Polish Logic One of the Best Traditions Still?

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Andrew Schumann: The Polish logical tradition is one of the best. How can you explain the fact that Polish philosophers and mathematicians have been a long way in logic and analytic philosophy? Which Polish scientific centers are still heavyweight in this subject?

Roman Murawski: One should look for roots in the interwar period. Polish logic and analytic philosophy at that time is an amazing phenomenon. The school has been founded by Kazimierz Twardowski and is called Lvov-Warsaw school of philosophy. A part of it was also Warsaw school of logic. There is a fundamental monograph (published by Kluwer) describing this school and its achievements − I mean Jan Woleński's *Logic and Philosophy in the Lvov-Warsaw School*. The standards of a scientific work in logic and analytic philosophy developed then certainly helped to reach the level you are talking about. According to Twardowski and his students, one should clearly and sharply distinguish world-views and the scientific philosophical work. This idea was particularly stressed by Łukasiewicz, the main architect of the Warsaw school of logic. He regarded various philosophical problems pertaining formal sciences as belonging to world-views of mathematicians and logicians but the work consisting in constructing logical and mathematical systems together with metalogical and metamathematical investigations constituted for him the subject of logic and mathematics as special sciences. Hence philosophical views cannot be a stance for measuring the correctness of formal results. Yet philosophy may serve as a source of logical constructions. One should disregard philosophical controversies (and treat them as a „private'' matter) and investigate (controversial) axioms as purely mathematical constructions using any fruitful methods.

An interesting phenomenon was also the close collaboration of philosophers, logicians and mathematicians (especially in Warsaw) which resulted in important achievements.

Andrew Schumann: Which contributions of Polish logicians to decidability theory and recursion theory could you notify as the most important?

Roman Murawski: One should start by mentioning the method of quantifier elimination studied by Tarski and his students. This method had various applications to the decidability problems. Using this method Tarski proved the decidability of the theory of Boolean algebras, of the theory of dense linear order and of the theory of discrete order. He applied it also to the study of geometry and to the field theory showing the decidability of the first order theory of reals. He proved also the decidability of the theory of real-closed and algebraically closed fields. Among the decidability results obtained by quantifier elimination method by Tarski's students the most famous is Moses Presburger's result on the decidability of the arithmetic of addition. Tarski, together with his student Andrzej Mostowski showed the decidability of the theory of well ordering.

Polish logicians considered also and showed the undecidability of various theories. One should again mention here Tarski and his work on general methods of establishing the (essential) undecidability of first order theories. Using those methods the (essential) undecidability of various theories has been shown. One should mention here also the finitely axiomatizable arithmetic Q developed by Tarski, Mostowski and Robinson which appeared to be very useful in decidability studies.

One should mention also works by Józef Pepis on reducibility. Unfortunately Pepis was killed by Gestapo, probably in August 1941.

What concerns the contribution of Polish logicians to the recursion theory one must mention first of all the paper by Andrzej Grzegorczyk where a hierarchy of primitive recursive functions has been introduced and studied. This hierarchy is called today Grzegorczyk's hierarchy. It has been carefully studied by various logicians, it has been extended and generalized. One found various applications of it also outside logic, in particular in theoretical computer science and the complexity theory.

As next contribution of Polish logicians to the recursion theory one should mention the classification of non-recursive relations constructed independently by S.C. Kleene and Andrzej Mostowski and called today Kleene-Mostowski hierarchy. Let us mention also Grzegorczyk's studies of computable functionals of higher types as well as Banach-Mazur's and Grzegorczyk's studies on constructive mathematics and Mostowski's and Grzegorczyk's studies on the complexity of models of theories.

Andrew Schumann: What can you state about the development of Hilbert's program? Is it failed as the majority think?

Roman Murawski: Gödel's incompleteness theorems indicated certain difficulties in carrying out the validation and justification of classical mathematics on finitistic grounds postulated by Hilbert. They struck Hilbert's program but they did not reject it. The natural consequence of it was the idea of extending the admissible methods and allowing general constructive methods instead of finitistic ones. It seems that Paul Bernays was among the first to recognize this need. The very concept of constructive methods is in fact not quite clear. Nevertheless the idea has been accepted and became a new paradigm leading to the so called generalized Hilbert's program. Investigations were carried out in this direction and several interesting results have been obtained. One should mention here studies that followed Gentzen's idea of using transfinite induction on a certain recursive ordering (Schütte, Takeuti), the program of predicative reductionism (Feferman) or the idea of using primitive recursive functionals of higher types (Gödel). One should add that all those attempts are in fact different from the original Hilbert's program. Hilbert postulated the justification and validation of classical mathematics by a reduction to finitistic mathematics. This had an important philosophical meaning: finitistic objects and reasoning have a clear physical meaning and are indispensible in all scientific thought. None of the proposed generalizations can be viewed as finitistic and they do not have a similar philosophical and methodological meaning. Nevertheless the generalized Hilbert's program is an interesting contribution and is compatible with Hilbert's reductionist philosophy.

Another consequence of Gödel's incompleteness theorems is the so called relativized Hilbert's program. If the entire classical mathematics cannot be reduced and justified by finitistic mathematics then one can ask for which part of it is that possible? In another words: what part of classical mathematics can be developed in formal systems that are conservative over finitistic mathematics with respect to real sentences. One of contributions to this program is the reverse mathematics initiated by Harvey Friedman. Results obtained within this research program lead to the conclusion that a large and significant part of classical mathematics is finitistically reducible. This means in fact that Hilbert's program can be partially realized.

Andrew Schumann: What are mechanized deduction systems in fact? Why are they being constructed? What can be provided by their implementations and where?

Roman Murawski: In 1936 Alan Turing and Alonzo Church proved two theorems which seemed to have destroyed all hopes of establishing a method of mechanizing reasonings. Turing reduced the decidability problem for theories to the halting problem for abstract machines modelling the computability processes (and named after him) and proved that the latter is undecidable. Church − solving Hilbert's original problem − proved the undecidability of the full predicate logic and of various subclasses of it.

On the other hand results of Skolem and Herbrand showed that if a theorem is true then this fact can be proved in a finite number of steps − but this is not the case if the theorem is not true (in this situation either one can prove in some cases the falsity of the given statement or the verification procedure does not halt). This semidecidability of the predicate logic was the source of hope and the basis of further searches for the mechanized deduction systems. Those studies were heavily stimulated by the appearance of computers in early fifties. There appeared the idea of applying them to the automatization of logic by using the mechanization procedures developed earlier. The appearance of computers stimulated also the search for new, more effective procedures.

The idea is here to use a computer to prove non-numerical results, i.e., to determine their truth or falsity. One can demand and expect either a simple statement „proved" or a human readable proof. We can distinguish also two modes of operation: fully automated proof search or man-machine interaction proof search.

Note that the studies of mechanized deduction systems were motivated by two different philosophies. The first one − call it logic approach − can be characterized by using of a dominant logical system that is delineated and in fact static over the development stage of the theorem proving system. The second philosophical viewpoint is called the human simulation approach. It is generally the antithesis of the first one. Here one attempts to simulate human techniques of solving problems. Of course the logic and human simulation approaches are not always clearly delineated.

Various mechanized deduction systems have been developed. Let us mention here systems of Davis, Newell-Shaw-Simon, Gilmore, Gelernter et al., Hao Wang and Davis-Putnam. Very important role is played in those research also by the resolution and unification algorithms of Prawitz and Robinson. They turned out to be crucial for the further development of the researches towards mechanization and automatization of reasonings.

What does one expect from mechanized deduction systems and from an automated theorem prover? First of all certain unification of reasonings and their automatization are obtained. If one has such a system or prover one can shift the burden of proof finding from a mathematician and a logician to the computer. In this way one is also assured that faulty proofs would never occur. There is a question whether such automated theorem provers are clever than people? Of course they can proceed quicker than a human being. But they can also discover new mathematical results. In fact some open questions have been answered in this way within finitely axiomatizable theories. On the other hand there are some limitations implied by theorems on the complexity of decision procedures.

Andrew Schumann: What is reverse mathematics and which philosophical meaning does it have?

Roman Murawski: Reverse mathematics is a research program formulated by Harvey Friedman in 1974. Its aim is to study the role of set existence axioms, i.e., comprehension axioms in ordinary mathematics. The main problem can be formulated as follows: Given a specific theorem T of ordinary mathematics (e.g., of analysis, of algebra, of functional analysis, of differential equations, etc.) ask which set existence axioms are necessary in order to prove T? The procedure used in the reverse mathematics (and explaining its name) is to show that the considered theorem T is in fact equivalent to the existence axioms used in the proof of T and the main and usually most difficult part of the proof is to show that T implies the axiom (hence the procedure is here in a certain sense a reverse of the usual procedure used in mathematics where one proves that a given axiom implies a theorem). Some specific systems have been considered here (they are in fact subsystems of the second order arithmetic with various forms of the comprehension axiom) and their role and meaning with respect to various theorems from „hard” mathematics have been investigated leading to many very interesting results. Unfortunately they are rather very technical and complicated and it is impossible to describe them here in detail. One of the consequences of those results was the corollary indicated already above that Hilbert’s program can be partially realized.

Andrew Schumann: Whether there can be a logical symbolism for anything? What philosophical background does symbolism have as a whole?

Roman Murawski: There are three kinds of motivation inspiring the development of symbolism in logic: (1) the attempt to create an ideal artificial language as a substitute for an imprecise colloquial language, (2) a tendency to reduce logic to the study of properties of language or, in extreme cases, to the theory of signs, (3) a nominalistic tendency according to which abstract terms do not denote objects but are only empty signs. One or more of those tendencies can be seen in all logicians trying to develop a symbolism in logic. For example Aristotle exemplifies the first tendency, in the Stoics one sees clearly linguistic tendencies and in mediaeval logic one sees some semiotic tendencies (Abelard, the nominalists, Ockham).

There is a problem of relations between symbols and reality. It has been solved in various ways by logicians. One should mention also the tendency to overestimate the role and significance of symbolism. In this context one can mention the great Polish philosopher, the founder of Lvov-Warsaw School of Philosophy, Kazimierz Twardowski and his paper “Symbolomania i pragmatofobia” [Symbolic mania and pragmatic phobia] where he emphasized that symbols represent always objects but cannot replace them. A symbol is only a tool. If one forgets these two things we have the attitude Twardowski called symbolic mania. It can be characterized by a faith in the infallibility of a symbolism, in an autonomy of operations on symbols and by a condemnation of opinions which are independent of any symbolism. This attitude is connected with another called by Twardowski pragmatic phobia and consisting of bias against objects denoted by symbols.

Andrew Schumann: What is mathematical or logical truth? Does a mathematical or logical reality exist outside of the life-world?

Roman Murawski: Well, the usual and in fact the unique method of establishing truth in mathematics and logic is to construct a proof. The very concept of a proof is not quite clear and rather vague. In mathematical research practice the role of a proof is to convince other mathematicians that a given statement holds. Logicians tried to make it more precise by introducing the concept of a formal (or formalized) proof. But is it an adequate counterexample of proofs from mathematical practice?

Gödel’s first incompleteness theorem shows that one should distinguish between provability and truth (in a given model). In fact what can be proved is true but not always vice versa. Hence there are sentences that are true (in a given model) but that are simultaneously undecidable, i.e., neither they nor their negations can be proved in a considered theory. What means here “true” (in a given model) was explained by Tarski in his famous definition from 1933. His definition is connected with the classical definition given by Aristotle and called the classical definition of truth or the correspondence definition. It says that a sentence is true if and only if it adequately describes the state of affairs in the reality. In the case of a mathematical sentence one should speak about the mathematical reality. But what it is? Here we come to the second part of the question. This is one of the most fundamental problems of the philosophy of mathematics and logic. Several answers have been given here. One can classify them into three main groups. The first says that mathematical objects exist in an objective way and are independent of time, space and human mind. They are given to a mathematician and logician whose aim is to discover them and to describe their properties and mutual relations. One calls this doctrine Platonism. Another one says that mathematical objects exist in fact in human mind and are mental construction of mathematicians. This idea is called conceptualism. The third one called nominalism claims that there are in fact no abstract and ideal mathematical objects − there exist only physical items and in mathematics (and logic) we have to do only with expressions that should be treated as physical objects. All those doctrines have their adherents. One should add however that normal mathematicians behave in their research practice usually as platonists being convinced that the mathematical reality is given to them and that they do not have an unlimited freedom in dealing with mathematical objects they are studying. Note also that a philosophical declaration with respect to the problem of existence in mathematics can imply a limitation of admitted methods and considered problems (as it is a case by intuitionism) or one can treat philosophical sympathies as a private matter and develop mathematics or logic using any correct methods (as it was by Polish logicians and mathematicians in the 1920s and 1930s).