



## Logical Simulations of Economic Phenomena and Computational Economics



*Viktor Winschel* is an economist at the University of Mannheim. He has majored in econometrics, international monetary and exchange rate policy and theory and political economics. During his PhD he has worked on optimal currency areas and computational macroeconometrics. After that he has searched the proper mathematical tools to approach his top level problems in the optimal currency area theory namely value, money and institutional theory and the econometric identification of theories about agents with belief formation. The result is a collaboration with computer scientists, logicians and mathematicians in order to develop a global semantic and reflexive approach to economics ultimately with the tools of category theory, algebraic geometry and logic and so far with coalgebras as a functorial interface for games in mathematical economics.

*Andrew Schumann:* Since the spreading of Keynesian and Neo-Keynesian ideas in economics, many mathematical tools such as game theory, econometrics, probability theory, data mining, etc. have been used in economic researches as wide as possible. Why have some new mathematical theories such as coalgebra and category theory been proposed to be applied in economics recently? Is it insufficient to use the conventional tools accepted within the Neo-Keynesian paradigm?

*Viktor Winschel:* The global and short answer is that new mathematics can always potentially unite, generalize, organize, proof and program old mathematical economics and provide new formalizations of old questions and solutions to them. We would not consider to program the Internet by machine language or multiply roman numbers. But in mathematical economics as possibly in any applied mathematics we always try that until new mathematics arrives. The language situation in economics looks to me like that: much of interesting economics and sociology takes place in work expressed in natural languages, the common language of mathematical economics is functional analysis, not much of discrete mathematics, hardly any logic as object language, model theory or category theory. The situation is that we use a too low level language for the higher level concepts in economics with the difficulty of a too large and error prone gap between concepts and mathematics. Of course, different fields use higher mathematics but it was not clear to me how to unite different formalisms, automatize and run that within software. There is much methodological work ahead since radically new tools arrive from computer science, logic and mathematics.

The local and longer story goes like that. In my first lecture in economics on household theory I have missed the units and the symbol for the household. This suspicious situation about mathematical economics made me study rather neutral econometrics and economics as much as

possible in natural language form in order to not miss the non formalized parts. In the PhD I wanted to formalize the optimal currency area decision. Soon the question arose how to formalize the economic reasoning itself since we need to model all kind of agents who do reason economically. At the same time economic policy questions are answered by the composition of various economic argumentation lines of different size and form into an answer that applies the composed theoretical construction analogously to the question at hand. So I went on to program a theory discussion software and see whether it can be made into a production function of me as an economist. Compositionality of economic theories occurred to me to be an impossibility with my tools. One core impossibility was how to use logic about differential equations that describe agents who do the same? I run into at that time an unsolvable meta and object language clash. A reusable programming style is considered to write the generalized problem solution with the actual problem being an instantiation, much like Grothendieck tended to characterize his work as “to open a shell is to dissolve it in water” or the functional programming approach to design layers of ever higher domain specific languages until one can express the original problem in a natural form. Is it a surprise that we need very abstract mathematics for the very abstract cost benefit analysis of economic decisions like the optimal currency area question? After all abstract mathematics is used for engineering problems that are much less complex than economics.

So, programming myself as an economist seemed to me as a digital native programmer to be a natural approach and it occurred to me only later, after the PhD, that the underlying topic of reflexivity is probably as old as the human discovery of their own identity and besides in economics also at the core of philosophy, sociology, mathematics and computer science and that I need to descend into rather deep mathematical and philosophical waters. I have reprogrammed meta circular interpreters in Lisp and Prolog where one uses a language that is able to program its own interpreter, which is the source of the need for reflexive figures in computer science, just like the fact that data and code is located in the same memory and that code can be data or input and output of other code. After this insight I knew that it captures as well something very important in economics but I did not know what and why until I understood in what sense lambda calculus and domain theory and the underlying functorial fixed points are similar to an infinite game and other mind boggling structures in economics.

So, after my PhD my tools became insufficient for my economic goal of a formal theory that is given mostly in a natural language form with parts fragmented in different mathematical subfields of all kinds of economics. My tools have been some functional analysis, statistics and Fortran hacking, no logic, model theory or sufficient amounts of topology, differential geometry or software technology. The modelling issues like the Lucas critique, that agents anticipate theory and policy, together with belief, theory and institution formation and the interaction of theory and model are at the core of the modelling issues in optimal currency areas and the underlying value and money theory. All that are rather deep conceptual waters as well and the question is whether and how more abstract mathematics might help. But not only functional analysis became insufficient but set theory itself is not sufficient for the synthetic, relational and reflexive structures in economics that is in need of rather high level theoretical operations. We need units, types, functors, their fixed points in recursive domain equations, solution concepts as first order citizens, global macro entities, aggregation schemes, relational calculi and much more logic and model theory. I am currently trying to interpret the macroeconomic structures as algebraic geometric constructions like homotopy, schemes, sites, coverings, cohomology, colimits, sheafs, glueing conditions, comonad, topoi, local languages or adjoint functors. Many economic concepts have a natural representation therein.

Coalgebras are, compared to the available, and I think urgently needed, machinery, to be seen as a starting point for functorial fixed points and categorical methods in economics. All of that have been so far used only in few economic papers. Category theory was used to my best knowledge for the first time in 1989 in some papers by Vassilakis with functorial fixed points on the category of domains in order to capture various reflexive or infinitely hierarchical phenomena

discussed in economics since long. We see our usage of coalgebras as a first typing of game trees, the computational machinery of current macroeconomic models as in my PhD is still to be typed. To my best knowledge no one has ever proved in computational economics (as opposed to economics) that his code is bug free, instead everybody knows they are buggy and we know that we know that, even so billions of Euros and whole nations depend on the decisions of let's say central banks or other international economic institutions. We may be able to use modern computer scientific technology not only in order to avoid crashing airbuses but also to avoid collapsing exchange rate systems.

Coalgebras unite modal logic, unobservable state transition systems and even calculus to some extent for macroeconomics. Vassilakis categorical Ansatz for some deep economic problems did not take off probably because his handful of papers were not enough for such a shift in the abstraction level of the used mathematics. There must be a bridge to usual mathematical structures and worked out examples that prove new theorems or generalize or simplify old ones. Kalman style system theoretical models are unobservable state transition systems, Kripke structures, automaton, largest fixed points on posets – all that is used in economics and all that are coalgebras. They unite and generalize existing mathematical tools in economics and make the tools of computer science accessible. Many of them are not even known to exist and not all economists know that most of what they do in theoretical discussions is abduction with counter example generation that can be automatized by model generation, model checking and theorem proving assistants. The Curry-Howard isomorphism and even lambda calculus are hardly used in economics and even computational economics.

Coalgebras are a kind of lower upper bound of the mathematics we have compared to the one we can get. It is an interface for mathematical economics. Final coalgebras as semantic domains of all behaviour of the functorial structures are functorial fixed points on the set category rather than on domains simplifying Vassilakis approach while they still allow for infinite, observational, reflexive, dynamic structures like sets that contain themselves and that arise for example in belief formation in economics. Corecursion is amazingly practical for programming potentially infinite structures in Haskell like natural numbers  $\mathbf{N}=[1,2,3,\dots]$  which makes nasty nested loops in software into elegant guarded co/recursions with the categorical compilers doing the mechanical work of translating into loops.  $\mathbf{N}$  is definable for example as the largest fixed point of  $n = 1 : \text{map } (+1) n$ . Corecursion allows for an ordinary differential equation solver programmed directly as the fundamental theorem of calculus in two lines of Haskell that compiles into a coefficient matcher on power series which is a mess to imperatively program that by hands as loops. Economic dynamics is likewise so far formulated only recursively and not corecursively and only at the function and not domain level for hierarchical systems like belief or institution formation. The coalgebraic formulation of simple games might look like an overkill but it is invariant over existing game theory and it can be integrated (I think for the first time) with macroeconomics via algebraic geometry and topology. A computational side effect of our coalgebraic framework is a running software engine that is more or less a directly written down version of the mathematics of the framework itself. Corecursion is the proper structure for infinite data types like hierarchies of belief, times or interest rates or repeated games but also real numbers, approximation and convergence arguments. In short, functorial dynamics on structured domains is meant to unify, simplify and generalize mathematical and computational economics.

Aren't sets that contain themselves an intuitive starting point for the fact that modelling in social sciences takes place in and changes the modelled system? So the question is what mathematics supports these kind of circularities and how to factorize directed economic production functions into causalities, epimorphisms, monomorphisms, relations, networks, rings, fields, global solutions of equations, graphs, axioms or code and finally in policy and institutions? In what language? What could be a categorical dual accounting? The best lesson of my first economic lectures is that household theory is dual and dualities are one of the strengths of category theory. I hope to see soon in the Edgeworth box of the interaction of two economic agents the algebraic

geometric information of the global solution of the contract curve. I think we can discover many new approaches and solutions in a categorified economics. There are many functorial and global structures in economics and sociology beyond sets that contain themselves like in languages that create new ones. We can start to rethink the type of a coconut that produces a coconut just as money that merges apples and oranges into dollars, validated by functional analytical arguments and (why not global) welfare theorems, creative accounting, bursting bubbles and black holes for central banker's and finance minister's moneys – these seem to be strange local languages and type translations.

*Andrew Schumann:* The new mathematics such as coalgebras, stream calculi, process calculi, labelled transition systems and so on with their applications in economics is called non-well-founded, because the set-theoretic axiom of foundation is violated there and, as a result, we cannot build inductive sets which have been traditionally used for mathematical simulations in physics, economics, etc. This new mathematics is unconventional. What advantages does this mathematics have with respect to conventional mathematical tools in economics?

*Viktor Winschel:* Coalgebras generalize and unify rather usual mathematics in economics and by that switch in theory and software from awkward and implicit coalgebraic constructions to their explicit formulation with available proper higher order tools. My goal is to capture, starting with non-well-founded tools, mathematically more naturally the open, infinite and self-generating processes of social systems. These problems are treated ever since in economics and related disciplines but for sure in some cases not with the proper since new mathematical tools. An important goal of abstracting from the economic application is to arrive at a mathematics that may connect to the available ones in mathematics itself and to avoid as much as possible the possibly unavoidable production of inferior local solution processes to economic structures where the economic semantics is given in natural language stories that loosely translate between axioms, results of formal methods and hardly between economists and non economists, including mathematicians. It might be some bug in the incentive system of economics to be uncontrollable by secretly deciding on undecidable problems but this then is a case for economists bashing in the political economics of economics and for theoretical, institutional, constitutional and existential reform of economics possibly including the diagnosis that a reflexive approach to economics is worthless or too costly.

What we need is a better division of labor with all other sciences, we need to type natural language economics and we need ontologies and databases of theories, practically available and composable in software tools. After all modelling is also a process of agreeing on the communication protocol with others trying to solve the same problem. Here is where economists need to work on. In fact my work can be headed as a search for a language of economics that connects us to the rest of the non economics world. The overemphasize in economics of “applications” is self-defeating since without foundations there are no applications in a changing world and without syntax and application independent structure identification there is no economics, and no division of labor, of economics as in any science. But yes, my application is still missing just as the Euro is still not doing well as there seem to be some holes where the money is pouring into and it seems that we do not know why and where these holes might be. They might be detectable by Betty numbers as an exchange rate between the hyperplanes of a relational picture of an economy versus a usual one, who knows? To what do the holes connect to? To the banking system? Does it extract the rent of double accounting?

The extraction of formal meaning from natural languages and retranslations will ever prevail in economics and economies, this is about condensing learned lessons into reusable, generalizable mathematics, business plans, arbitrage opportunities and rent extraction and generation activities. I hope that category theory, as a way to translate in between different mathematical formalisms, as a

semantics for mathematics itself, can help to import powerful tools into economics and relate it to its local, existing syntax, semantics and pragmatics.

We can find in categories many economic stories and economics at its core is a rather universal cost benefit analysis similar to complexity and semantics in computer science. Categories are very suited for social science by allowing to define local languages and infinitely many truth values as approximation. Allowing for properties only by embedding objects into their environment is very but so far not valuable. The ability to define properties without introspection into their carrier is useful for theories of introspection, reflexively enough this sounds rather strange. I think about the categorical self-participating universals and adjoints as universal or golden social rules just like Kant's categorical imperative or Ellerman's helping people to help themselves. By switching from content to pure form, I guess we can better discuss modularization and decentralization, private versus state run production, why there are firms and markets, representative democracy or whether a common currency is to be used. Can in times of iPhones the question of currencies be reduced to an algorithmic problem? This is the question of rules versus discretionary based monetary policy. The economics and sociology not yet formalized but already in natural language form is full of challenges to mathematics and often in need of rather abstract structures, think of constitutions of how to find good constitutions or how to price cohomologies in economies? What is the profit from teaching a mathematician natural language economics and vice versa?

*Andrew Schumann:* George Soros was one of the first experts in finance who proposed to apply the notion of reflexivity in economics. On the one hand, in German (transcendental) philosophy there is a long tradition of logical, philosophical, sociological studies of reflexivity. On the other hand, this notion is formalized within unconventional mathematics. Whether this means that new mathematical tools in economics might combine continental philosophy with the paradigm of non-well-founded mathematics? What is reflexive economics?

*Viktor Winschel:* Soros represents the math of his approach as participating and observing functions  $y=p(x)$ ,  $x=o(y)$ , in one of the *Alchemy of Finance* editions. They are like the two corecursive equations that we use with Samson Abramsky or as Pierre Lescanne to define infinitely alternating moves of strategically interacting agents. Soros writes that they solve into never ending sequences of change and not equilibria. This captures in fact the duality of participating as algebra and construction versus observing as the coalgebra of the infinite. I have called it the do-see duality of econometrics of non experimental macroeconomic data where contexts cannot be held fixed in order to easily infer causalities from observations. His remark on equilibria depends on what it might be. I think of an equilibrium as a solution to an interactive problem. I agree if he means that equilibria in economics do often smell static rather than dynamic and interactive. There is a severe mathematical problem in functional and not domain recursive economic dynamics. Solow proposed to Soros to recap his knowledge about solutions of systems of equations. But Soros is right in that content and context interact in economics and this gives a never ending change, as a kind of dialectics (and remember there is synthesis after thesis and antithesis), if one thinks about largest fixed points and not as, presumably Solow did, about Brouwer typed ones. In usual economics we arrive at an equilibrium and the question is what happens then. We usually need some exogenous shocks and adjustments to it in order to generate dynamics, which is obviously rather strange as a picture of societies that generate shocks from within. We can always ask what is an equilibrium of equilibria and then we are in a world of many possible equilibria and some process to select one in theory that we actually observe. However, the word equilibrium is a highly unlucky one for the concept of coordination that we have in mind and it is still the heritage from mathematical physic analogies of around 1870.

Besides that, mathematics is always unconventional and according to von Neumann never understood, we only become used to it, I guess by retranslations and generalizations to and from the

old habits as a form of understanding. Morgenstern has written about self-fulfilling prophecies and the interaction of the theory and the modelled system in the 1920s. And I guess, yes, non-well-founded mathematics and categories in general provide ideas to answers to challenging parts of the continental philosophy. The closest connection to my thinking that I have found in sociology and philosophy is Luhmann's system theoretical sociology that is very much related to coalgebras, non-well-founded structures and in fact topos theory. He translated many results of computer science, system theory, cybernetics and explicitly builds on Spencer-Brown's (rather isolated and idiosyncratic) mathematics of the Russell paradox. There must be a coalgebraic formulation of much of Luhmann's work since Spencer-Brown's complex truth value  $v$  is an infinite sequence of True and False,  $v = \text{True} : \text{map not } v$ . Moreover, there is a new logic of David Ellerman built on the partition dual of the usual logic by subsets. This looks like a logic for the observing of observers where the unit of observing is to make a difference. Johannes Heinrich's philosophy is similarly a modern account of taking reflexivity as the foundational figure of societies. His notion of mutual thinking about each other's thinking is similar to the belief hierarchies of Harsanyi which is definitely a coalgebraic construction just like the Brandenburger-Keissler paradox of Alice's beliefs about Bob's assumption is a first  $n$ -players Russell paradox, hence sets that contain themselves – again a coalgebra. In short, we need to endogeneize theories into the system they are about and see how different theories aggregate into the dynamics of the system itself. I call that quantum physics to the power of quantum physics. If the observed system is changing the physicist then together they form a social system.

Reflexive economics has to provide a model theory for social science, where theory, syntax, content, form drives the model, semantics, content, function. In logics itself this interaction of logic as a description language and as a structure in its own right is not often discussed. This might be a new challenge that social science can approach together with the help from logic, computer science and mathematics. Maybe we proceed to dynamically varying sets, vibrating strings but for sure to some existing or new mathematics of unseen economic form. Any help is welcome from anyone who asks for the well being of our top level resources for life on earth as the ancient goal of housekeeping in times where the house becomes the whole earth.

In what ways for example could topos theory, that has recently been proposed for the physics of endogenous space and the rest (including economics?) be useful for the endogeneization of truth values, domains, languages, rules, dogmas and institutions in social systems as windows into a reality where it is never obvious whether it is the window itself.

In my economic problem the language of the economy is money. But reflexivity is only the first step of two observing agents who observe each other. The next question is a global one namely how do they observe themselves together or why, how and what society is emerging from these mutual observers and how to evaluate their exchange of work and money from all three perspectives and in different languages and truth values. I would like to combine micro economics in coalgebras and logic with the algebraic geometry of macroeconomics where the local to global transition is mathematically taking place and which is what category theory was developed for. After all money makes the world go around and it is an improbable geo meter. Macroeconomics is heading towards algebraic geometry in applications of homotopy theory for the global solutions of polynomials in general equilibrium theory. Cohomology gives us the calculations and I think we can also find around these structures the proper homes for the wholenesses, globalities, syntax and semantics of theories and models, solutions, entities, identities, persons, agents, households and values that we are talking about in economics and social sciences all the time – in short we need a synthesis from “I” and “you” to “we” as the embedding of “I” and “you” into the “we” and vice versa. People communicate by taking alternative points of view in all over the common space and they try and succeed or not to understand the different truth values arising from that.

*Andrew Schumann:* Is it possible to construct in the future computational economics, where all economic phenomena will be simulated, programmed, and predicted? How will it look like?

*Viktor Winschel:* Economic and social theory is about predictions which might change the predicted and about changing the rules as the best way to predict the future and to interpret the history. If we find out something about the informational structures that govern societies as their nervous systems we extend them into a new form. Economics is a “organize yourself and your household” theory with predicting the future as one of its tasks. For the finance minister this includes changing the rules and even changing the constitution which is about rules to find good operational rules or laws. My mental image is a software that runs mathematics or theorem proving as type transformations, for a kind of self-organized SAP system for national states and communities, the finance minister's workflow rethought, decentralized and integrated, if you want. We have that kind of software in the economy in chip design and verification of security critical software. Similarly, the research called Social Software mainly in computer science and logic looks at societies by means of algorithmic and semantic tools. So the boundaries between theory, software, model, economics and computer science are blurred and traditional economic concepts are about to be re-examined.

As our first code is up a running my goal for the next steps of a computational economic system are logical specification languages for theory and system specification and verification with model checking and generation and econometrics as code and automaton generation for an automatized production or synthesis of economic theories and the analysis of their theoretical behaviour and the same for the agents in my theories which are in fact my principals and I am their agent. So the content and the context interact in my own type of work and even change their roles, just like in our corecursive or Soros functions, where it is not clear what is the context and what is the content, both are both, depending on the point of view. They are both, alternatively changing their roles, infinitely, just like Spencer-Brown's complex truth value or  $-1, +1$  if plugged in  $x = -1 / x$ .

Software can visualize economic theories as theatre plays, synthesized movies and all kind of various media and data and theory builder may even interact with sensors within virtual worlds like SimCity. The theatre play, graphical and symbolic format is what I have often used to teach myself mathematical and informal economic argumentation lines, figures and patterns. My motto right now is that the theory is the code and formal methods of computer science are used to analyse their properties. Our coalgebraic framework in the, almost finished, paper with Achim Blumensath explicitly uses this metaphor, where, as in formal bialgebraic language semantics, we care about the behaviour of the whole code arising from the behaviour of the individual operators. We use natural transformations of functorial games and strategies for compositionality and hence aggregation as the first step to macrostructures and their identities, it seems as colimits. However, since we need micro and meso and macro structures the bialgebraic semantics turned into a sub modular one with two instead of one natural transformation. Syntax and algebras can be taken as network structures that are strategically constructed within the system, by that it organizes itself. The need for meso structures makes econophysics based on statistical mechanics useless. Complex systems are like that because they have intermediate structures that moreover reason about the aggregated structures. In complex systems there are neither case based singularities at work nor laws of large numbers. It points into a fractal repetition of the same structures at varying levels just like category theory reveals the fractal organization of mathematics.

A related general problem is the prevailing usage of only extensively interpreted functions as input output boxes in computational and general economics. Non intensionally, without looking at the rule or algorithmic content of functions, it becomes complicated in not impossible to build a theory of the composition of production and utility functions and to see how synergies and added value evolve and are distributed in economies. At the same time, we need to model behavioural phenomena at the interfaces to unobservable spaces and hence a clear notion of automatons and unobservable state based systems for epistemic and ontological states. This is what the coalgebraic framework provides, unobservable state space systems and automatons, together with the implementation and the access to formal verification systems and much more. Lambda calculus for example tells us how the evaluation of function arguments relates to function composition.

Extensionally we cannot distinguish between  $f(g(x))$  and  $(f.g)(x)$  but economically we have to since intensionally both expressions are possibly subject to very different costs or complexities and that depend on the order of argument evaluation and function composition. In the end more general notions of morphisms than functions are needed to construct the category where the invisible hand arises is a colimit. The question of economic value seems to be strangely outside of economic theory as rather arbitrary cost functions that accompany production functions. The units of the operations are not specified and accordingly much of economic semantics evaluates natural language concepts ultimately to real numbers. So we neglect the computational and algorithmic content of production and most of all the compositional effects. We simply do not use the proper algorithmic tools for the compositionality and the processes of economies. This is what our coalgebraic framework is about. We aim at compositionality by natural transformations of games and strategies into aggregated ones.

Take any Internet company which is about producing software or management that is about producing similarly algorithms or rules of transformations of some types. What is the type of an economist, who is producing consultants who are taking production functions and produce better ones? We can speculate whether an economist without an algorithmic interpretation of functions would succeed as a manager of Adam Smith's nail factory, where one needs to detect sub modular opportunities for the division of labor that preserves the whole product but at lower costs, coverings and normalizations. This can be taken as a coalgebraic form of graph minimization under bisimulation as behaviour preserving equivalence relations or as a normalization on a wholeness. In turn, reflexively, due to a lack of understanding of the division of labor or value theory, we have a fundamental problem of composing economic theories efficiently. The division of labor situation in economics and with the rest of the sciences is not Pareto efficient. Compositionality is a field where much work was devoted on in computer science and mathematics and from where we can learn very much for some of the core questions of economics. A related question is why have these tools not been developed in economics? How do we need to educate our students in order to do so?

I think computer science and economics share some similar and foundational concepts and problems like the need for introspection or reflexivity, explicit syntax and semantics or value theory, encapsulation and global solutions or centralization and parallelization. Moreover computer science moves more and more into traditional social science domains implementing our societies and human-computer interfaces. However, computer scientific and logical results are about to improve the economic reasoning process itself independent of the entertained theory. But we need to discuss how that connects to existing economics.

*Andrew Schumann:* Many experts claim that the financial crisis of 2008 was caused by the insufficiency of conventional economic paradigm including Neo-Keynesian mathematics. Can we assume that the new mathematics in economics allows us to solve much more problems in the future?

*Viktor Winschel:* The economic problem of the current and other financial crisis is most likely the result of an insufficient understanding of relational structures in economies and economics. Take double accounting and Walras' Law, my colleagues tell me hand wavingly that this is simple, at the same time it is unclear whether dynamics in macroeconomics is consistent with national double accounting, implemented by the banking sector. More technically it is unclear whether the postulated or real dynamics takes place at some hyperplanes that cannot be reached without some creative accounting, invention of nonexistent types in economic theory or securities in financial markets. I am not even sure whether we know the truth value types for these kind of questions. Where does cheating starts – already in the syntax? Are there social structures where cheating is a way to do anything? For sure it is easier to cheat and err in a theory if there are no units of kg or dollars of resources. What units do the indices of consumer prices have? And what is the type of a financial contract that is written in 100 pages of a juristic language talking about prices composed



from various assets priced by nonlinear stochastic differential Black-Scholes equations? It may not be a standard contract to be traded over the counter, would you buy it?

The old stock-flow problem in monetary economics as far as I know is still not resolved which is about constraints propagation in hierarchical knowledge based systems and amounts to a proper treatment of time points and spans. Process logic, embeddable into our coalgebraic framework, might be a language for economic theory to ask how to approach the measurement and control of decentralized structures. It is one of the most complex problems in computer science, engineering, economics, banking and management. Most of all control is either dictatorial or emergent or composed from the control of subparts. For sure emergence is not discussable in the mathematics we have in economics and we need global and geometric methods.

My own understanding is that money theory and economic theory in general is about generalized double accounting thus a relational system with adjoints that give us universal values. Accordingly, my inner problem of economics are endogenous, sub modular hierarchies or meso network structures that create and distribute value. The mathematics we were talking about is the result of my modelling problems that I had within optimal currency areas. It needs by subject much of economics and from my point of view some latest, local and global mathematics.

We will see where the new mathematics, logic, computer science and programming languages which drive the internet and the economy can be of use in economics. The work ahead is to type the theory of optimal currency areas to arrive at the value of a common decision about the constitution of a central bank written on some pages in the language designed by the mathematics we were talking about and interpreted by a proof engine and model generator for a scenario and counter example exploration that evaluates the contract on available data. This is the general problem solving and contract generation machinery of economics if one takes optimal currency areas and central bank as variables of the type of global game and strategy, respectively.