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The Semantics and Pragmatics of the Conditional in al-Fārābī's and Avicenna's Theories

Saloua Chatti

Faculté des Sciences Humaines et Sociales de Tunis,
University of Tunis,
Tunisia

e-mail: salouachatti11@gmail.com

Abstract: In this paper, I examine al-Fārābī's and Avicenna's conceptions of the conditional. I show that there are significant differences between the two frames, despite their closeness. Al-Fārābī distinguishes between an accidental conditional and two "essential" conditionals. The accidental conditional can occur only once and pragmatically involves succession. In the first "essential" conditional, the consequent follows regularly the antecedent; pragmatically it involves likeliness. The second "essential" conditional can be either complete or incomplete. Semantically the former means "if and only if"; pragmatically it means "necessarily if and only if". The latter is expressed by 'if, then' and means entailment; pragmatically, it involves necessity and the inclusion of the antecedent into the consequent. As to Avicenna, he rejects explicitly al-Fārābī's complete conditional and distinguishes between the *luzūm* (real implication) and what he calls *ittifāq*. He quantifies over situations (or times) to express the various conditionals. The two universals $\mathbf{A}_C$ and $\mathbf{E}_C$ are expressed by "In all situations, if…, then…", while the two particulars $\mathbf{I}_C$ and $\mathbf{O}_C$ are expressed by "In some situations, if…, then..". This gives them a modal connotation, and makes the universals close to strict implications. Pragmatically, $\mathbf{A}_C$ presupposes the truth of the antecedent, but there is no such presupposition in $\mathbf{E}_C$, while what is presupposed in both $\mathbf{I}_C$ and $\mathbf{O}_C$ is a (possible) conjunction.
Despite these differences, in both systems, the conditional is not truth functional, unlike the Stoic conditional.

Keywords: Essential *vs* accidental conditional, *ittifāq*, *luzūm*, entailment, strict implication, strict equivalence, quantified conditionals.

1. Introduction

In their respective hypothetical logics, al-Fārābī (873-950, AD) and Avicenna (980-1037, AD) present syllogisms containing conditional and disjunctive propositions. These syllogisms express explicitly and implicitly their conceptions of these connectives.

In the following paper, we will focus especially on the meanings of conditionals and will try to answer the following questions: How do both philosophers define the connective of conditional? How do they use this logical constant in their theories? Do these definitions contain a pragmatic aspect, given that pragmatics deals with presuppositions, contexts, intentions and implicit meanings that go beyond what is literally said? Are there any differences between the two theories?

In answering these questions, we will clarify the two conceptions defended by these authors and the differences between these conceptions which are made explicit through the analysis of their respective hypothetical syllogisms and the rules admitted in both frames.

2. The Meaning(s) of Conditionals in al-Fārābī's Frame

Before starting the analysis of al-Fārābī's doctrines about conditionals, let us first clarify the words 'semantic' and 'pragmatic'. As it is usually defined, semantics has to do with the explicit meanings of words and propositions. These meanings help determine the truth-values of sentences. While pragmatics has to do with contexts, presuppositions, implicit meanings and the intentions of the utterers. It is by taking into account these presuppositions, intentions and contextual circumstances that the truth-values of sentences can be settled.

Now we don't find any explicit definition of semantics and pragmatics in al-Fārābī's or Avicenna's texts, but the examples they provide in their respective hypothetical logics are based implicitly or explicitly on semantic and pragmatic considerations. This is why it makes sense to talk about the semantics and pragmatics of the logical conditional in their respective frames. Let us start by al-Fārābī.

Al-Fārābī's hypothetical logic is presented in his *al-Qiyās* (the counterpart of *Prior Analytics*) and *al-Maqūlāt* (the counterpart of the *Categories*). The hypothetical syllogisms contain either conditional propositions or disjunctive ones. In these two treatises, conditionals as well as disjunctions are defined and considered in the context of the valid hypothetical syllogisms. These valid syllogisms are developed mainly in *al-Qiyās*.

In *al-Maqūlāt*, the conditional proposition (*mutalāzima*) is such that "if one of its elements exists, the other one exists too by means of that [*idhā wujida aḥaduhumā, wujida-al-'ākharu bi wujūdihi*]" [4, p. 78]. There is thus a dependence relation between the antecedent and the consequent of the conditional. But this relation is not exactly the same from one kind of conditional to another.

There are basically three kinds of conditional propositions:

1. The *accidental* conditional, where the consequent and the antecedent are related by accident (*bi-l-'araḍi*). As an example, al-Fārābī provides the following: "If Zayd comes, 'Amr leaves" [2, p. 127]. Here, the consequent may follow the antecedent, but not always, nor often. But 'follows from' may just mean a succession in time. We can note, however, that the two verbs 'come' and 'leave' are semantically related.

2. The *essential* conditional, which is of two kinds:

2a. A first kind of conditional where the consequent follows the antecedent "most of the time" [2, p. 127] but not always. This is illustrated by the following example: "When Sirius rises in the morning, the heat will be severe and the rains will cease" [2, p. 127, translation Wilfrid Hodges [10, p. 247.]

2b. A second kind of conditional where the consequent *necessarily* follows from the antecedent. In this case, the consequent always follows the antecedent.

The last conditional is either *complete* or *incomplete*. The complete one is a biconditional (or an equivalence): necessarily when the antecedent holds, the consequent holds, and conversely. This kind is illustrated by the following sentence:

$2b_1$: 'If the sun rises, it is daytime' (and vice versa)

As to the incomplete one, it is a single conditional which does not convert. For instance, we can say:

$2b_2$: 'If this is a human, it is an animal' (but not conversely) [4, p. 78].

The second example looks like a strict implication, while the first one looks like a strict equivalence. Both involve a semantic and necessary link between their elements, which might also be causal. The semantic aspect is related to the explicit meanings of the antecedent and the consequent. Given these meanings, the conditional relation holds.

Note that even in (1) and (2a), there is either a semantic or a causal link, for in case (1), the words 'comes' and 'leaves' are semantically related, while the events evoked in (2a) happen successively and might be related causally, even if there is no necessity in the link between the antecedent and the consequent. The two implications are thus intensional because of the semantic relations between the antecedent and the consequent. However, these semantic relations cannot determine alone the truth-values of the sentences, for in sentence (1), for instance, there is no necessary link between the coming of Zayd and the leaving of 'Amr; so if the sentence is true, its truth is not due to the meanings of the words 'coming' and 'leaving'; rather it would be due to the facts that really happened. So this kind of conditional is not comparable to those expressed by sentences which are "true solely by virtue of the meanings of the words" as Carnap characterizes them. In other words, this conditional is not "analytically true" in the Carnapian or modern sense, despite the semantic link between the antecedent and the consequent. Rather the truth of the whole conditional has to do with the context of utterance of such a sentence, hence it has also a pragmatic aspect.

As to sentence (2a), its alleged "essential" character raises a problem, for if the consequent follows the antecedent only "most of the time" but not always, how could the relation be "essential"? What does the word "essential" mean in that particular case, given that "essential" is usually connected with the notion of necessity which is stronger than what seems to be involved here? This particular case seems strange to Wilfrid Hodges too, who says in his book *Mathematical Background to the logic of Avicenna* (2016): "Curiously he allows that some 'essential' relations hold only for the most part" [10, p. 248].

A possible answer would be to interpret (2a) as expressing some kind of *natural* connection, i.e. a connection that holds in nature and can be observed most of the time. "Essential" would then be related to the context of utterance of the sentence, since we cannot consider that the linguistic meanings alone make the sentence truth. We can perhaps also evoke some kind of "non-technical" or "broad" meaning of the words "essential" and "essentially" which makes them close to "mostly" as one reviewer suggests. In that case, the word "essential" does not require necessity; it would have to do with the fact that the two elements are fundamentally, though not necessarily related. As a matter of fact, al-Fārābī's analysis relies on ordinary meanings of words in Arabic and this could explain and justify some of what he says about the conditional operator. Here too, we can also evoke a pragmatic aspect, which results from the consideration of the contextual circumstances and facts that make the sentence true.

The relations in (1), (2a) and (2b) are different, for there is no necessity and no reciprocity in (1) and (2a), while in (2b) the relation is clearly symmetric, given that what is presupposed by al-Fārābī is a biconditional, although he does not use a specific (and different) word to name it. What then, is meant by 'implication' in (1) and (2a)? In particular, given its "accidental" character, can we say that (1) is a material conditional? The answer is: No. First because the material conditional is extensional since there is no semantic link between p and q, which may be entirely independent semantically, while (1) is intensional, for it depends on the meanings of its elements. Secondly because there seems to be a temporal succession between the antecedent and the consequent, which is not always the case in the material conditional. Thirdly because al-Fārābī does not give the whole truth conditions of this conditional, since the cases where the antecedent is false are not sufficiently clarified, as we will see below.

What about (2a)? Unlike (1), which is said to be "accidental" by al-Fārābī, for it may happen just once, (2a) expresses a regular and frequent succession in time. But this regularity does not mean that the relation between the antecedent and the consequent always holds. For this reason, the relation lacks necessity. So it cannot be a strict implication. However, this link is not accidental either, for it is natural, observable and mainly empirical. We could say that it stands between "necessary" and "accidental" just as "general" stands between "universal" and "particular" and "most" stands between "every" and "some" in ordinary languages. This also means that in that case too, the relation is not a material conditional. It could be expressed by means of a probable relation between the antecedent and the consequent as follows: 'If p then probably q', or 'Probably (if p then q)', where 'p' and 'q' are semantically related. But such a kind of probability is better expressed by the word "likely" and is not comparable to the standard and mathematical account of probability. Rather it is more like some kind of *imprecise* probability.

In both cases, the meanings explicitly carried are different, despite the use of the single expression "if…then", for in (1), the relation between the antecedent and the consequent seems to be contingent or chancy, while in (2a), this relation is not chancy ; rather it is regular and likely to occur, even if it is not really necessary.

As to presuppositions, they are also different, for what is *presupposed* in (1) seems to be a *possible* implication involving a succession in time, since the antecedent precedes the consequent, but not conversely, while (2a) seems to implicitly express a *probable* implication, despite the fact that al-Fārābī does not use an explicit modal word in that context. It also involves a succession in time, which is regular and considered as "essential" by al-Fārābī. This succession is empirical and suggests a natural link between the antecedent and the consequent, even if this link is not necessary. Possibility or probability are thus more implicitly suggested by the examples chosen than explicitly expressed. They may be part of the *pragmatic* meaning of these kinds of conditionals, which convey different presuppositions related to the expression "if… then".

As to (2b), it is said to be necessary, whether it is complete or incomplete. The semantic meaning of "if…then" in the incomplete case is a non-convertible implication, where the antecedent always precedes the consequent, while the meaning of the complete implication is rather equivalent to the meaning carried by "if and only if". In both cases, what is presupposed is a necessary link between the antecedent and the consequent. So what is *pragmatically presupposed* in ($2b_1$) is "necessarily if […] then […]", while the presupposition conveyed by ($2b_2$) is "necessarily […] if and only if […]".

In both cases, the expression explicitly used by al-Fārābī is "if … then", but he does say that the complete meaning is "convertible", which means that the double implication is explicitly assumed, even if there is no additional *word* translating the convertibility. This kind of '[bi] conditional' may be rendered by the following complex expression: "If p then q and if q then p" which is strongly suggested in the arguments provided by al-Fārābī.
This essential *complete* meaning validates the following syllogisms:

1. 'If p then q; but p; therefore q'

2. 'If p then q; but q; therefore p'

3. 'If p then q; but ~p; therefore ~q'

4. 'If p then q; but ~q; therefore ~p' [4, p. 79, my formalization]

In these hypothetical syllogisms, what is presupposed is 'if and only if' rather than simply 'if…then', as appears in the following quotation: "And those expressing a complete implication are those where if *whatever* element holds, the other one necessarily holds too by means of it (*bi* *wujūdihi*), for if the first one holds, the second one necessarily holds, and if the second one holds, the first one necessarily holds too.” [2, p. 127, my emphasis].

The example illustrating this case is the famous Stoic example: “If the sun is up, it is daytime”. Given the inseparability of both events, they always hold together, which justifies the completeness of the implication. Note that the same example expresses a simple (and truth-functional) conditional in the logic of the Stoics, according to Suzanne Bobzien, who says that the conditional is expressed in Stoic logic by means of the following negated conjunction: ‘Not (p and not q)’ [8, § 5.3]. This makes the Stoic account of the logical conditional different from that of al-Fārābī, since it does not contain any kind of modality. On the contrary, al-Fārābī’s necessary kind of implication would be closer to what is now called “strict implication”, which is expressed by “Necessarily (If p then q)”, although al-Fārābī does not use its equivalent formulation “Necessarily not (p and not q)” or “Impossibly (p and not q)”, given that he does not use a conjunction to express the conditional operator. The complete meaning of that implication, which validates the inferences 1-4 above and is illustrated by the classical example “If the sun is up, it is daytime”, would be more like a necessary biconditional, which we could express by the following “Necessarily (If p then q) and (If q then p)”.

As to the incomplete conditional exemplified by ($2b_2$), it admits the following syllogisms:

(1). ‘If p then q; but p; therefore q’

(2). ‘If p then q; but ~ q ; therefore ~ p’ [4, p. 79].

(1) corresponds to the *Modus Ponens*, while (2) corresponds to the *Modus Tollens*.
But it does *not* validate :

- ‘If p then q; but ~ p, therefore ~ q’

- ‘If p then q; but q; therefore p’ [5, p. 138]

given that the relation is not convertible.
The rejection of these two cases means that from the falsity of p, one cannot deduce the falsity of q, and that from the truth of q one cannot deduce the truth of p. So, when p is false, q could be either false or true. Similarly, when q is true, p could be either true or false.

In addition, he rejects the case where the antecedent is true and the consequent false, for when the antecedent is true, the consequent cannot be false.

In ($2b_2$) [‘If this is a human, it is an animal’] which illustrates this kind of incomplete implication, what is presupposed is the *inclusion* of the antecedent into the consequent for in the example provided, the class of humans is part of the class of animals.

So (1), (2a), ($2b_1$) and ($2b_2$) do not have the same semantic meaning nor the same pragmatic meaning. For in (1), there is probably only a succession of events, which may happen only once, while in (2a) this succession is regular and frequent, and in ($2b_1$) the link is causal while in ($2b_2$), it is an inclusion, hence it is conceptual. Here, the example chosen [‘If this is a human, it is an animal’] seems to pertain more to the Aristotelian categorical logic than to the hypothetical (or propositional) logic, for this sentence is another way to say that “All humans are animals”. This is why we could use the notion of inclusion to characterize the relation between the antecedent (the subject of the categorical sentence) and the consequent (the predicate of the categorical sentence). This means that the universal categorical sentence of the form **A** is expressible by a conditional in al-Fārābī’s frame. But in another book entitled “*al-Alfāḍ al-mustaʿmala fī al-mantiq*” (*The expressions used in logic*), al-Fārābī uses the word “inclusion” too, when talking about the hypothetical connected sentences which start by “If” or “whenever” or “when” and says that the antecedent in these sentences includes (*yataḍammanu*) the consequent, for he says that in the sentence “If the sun is up, it is daytime” “The rising of the sun includes (*taḍammana*) the succeeding emergence of the day" [1, p. 54], given that both events are closely related and that there is no daytime without rising of the sun. So although al-Fārābī distinguishes between the hypothetical sentences and the categorical ones, the notion of inclusion is involved in both kinds of conditional sentences, i.e. the hypothetical ones and the categorical ones. The consequent is included in the antecedent of a hypothetical connected sentence, as the predicate is included in the subject of the universal categorical one. This closeness will be acknowledged by Avicenna too as we will see in what follows.

Can we say that this conditional is truth-functional, given the truth-conditions provided by al-Fārābī, as is the case with the Stoics' conditional?

As a matter of fact, al-Fārābī provides two cases where a conditional is true and where the truth-values of either its antecedent or its consequent are deductible, i.e. known with certainty. These cases are the *Modus Ponens* and the *Modus Tollens* above. But does this mean that one can determine the truth-value of this conditional operator starting from the *values of its elements alone*? In other words, is the logical conditional truth-functional in his frame?

If we consider the examples given, there is always a natural or a semantic relation between the antecedent and the consequent. So the truth of the conditional operator depends also on the meanings of its elements, which are crucial to determine its truth or its falsity. As to the restrictions provided by al-Fārābī, they mean that it *is possible* for a conditional to be *true* when its consequent *alone* is true and also when its antecedent *alone* is false. But is the truth of this operator *warranted* in these two cases?

If we consider the examples provided, could we say, for instance, that sentence (1) ["If Zayd comes, 'Amr leaves"] is true if its antecedent is false, i.e. if Zayd does not come? Nothing indicates that this truth is warranted, nor even seriously considered by al-Fārābī.

As to sentence (2a) ['When Sirius rises in the morning, the heat will be severe and the rains will cease'], which involves a regular but not necessary link between the antecedent and the consequent, the truth of the sentence seems to presuppose the truth of its antecedent, since the case where the antecedent is false is not intuitively a case of truth for the whole conditional (or implication). Why should we say that this particular implication is true when its antecedent is false, i.e. when 'Sirius does not rise in the morning'? So, this kind of implication does not seem to be truth-functional, given that its truth-value depends on our intuitions and on the facts involved, not only on the truth-values of the elements. As we know, in the intuitive and ordinary sense, the implication or conditional does not seem to be true when its antecedent is false. In that case, its 'truth' would be very counter-intuitive. So the meaning of this kind of conditional is not determined by its truth conditions.

What about the necessary versions of the conditional, which are either complete or incomplete? The incomplete case is exemplified by the sentence 'If this is a human, then it is an animal'. It involves the notion of inclusion (to the class of humans inside the class of animals), which means that the sentence is true if such an inclusion holds, and false if it does not. But what happens if there are no human beings, that is, if the antecedent is false or in other words if the class of humans is empty? Would the sentence be true in that case? If we consider the fact that this sentence is an instance of the universal proposition 'Every human is an animal' [since we can express it by saying 'if *a* is a human, then *a* is an animal'] and if we take into account the fact that the universal affirmative cannot be true if its subject does not exist, in al-Fārābī's frame, as he says in *al-Maqūlāt*, where talking about the affirmative propositions, he claims that "when their subject does not exist, they are all false" [2, p. 124.13-14], then we may consider that the conditional proposition whose antecedent is false would not be true in al-Fārābī's theory. At least, its truth is not warranted. Consequently, this kind of conditional is not presumably truth-functional. If we compare al-Fārābī's position with modern ones, we can see that it is different from that of Strawson, a contemporary author who says that the sentence *lacks* a truth value whenever its subject does not exist as appears in what follows: "The more realistic view seems to be that the existence of children of John's is a necessary precondition not merely of the truth of what is said but of its being either true or false" [15, p. 174]. Unlike Strawson, who endorses the position that all sentences whose subjects are non-existent *do not have a truth value*, al-Fārābī says that this kind of sentences are *all false*. In this respect, his position is more Russellian than Strawsonian, despite the important differences between Russell's theory about the logical conditional, which is extensional, formal and mathematically expressible, and al-Fārābī's one.

What about the complete one? This is a necessary equivalence and the four valid moods involving it provided by al-Fārābī show that a [bi]-conditional is true when both its elements are true and when both its elements are false. So we know that this [bi]-conditional is true when its two elements have the same value. Consequently, we can deduce that it is not true when their values are different. The truth conditions of the complete implication seem thus to be settled. However, the very reason why these truth conditions are given depends ultimately on the meanings of the elements considered. So, here too, the truth-functionality of the [double] implication is not really clear.

What about Avicenna's theory on the conditional? This will be examined in the next section.

3. The Meaning(s) of Conditional(s) in Avicenna's Theory

According to Avicenna, conditionals may be weak, medium or strong. All these kinds of implication are expressed by specific and different words in the ordinary language (currently the Arabic language). The strong implication is the one expressed by '*in...fa*' (= if...then); the weak implication is expressed by '*matā*' (= when), while the mediate one is expressed by '*idhā*' (= if) [6, p. 235]. This classification suggests that these words carry different significations and do not involve the same presuppositions. For instance, the word '*matā*' (when) clearly has a temporal connotation, which may not be present when one uses the particle '*in*' (if). Avicenna also uses in some kinds of conditionals the word '*kullamā*' (= whenever) which evokes universality.

In his informal analysis of the logical conditional, he evokes al-Fārābī's distinction between a complete implication and an incomplete one without endorsing it, but his discussion of the matter clarifies the difference between the two conditionals involved and their nature in the complete implication. The complete implication is defined as: 'if p then q and conversely' (symbols added) and illustrated by 'whenever the sun rises it is daytime and whenever it is daytime, the sun rises' [6, p. 232]. As we can see, the two elements in the complete implication are clearly expressed and separated. In both cases, the link between the antecedent and the consequent is causal, but the causal relation occurs differently. For when one says: (1) 'If the sun is up, it is daytime', he means that 'the sun is the *cause* of the daytime', while when he says (2) 'if it is daytime, then the sun rises', he expresses rather the *inseparability* of the cause and the effect [6, pp. 233.17-234.1].

So in case (1), the word 'if' means that the antecedent is the *cause* of the consequent which follows from it for this reason, but in case (2), it means that they are (necessarily?) *concomitant*, given that the antecedent is not really the cause of the consequent. The conditionals involved carry then different meanings in both cases. In (1), the condition seems to be necessary, while in (2), it is rather a sufficient condition. The first condition is strong, while the last one is rather weak. There is thus a kind of asymmetry between both elements of the complex relation.

Note that, even in the modern sense, 'if and only if' contains the same asymmetry, for 'only if' is stronger than 'if', since it expresses a necessary condition.

But al-Fārābī's distinction is criticized by Avicenna who says that the complete implication does not respect the syntactical structure of the conditional, which is shown by the fact that the antecedent *precedes* the consequent in all cases. For saying that an implication might be convertible in some cases is like saying that the copula in an affirmative universal proposition might express an identity in some cases. But according to Avicenna, this is not compatible with the formal character of logic and is not the way al-Fārābī himself, as all traditional logicians, treats the universal affirmative in his syllogistic, given that he never considers the difference between a case where "the predicate is equivalent to the subject" and the case where it is not, when dealing with the universal affirmatives in the context of his discussion of the syllogistic moods. Avicenna observes that if the subject and the predicate were convertible in the universal affirmative, then *Darapti*, for instance, would have a universal conclusion, for we would say “If the predicate is equivalent to the subject, then [the proposition] would convert as itself; and if it is not, then it will convert as a particular” [6, p. 392]. But nobody, including al-Fārābī, ever considers this case or admits a third figure mood where the conclusion is a universal affirmative proposition following from two universal affirmative premises. Given this fact, one should take into account the form of the propositions in the hypothetical logic too.

The weak implication is illustrated by the following example: ‘If every man is speaking, then every horse is whinnying’ [6, p. 268]. Here, there is no real semantic relation, but only a kind of (contingent?) concomitance, for the antecedent cannot be said to be the cause of the consequent; they are rather independent, given that each of the two propositions may be true on its own, without being really *entailed* by the other one. This example illustrates what Avicenna calls ‘*ittifāq*’, a word translated as ‘chance connection’ by Nabil Shehaby. This translation expresses the idea that the link between the antecedent and the consequent in an ‘*ittifāqī*’ conditional is accidental; it is not strong, not natural and may be present only once. However, there is no consensus about the interpretation of the word ‘*ittifāq*’, for it is understood in a different way by other commentators. For instance, Wilfrid Hodges says that the notion of chance (or of accident), evoked by N. Shehaby, is not relevant and does not account for the examples provided by Avicenna. According to him, ‘*ittifāq*’ should be translated by the word ‘agreement’, which is more in accordance with the examples used by Avicenna. Agreement may be understood as an agreement with the facts, i.e. with what happens in the real world. This interpretation could be found in Wilfrid Hodges’ recent book entitled *Mathematical Background to the Logic of Avicenna* (2016). It is supported by many examples like the one above [6, p. 268], since the whinnying of horses cannot in any sense be said to *depend* on the fact that men are speaking. This is what Avicenna calls “agreement in the truth” (“*al-muwāfaqa fī al-ṣidqi*”) [6, p. 265.11]. In this sense, one might consider that ‘*ittifāq*’ expresses in some way a conjunction rather than a conditional, since its elements do not depend on each other and since they are both true. Thus, *ittifāq* would express the weakest meaning of the conditional, since it does not involve any kind of dependence between the antecedent and the consequent. It would thus be close to (1) in al-Fārābī’s classification. But Avicenna provides also other kinds of examples to illustrate *ittifāq* (or *muwāfaqa*) in which *only the consequent* is true, for instance, the following: “If every donkey is talking then every man is talking” [6, p. 270], and he even says that “Agreement (*muwāfaqa*) is nothing but (*laysa illā*) the configuration in which the consequent is true (*wa al-muwāfaqa laysa illā nafsu tarkīb al-tāli ‘alā annahu haqqun*)” [6, p. 279.15]. So the question is the following: Can one interpret *ittifāq* as a conjunction? If not, what is its real logical meaning? We find an answer to this question in Wilfrid Hodges’ article “Ibn Sīnā’s propositional logic” (2014), where the author says that the interpretation of the proposition ‘If p then q’, where ‘if…then’ expresses an *ittifāq* may “come from Peripatetic speculations about how we can know that a sentence ‘If p then q’ is true. Two suggestions were: (a) We can know it because we know that q is true; (b) We can know it because we deduce q from p. Ibn Sīnā reads the *ittifāqī* case as (a) and the *luzūmī* case as (b).” [11, slide 25]. But he says that this notion is “strictly not logical at all” [11, slide 25]. This last judgment may be justified by the fact that when interpreted as true *only* when the consequent is true, it does not correspond to a conjunction, which is true only when *both* propositions are true, nor even to the usual material conditional, which is true in two other cases, besides the one considered by Avicenna.

The notion of *ittifāq* is distinguished from the *real* implication which is called *luzūm* [14, p. 37]. For in the real implication (*luzūm*), the consequent really *follows* from the antecedent either causally or semantically, while in the *ittifāq*, the link between the antecedent and the consequent is not causal. In sum, the real implication or *luzūm* involves the idea that the truth of the consequent *depends on* that of the antecedent, given that the consequent is true *only because* the antecedent is true, while there is no such dependence in the *ittifāq*, since both propositions can be true independently of each other, only by means of their agreement with reality. What Avicenna calls ‘*luzūm*’ expresses thus some kind of *entailment*. In this sense, it may be true even if *both* its elements are *false* provided that the consequent *follows from* the antecedent.

The non-convertible implication may also be expressed by the word “whenever”, which means ‘in all cases’ or ‘in all situations’ or ‘in all times’. In that case, implications would convey a universal content as in ‘Whenever (*kullamā kāna hāḍha*…) this is a man, it is an animal’ [6, p. 232]. The implicit relation, here, is clearly an inclusion, which is not casual, but rather necessary.

In addition, Avicenna also mentions the counterfactual conditional which can be used to deduce the impossible from the impossible, as in ‘If humans were not animals, they would not be sensitive’ [6, p. 238]. This presupposes a necessary semantic relation between the antecedent and the consequent, for being sensitive implies being an animal, so that if one is not an animal, one cannot be sensitive. Pragmatically, as in all counterfactual conditionals, the antecedent is clearly *presupposed* to be *false*, and the consequent is what follows from that falsity. This makes this kind of conditional different from the indicative ones, where the antecedent might be false, undetermined or true, depending on the sentence. Despite this difference, however, it seems that in this kind of conditional too, there is clearly a dependence relation between the antecedent and the consequent. In this respect, its contrapositive form is also valid, for “If humans are sensitive, they are animals” is true too.

Generally speaking, conditional (or implication) is not truth-functional in Avicenna’s frame, for he does not provide its whole truth conditions. The only settled cases are the ones where the antecedent is true. In that case, if the consequent is true, the implication is true, while if the consequent is false, the implication as a whole is false. In the cases where the antecedent is false, the implication *may* be true, but its truth is not warranted. When both propositions are false, a conditional or implication may be true but not necessarily in all cases, for it is the semantic link between its elements that makes it true, despite their falsity, for instance, when one says “If this is a stone, then it is inert”, the sentence is true because being a stone implies being inert, whether the thing is really a stone or not. When the truth-values of the propositions are not known, a conditional may also be true, but not perforce in all cases. The example provided by Avicenna is the following: “If Abdullah is writing, then he is moving his hand” [6, p. 260]. This is true because even if we don’t know if this man is really writing or not, we know that if he is writing, then he surely is moving his hand, by definition. The two examples are reminiscent to al-Fārābī’s kind of conditional ($2b_2$) above, which expresses an analytic implication, true by definition.

It seems then that the truth or the falsity of a conditional are determined on the basis of the meanings of its components, as all the examples show. Thus an implication, in Avicenna’s frame, is not a material conditional. It is then intensional for it is based on the meanings of its elements.

In addition, Avicenna uses quantifications to express the several kinds of implications. These quantified implications are the following:

Ac: Whenever (*kullamā*) A is B then H is Z [6, p. 265] [whenever = always, if, then]

Ec: Never (= *laysa al battata*) (if A is B then H is Z) [6, p. 280]

Ic: Maybe (*qad yakūn*) if every A is B then every H is Z [6, p. 278]

Oc: Maybe not (if… then …) (*qad lā yaqūn*) [7, p. 235]

These quantifiers have been interpreted in two ways by different authors. Nicholas Rescher [13] says that they range over times, while Zia Movahed [12] says that they range over situations. In the first case, one can interpret them as temporal implications; in the second case, they would be more like modal implications, for the situations are not necessarily temporal, since they could also account for conceptual, for instance, mathematical cases. The temporal interpretation seems too weak to account for the conceptual and analytic link between the antecedent and the consequent; so the interpretation in terms of situations seems better because it is more general and more in accordance with the examples provided by Avicenna. However, one could not credit Avicenna with a theory similar to Possible Worlds Semantics, which is too sophisticated and mathematically inspired to be endorsed by a medieval author, nor even with a frame comparable to Carnap's theory of "state descriptions", which is also too related to modern (logical) probability theory to be evoked by Avicenna. The only possible worlds that Avicenna explicitly talks about are future worlds, but he evokes them in his modal logic rather than his hypothetical one, in the context of his analysis of the concept of possibility [6, p. 141].

Now what do the expressions (1) 'whenever if...then', (2) 'maybe if...then', (3) 'not whenever if...then' and (4) 'never if...then' really mean in Avicenna's frame?

(1) seems to express the *real* implication, i.e. the relation of 'following from' or of *entailment*, which is necessary and warrants the deductibility of the consequent. It is used in a universal affirmative proposition, that is, $\mathbf{A}_C$; (3) is $\mathbf{A}_C$'s contradictory and corresponds to $\mathbf{O}_C$, which contains a conjunction. For although what is literally said is 'if...then', what is *implicitly meant* is 'and' or something close to it. As to (2) and (4), they are also contradictory, for (4) is used in the universal negative proposition $\mathbf{E}_C$, while (2) is used in its contradictory $\mathbf{I}_C$, and expresses rather a conjunction.

These interpretations are clearly expressed in the following formalizations (where 's' stands for 'situation(s)'):

$$\mathbf{A}c = (\forall s)(Ps \supset Qs) \;/\; {\sim}\mathbf{A}c = {\sim}(\forall s)(Ps \supset Qs) = (\exists s)(Ps \wedge {\sim}Qs) = \mathbf{O}c$$

$$\mathbf{E}c = (\forall s)(Ps \supset {\sim}Qs) = {\sim}(\exists s)(Ps \wedge Qs) = {\sim}\mathbf{I}c, \text{ so } \mathbf{I}c = (\exists s)(Ps \wedge Qs)$$

Thus it seems that 'if ... then' in $\mathbf{I}_C$ and $\mathbf{O}_C$ is close to 'and', which seems to be its *implicit* or *presupposed* meaning, that is, its *pragmatic* meaning; while in $\mathbf{A}_C$ and $\mathbf{E}_C$, 'if...then' really expresses the relation of following from or of entailment, which is necessary, intensional and sometimes causal. The quantification on situations gives a modal connotation to all these propositions, for "$(\exists s)$" means "for some situation", while "$(\forall s)$" means "for all situations". So $\mathbf{I}_C$ would be interpreted as follows: "There is one (or more) situation(s) where P is the case and Q is the case", while $\mathbf{A}_C$ would be interpreted as saying: "In all situations, if P is the case, then Q is the case". These interpretations account for the *weak* and the *strong* meanings of the conditional. As to the *medium* meaning, Avicenna does not render it in a formal way and he does not seem to give it much importance, although he evokes it in his informal discussion of the conditional. Nevertheless, if one wants to express it in a more precise way, one could perhaps appeal to the concept of *imprecise probabilities* (expressed by the word "likely") and interpret it as saying something like: "In most situations, if P is the case, then Q is the case". But Avicenna himself never used the word "most" in his quantifications.

Now what is presupposed in the meanings of the universal propositions? The answer lies in the hypothetical syllogistic constructed by Avicenna with the four propositions above, which duplicates the categorical syllogistic, for we can find in the hypothetical syllogistic the counterparts of all valid categorical syllogisms.

For instance, the hypothetical *Barbara* is expressed thus:

- Whenever A is B, then C is D (= $\mathbf{A}_C$)

- Whenever C is D, then H is Z (= $\mathbf{A}_C$)

- Therefore whenever A is B then H is Z ($\mathbf{A}_C$) [6, p. 296]

Among these syllogisms, we find the hypothetical analogue of *Darapti*, for instance, which is expressed formally as follows:

$[(\forall s)(Ps \supset Rs) \wedge (\forall s)(Ps \supset Qs)] \supset (\exists s)(Qs \wedge Rs)$.

As it stands, however, *Darapti* should not be valid, because the main conjunction may be true when the two premises are true, and the premises, which contain conditionals, might be true when their two propositions are false, in which case the conclusion which contains a conjunction, would be false. So why does Avicenna hold this mood valid, then? This is so because he presupposes that the antecedent of the two premises (of the form $\mathbf{A}_C$) must be true too. For when these antecedents are true, the consequents in the two universal premises of *Darapti* will be true; consequently the conclusion will be true too. This presupposition appears in the following quotation:

"When we say: 'If A is B, then H is Z', we assume from this (*nūjibu min hādhā*) that at any time where 'A is B' is the case and when A is B then H is Z, as if the fact that H is Z follows the fact that A is B, in so far as in effect A is B (*min ḥaythu hūwa kā'inun A [hūwa] B)*" [6, p. 263. 8–9]

This means that the real implication in $\mathbf{A}_C$ *presupposes* that the antecedent of the universal affirmative is true. Only in that case, the hypothetical *Darapti* could be valid; otherwise it is not. The first premise of *Darapti* above would then be expressed by the following conjunction '$(\exists s)Ps \wedge (\forall s)(Ps \supset Rs)$], which stipulates that the antecedent is true [9, p. 194].

What about $\mathbf{E}_C$? Does it require the same presupposition?

Given the syllogisms containing $\mathbf{E}_C$, no presupposition of that kind is required in $\mathbf{E}_C$. For instance, the hypothetical *Felapton*, which contains $\mathbf{E}_C$ and $\mathbf{A}_C$ is valid only when we presuppose that the antecedent of $\mathbf{A}_C$ is true. Nothing else is required.

Felapton is formalized as follows:

$$[\sim(\exists s)(Ps \wedge Rs) \wedge (\forall s)(Ps \supset Qs)] \supset (\exists s)(Qs \wedge \sim Rs).$$

Given the presupposition related to $\mathbf{A}_C$, 'Ps' is true. In that case, Qs must be true, in order for the whole implication to be true; so ~Rs must be true too, in order for the conclusion to be true; consequently Rs is false, being the contradictory of ~Rs. Since Rs is false, $\mathbf{E}_C$, here, is true without any further requirement.

Does this mean that the meanings of the implications in $\mathbf{A}_C$ and $\mathbf{E}_C$ are different? We might say that $\mathbf{A}_C$ and $\mathbf{E}_C$ have the same *semantic* meaning ('q *necessarily follows* from p' in the first case, and 'not q *necessarily follows* from p' in the second one) but not the same *pragmatic* meaning, for they do not require the same presuppositions.

4. Conclusion

Implications are intensional in both frames, for they depend on the meanings of the elements involved. However, the two authors analyse the different kinds of implication in different ways. Al-Fārābī distinguishes between an accidental implication, a regular and frequent one and a necessary one, which is itself sub-divided into two kinds: an incomplete one and a complete one. The semantic as well as the pragmatic meanings of all these kinds are different for they are not true in the same conditions and do not carry the same presuppositions. All of them seem to be intensional for their elements in all cases are related semantically and / or causally, but while the accidental implication exemplified by (1) above involves mainly a succession in time which may occur only once, the regular implication illustrated by (2a) involves a frequent succession in time and a *probable* causal or at least natural link between the antecedent and the consequent. As to necessary implications, they involve either a causal or a conceptual link between the antecedent and the consequent. When the implication is complete, the relation is convertible and its pragmatic meaning is an intensional and necessary biconditional. When it is not complete, the implication pragmatically expresses a necessary inclusion. In both cases, the relations have a modal connotation, but al-Fārābī does not use explicitly the word 'necessarily' to express them.

As to Avicenna, he does not admit the complete implication endorsed by al-Fārābī for formal and syntactical reasons, but he does distinguish between a weak, a medium and a strong implication which are expressed by different words in the natural language he is considering. His analysis of these different implications is more developed than that of al-Fārābī, although it is clearly influenced by it, for he enters into more details when analysing the causal links between the antecedent and the consequent in implications. According to him, the weak implication does not really involve a semantic relation between its elements, for it expresses what he calls '*ittifāq*', which has been translated as 'chance connection' or as 'agreement' by different authors, but seems rather close to the agreement with the facts if we consider Avicenna's explanations. The '*ittifāq*' is opposed to the real implication, called '*luzūm*', which is close to the notion of *entailment* or the relation of 'following from'. This last relation is intensional, necessary and universal, for it can be expressed by the word 'whenever'. But it is not truth-functional.

Avicenna expresses the implications used in his hypothetical logic by means of existential and universal quantifications. These quantifications give them modal connotations, for the quantifiers may range over situations. When formalized, the existentially quantified propositions express a conjunction, while the universally quantified ones express intensional entailments.

So the implicit and pragmatic meanings of the particular conditionals $\mathbf{I}_C$ and $\mathbf{O}_C$ seem close, since they both are expressible by conjunctions, while $\mathbf{A}_C$ and $\mathbf{E}_C$, although they carry the same semantic meaning, which involves a universal and necessary dependence relation, do not have the same pragmatic meaning, for $\mathbf{A}_C$ presupposes the truth of its antecedent. This presupposition is strongly suggested in Avicenna's hypothetical syllogistic, for it is required to validate some kinds of third figure hypothetical syllogisms. Such a presupposition is not required for $\mathbf{E}_C$, which is thus pragmatically different from $\mathbf{A}_C$.

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References

1. Al-Fārābī, Abū Nasr. *al-Alfāḍ al-mustaʿmala fī al-mantiq*, second edition by Mohsen Mehdi, Beirut: Dar al Mashriq, 1968.
2. Al-Fārābī, Abū Naṣr. Kitāb al-Maqūlāt, In. Rafik Al Ajam (ed.), *al-Manṭiq 'inda al-Fārābī*, Vol. 1, Beirut: Dar el Machriq, 1986a, pp. 89–132.
3. Al-Fārābī, Abū Naṣr, Kitāb al-Qiyās, In. Rafik Al Ajam (ed.), *al-Manṭiq 'inda al-Fārābī*, Vol. 2, Beirut: Dar el Machriq, 1986b, pp. 11-64
4. Al-Fārābī, Abū Nasr. *al-Maqūlāt*, In. M. Tekī dench Proh (ed.), *Al Mantiqiyāt lil Fārābī*, vol. 1, Maktabat Ayat allah al-udhma al-mar'achi al-najafi, 1988a.
5. Al-Fārābī, Abū Nasr. *al-Qiyās*, In. M. Tekī dench Proh (ed.), *Al Mantiqiyāt lil Fārābī*, vol. 1, Maktabat Ayat allah al-udhma al-mar'achi al-najafi, 1988b.
6. Avicenna. *al-Shifā', al-Mantiq 4: al-Qiyās*, S. Zayed (ed.), Cairo: Wizarat al Thaqafa wa-l-Irsad al qawmi, 1964.
7. Avicenna. *Al-Ishārāt wat-tanbīhāt*, with the commentary of N. Tūsi, Part 1, Third edition, Cairo: Dar el Ma'ārif, 1971.
8. Bobzien, S. Ancient Logic, *Stanford Encyclopedia of Philosophy*. Ed. Edward N. Zalta. 2006. Available online at: http://plato.stanford.edu/entries/logic-ancient/.
9. Chatti, S. Syncategoremata in Arabic Logic, al-Fārābī and Avicenna, *History and philosophy of logic*, 35:2, 2014, pp. 167-197
10. Hodges, W. *Mathematical Background to the Logic of Avicenna*, 2016. Book, Unpublished. Available at: http://wilfridhodges.co.uk/arabic44.pdf
11. Hodges, W. Ibn Sīnā's Propositional Logic, 2014, talk available at: http://wilfridhodges.co.uk/arabic43.pdf

12. Movahed, Z. A critical examination of Ibn Sīna's Theory of the Conditional syllogism, *Sophia Perennis*, Vol 1, N° 1, 2009. Available online in: www.ensani.ir/storage/Files/20120507101758-9055-5.pdf
13. Rescher, N. *Studies in the history of Arabic logic*, University of Pittsburg Press, 1963 (Arabic translation by Mohamed Mahrane, 1992).
14. Shehaby N. *The propositional Logic of Avicenna. A translation from al Shifā, al Qiyās*, Dordrecht: Kluwer, D. Reidel, 1973.
15. Strawson, P. F. *Introduction to Logical Theory*, London: Methuen, 1952.