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Role of Logic in Cognitive Science

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Abstract:
In their work McCulloch and Pitts describe an idea of representing all of nervous activity in terms of propositional logic. This idea was quickly challenged. One of reasons for this challenge was rising believe that logic is unable to describe most of human cognitive processes. In this paper we will analyse premises of original McCulloch and Pitts proposition. Following that, we will ask about ability of symbolic (logical) systems to represent human cognition. We will finish by analysing relation between symbolic and subsymbolic computing, in hope of bridging the gap between the two.
Keywords: nonmonotonic logic; neural networks; human reasoning.

1. Introduction

The gap between symbolic and subsymbolic (neural network) modes of computation is a riddle for the philosophy of mind. Complex symbolic systems like those of grammar and logic are essential when we try to understand the general features and the peculiarities of natural language, reasoning and other cognitive domains. On the other hand, most of modern theories assume stance seeing that cognition resides in the brain and that neuronal activity forms its basis. Yet neuronal computation appears to be numerical, not symbolic; parallel, not serial; distributed over a gigantic number of different elements, not as highly localized as in symbolic systems. Moreover, the brain is an adaptive system that is very sensitive to the statistical character of experience. “Hard-edged” rule systems (classical logic) are not suitable to deal with this aspect of behavior. We will start with analyzing the roots of neural network approach, seen here as paradigmatic example of subsymbolic computation approach. It is widely accepted that this method started with the work by Warren S. McCulloch and Walter H. Pitts titled *A Logical Calculus of the Ideas Immanent in Nervous Activity* [16]. We will try to show connections between this approach and logical description of reasoning processes.

In the early days of cognitive science, logic was taken to play both a descriptive and a normative role in theories of intelligent behavior. Descriptively, human beings were taken to be fundamentally logical, or rational. Normatively, logic was taken to define rational behavior and thus to provide a starting point for the artificial reproduction of intelligence. Both positions were soon challenged. As it turns out however, logic continues to be at the forefront of conceptual tools in cognitive science. What is embodied by competitive to connectionist (neural network) AI approach. Rather than defeating the relevance of logic, the challenges posed by cognitive science have inspired logicians to enrich the repertoire of logical tools for analyzing reasoning processes and computation. We will examine the role of nonmonotonic logics in this endeavor. This kind of logic allows to overcome logics problem to deal with "soft-edged" rules, that neural networks excel at.

2. Logic and Neuroscience

Logic is a brand of science that deals with studying of the *correct reasoning*. Reasoning is a mental activity and as such is seen as at least closely related to the way the mind works. Classically, in logic the correct reasoning was synonymous with deductive reasoning and ordinary deductive reasoning takes place in natural language. That is why, to answer the question about the role of logic in science about cognition, we have to first ask about the relation between natural and formal language. As stated above, logic had two dimensions to its research, descriptive and normative theories of intelligent behavior. Those two dimensions find their explication in two kinds of answers to the question about natural-formal language relation. First view states that, at least some sentences of natural language have underlying logical form and these form are represented by formulas of formal language – this view is compatible with the descriptive dimension of logic. Since reasoning is an activity performed in language, logic provides deep structure of correct reasoning. This view is represented by philosophers such as Davidson [3]. The second view is that natural languages are ambiguous and vague and as such should be replaced by formal language lacking these features – this view is compatible with normative dimension of logic. According to a view like this, logically correct reasoning represents ideal sought after activity in natural language. In philosophy this approach can be found in works of W.V.O. Quine [19]. With the rise of cognitive science both of those roles were put into question. Instead of eliminating logic out of cognitive science it motivated logicians to expand tools in their repertoire.

Parallel to modern logic, a different type of science has begun its emergence since late 19th century. One that examined physical basis, rather than abstract rules governing the work of human mind. It was called neuroscience and it seemed as if nothing connected the two activities. It began to change with the publication of *A Logical Calculus of the Ideas Immanent in Nervous Activity* at the end of first half of 20th century. This paper is often cited as the starting point of research in artificial neural networks; for us it is the first moment in which research fields of logic and neuroscience meet. McCulloch and Pitts state in their paper that activity of any neuron may be represented as a proposition. We can assert that relations existing among nervous activities can be represented as relations between propositions. They notice two difficulties immanent in this approach, both problems rising from the physiological aspects of nervous activity. The first concerns the effects of previous excitations on future activations of nervous cells. The second notices that learning has to be a permanent change in neural structure. Nonetheless, they see this only as problematic in the case of asserting factual equivalency (or identity) between calculus of logical propositions and neural structures. Their statement is of much weaker kind; physiological aspects of neural systems do not affect the fact that relations of propositions corresponding to certain nervous activities are that of propositional logic.

Because of that they make certain assumptions about their calculus. These assumptions are aimed at simplifying of the behavior of real neurons.

(1) Activity of neurons is binary, they are either on or off.
(2) The threshold of neuron activation is independent of previous activations of a neuron.
(3) The only delay significant for nervous activity is the synaptic one.
(4) Inhibitory synapses absolutely prevent activation of neuron at certain moment.
(5) The structure of neural net does not change in time.

All of the above assumptions seam necessary to represent the neural activity in logical calculus. Additionally they arise as a result of the difference between formal and factual equivalency, authors distinguished. The actual neural activity would not comply to such rules, but the idea is – as stated before – that they talk about the abstract calculus of "mind".

The authors divide neurons into two categories. One that they name *peripheral afferents* - input neurons that do not receive signals from any other neuron in the net. Second consisting of all other neurons. Next step they take, consists of developing a logical apparatus necessary to define basic concepts of their calculus. As noted by Stephen C. Kleene [12] the approach and notation used by McCulloch and Pitts are obscure and hard to understand, that is why we will try to streamline it and present in a more approachable manner. Let us consider two problems presented by the authors: "(…) first, to find an effective method of obtaining a set of computable *S* constituting a solution of a given net [16, p. 103]."

In other words, an answer to the question: what does a given net compute (How to calculate behavior of the net)? This is called the *solution* of a net. We can define the solution of a net as a set of logical sentences of the form: neuron *i* is firing if and only if a given logical combination of the firing predicates of input neurons at previous times and some constant sentences including firing predicates of these same neurons at $t=0$ is true. These sentences are the solution for a net if they are all true for it.

The second problem is characterized as follows: "(…) to characterize a class of realizable *S* in effective fashion (ibid. 103)." The question here can be summarized as: can a certain net compute a given logical sentence (How to find a net that behaves in a specific way)? A sentence is *realizable* for a net if it is true for that net, or in other words when a net can compute it.

Following Stenning and van Lambalgen [21, pp. 218-219] we can define net, in modern fashion, as follow:

Definition 1 *Net is a graph on a set of computational units, connected with weighted links that can be either excitatory of inhibitory.*

Accordingly units can be defined:

Definition 2 *Computational unit (unit) is a function with the following behavior:*

- *Inputs are delivered through weighted links* $w_j \in [0, 1]$.
- *Links can be either excitatory* ($x_{1,\ldots,}x_n \in \mathbb{R}$) *or inhibitory* ($y_1,\ldots, y_n \in \mathbb{R}$).
- *If an inhibitory link is active* ($y_i \neq 0$)*, connected unit is shut off, and outputs 0.*
- *Otherwise, quantity* $\sum_{i=1}^{i=n} x_i w_i$ *is calculated; if it equals or exceeds threshold* (Θ) *unit is active and outputs 1; otherwise, unit rests and outputs 0.*

We can represent logical connectors in terms of units and connections. Conjunction can be represented by unit witch two excitatory inputs and threshold of 2; alternative can be represented by unit witch two excitatory inputs and threshold of 1; negation can be represented by unit witch one excitatory input and one inhibitory.

Authors propose a class of expressions representing solution of net, called *temporal propositional expressions* (TPE). TPEs have a single free variable, identified as discreet time.

Definition 3 *TEPs are defined by the following recursion:*

- *Predicate of one argument is a TPE.*
- *Logical disjunction, conjunction and negated conjunction (and not) of TPEs with the same free variable are by themselves TPE.*
- *Nothing else is a TPE.*

Theorems 2 and 3 of the discussed work give us a version of a rule of substitution for neural nets and a set of basic expressions from which those expressions can be constructed. Rule of substitution can be summarized as follows: *replacing peripheral afferent in a realizable net by a realizable net is in itself a realizable net.* By that definition all TPE are realizable. Set of basic realizable expressions follows then from definition of TPE and consist of nets representing operations of precession, disjunction, conjunction and negated conjunction. Respectively each net is represented below by figures 1a-d. Lines witch arrows at ends represent excitatory connections, lines witch circles at the ends represent inhibitory connections.

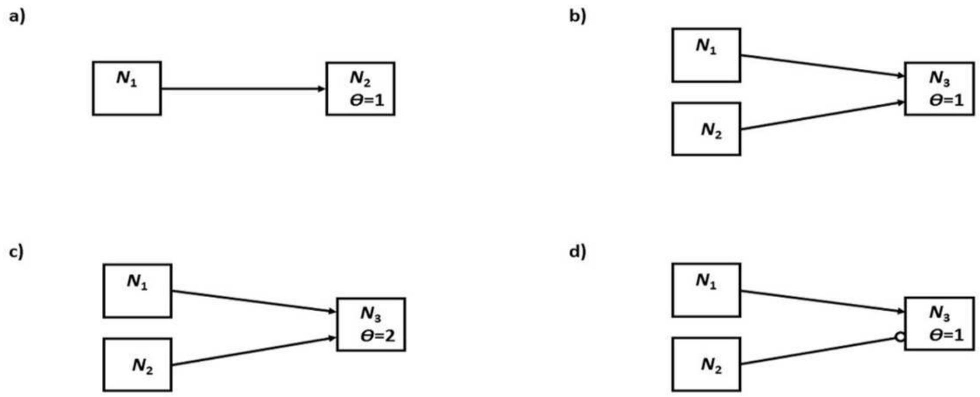

Figure 1. a) precession; b) disjunction; b) conjunction; c) negated conjunction. Version of nets presented in McCulloch and Pitts [16] adapted to presented definitions.

It can be described by following expressions:
a) $N_2(t) \equiv N_1(t\text{-}1)$
b) $N_3(t) \equiv N_1(t\text{-}1) \vee N_2(t\text{-}1)$
c) $N_3(t) \equiv N_1(t\text{-}1) \wedge N_2(t\text{-}1)$
d) $N_3(t) \equiv N_1(t\text{-}1) \wedge {\sim}N_2(t\text{-}2)$

The rule of substitution, following from mentioned theorems gives us a simple procedure of constructing neural nets. The authors propose to consider an example of heat sensation evoked by a short time cooling [16, pp. 106-107]. If a cold object makes contact with the skin and is instantaneously removed, the sensation of heat will occur; if the same object will not be removed, the sensation of cold occurs without the preliminary heat sensation. This happens for cold receptors but not for heat receptors. We assume there are different receptors responsible for heat and cold detection, but the same neuron is responsible for heat sensation in both cases. Because of that, the synaptic delay for the sensation of cold must be greater by one then for the sensation of heat. We can reproduce this effect using the described method by transforming the above mentioned expressions using the rule of substitution. We receive:
e) $N_3(t) \equiv N_1(t\text{-}1) \vee [N_2(t\text{-}3) \wedge {\sim}N_2(t\text{-}2)]$
$N_4(t) \equiv N_2(t\text{-}2) \wedge N_2(t\text{-}1)$

We can notice this net has 2 solutions, one for heat and one for cold respectively. Figure in which both of those expressions are realizable can be constructed from figures 1a-d in the following manner.

Beginning in the standard logical manner, we first consider the function enclosed in most brackets. We receive a net of form 1a representing expression:

$$N_a(t) \equiv N_2(t\text{-}1) \qquad (1)$$

Proceeding outwards, we introduce two nets, both starting from nodes N_a and N_2. One of form 1c ending in N_4. We receive:

$$N_4(\mathrm{t}) \equiv N_a(t\text{-}1) \wedge N_2(t\text{-}1) \qquad (2)$$

We must advance time variable for previous expression where we substitute it in this formula. Which is equivalent to:

$$N_4(t) \equiv N_2(t\text{-}2) \wedge N_2(t\text{-}1) \tag{3}$$

Second of form 1d ending in N_b. Giving us:

$$N_b(t) \equiv N_a(t\text{-}1) \wedge \sim N_2(t\text{-}1) \tag{4}$$

Substituting N_a for its equivalent in proper time interval we receive:

$$N_b(t) \equiv N_2(t\text{-}2) \wedge \sim N_2(t\text{-}1) \tag{5}$$

Finally we run net of form 1b starting in N_1 and N_b to neuron N_3. So that:

$$N_3(t) \equiv N_1(t\text{-}1) \vee N_b(t\text{-}1) \tag{6}$$

Again, due to substituting N_b for equivalent formula, (6) can be expressed as:

$$N_3(t) \equiv N_1(t\text{-}1) \vee [N_2(t\text{-}3) \wedge \sim N_2(t\text{-}2)] \tag{7}$$

The whole net can be represented by figure 2.

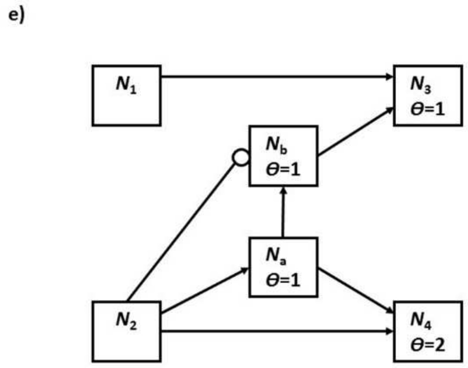

Figure 2. Net realizing expressions e). Modified from McCulloch and Pitts [16], to adapt to presented definitions.

That way we can create nets realizing underlying logical functions. We can clearly see that McCulloch saw propositional logic as an underlying structure of human mind. He writes:

> To psychology, however defined, specification of net would contribute all that could be achieved in that field – even if analysis were pushed to ultimate psychic unit or "psychon", for psychon can be no less than the activity of a single neuron. Since that activity is inherently propositional, all psychic events have an intentional, or "semantic" character. The "all-or-none" law of these activities, and the conformity of their relations to those of the logic of propositions, insure that relations of psychons are those of two-valued logic of propositions [16, pp. 113-114].

This sentence presents author's intentions of proving logical character of human mind activity. The nervous system is described as based on mechanics equivalent to propositional logic. Unfortunately, it highlights weak points of both logical approach and neural nets of McCulloch-Pitts type. This effort to "marry" logic and neuroscience marks the first and last attempt to do so by way of classical propositional logic. It may be because it highlighted certain weaknesses of logical approach – weaknesses we will analyze in the following paragraph.

3. Logic and Human Cognition

The above described neural networks meet with plenty of critique. Some of it is coming from biological background. For example, it was quickly noticed that the assumption about neurons always being in one of two possible states is biologically inadequate. In the context of discussion presented in this paper, what is more important is the fact that some developments in research of human cognition put descriptive dimension of logic under doubt. It remained a possibility that logic described a normative system of what certain types of reasoning should be, but it no longer could be perceived as a representation of natural cognitive processes.

If we accept descriptive dimension of logic, then at some level human reasoning should be based upon a set of simple logical procedures. However, humans tend to do surprisingly poorly when faced with tasks of performing simple logical procedures. This phenomenon was noticed and described by Wason in, named after him, *Wason Selection Task* [23], [24]. The task puts a subject in choice situation guided by a simple rule. The choice is made between cards. Each card has on it either a number or a letter. Cards, on a side visible to subject, read D, K, 7 and 3. The subject is then familiarized with singular rule of the task; "Every card which has D on one side must have a 3 on other". After that the question is posed; "Which if any, of the cards must be turned over to judge if the rule is true". From the classical logic standpoint the "if" in the rule should be read as material conditional, making the rule D → 3. Hence, using modus ponens (MP), we may deduce that D has to be turned to check if there is 3 on back side. Likewise, using modus Tollens (MT), we deduce 7 has to be turned over to ascertain if there is no D on the reverse. Making, assumed, correct answer D and 7. The most popular answer given is however, D and 3. In fact D is almost always given as one of the answers. Conversely, 7 is rarely seen as necessary to turn over. Some researchers, including Wason, see that as an evidence that humans are poor at even simple tasks. If we would accept Wason's interpretation of "D → 3" rule, we have to accept that people are bad at using MT, so tasks requiring it as reasoning schemata lead to fallacious reasoning.

Interesting development appeared out of certain rephrasing of Wason task [8], [11]. The original selection task took place in abstract domain of letters and numbers. Rephrasing the problem in a domain familiar to subjects changed outcome drastically. In the mentioned rephrasing, numbers and letters were replaced by ages and kinds of drinks. When the task is to confirm a rule "if person drinking beer, then that person is 19 or older", subjects performed nearly perfectly. Noticing the fact that rephrasing Wason's task in a familiar domain brings error rate down contradicts formal-logical model of reasoning.

The fact that context has an effect on the ability of subjects to deduce a correct answer may be explained by the theory of two competing systems of reasoning. It can be reasonably doubted that experiments like *Wason selection task* test what authors actually believed they did. Question can be posed: what does actually count as reasoning in natural environment? Proposing dual process theory of reasoning can explain the described situation. Here we assume reasoning consists of two systems supplementing each other. Describing *system 1* Evans writes:

> System 1 is (…) not a single system but a set of subsystems that operate with some autonomy. System 1 includes instinctive behaviors that would include any input modules of the kind proposed by Fodor.(…) The System 1 processes that are most often described, however, are those that are formed by associative learning of the kind produced by neural networks.(…) System 1 processes are rapid, parallel, and automatic in nature; only their final product is posted in consciousness [5, p. 454].

By contrast, *system 2* is slow, sequential and symbolic in nature. Logical reasoning belongs in system 2, because of that tasks performed by system 1 do not conform to rules of logic. This is also a reason why neural networks cannot be logical machines – system 1 is equivalent to a subsymbolic computing system.

We then have two approaches to reasoning. Let us call the first algorithmic: it states MP-MT asymmetry in Wason selection task is an effect of MT being harder to implement on algorithmic level. A sample of this approach can be found in Oaksford and Chater *The Probabilistic Mind: Prospects for Bayesian Cognitive Science* [17]. That is why reasoners trying to reason deductively have problems with finding the correct solution. The second, called non-logical reasoning, argues that subjects do not attempt to deductively find solutions to posed questions. That way MP-MT asymmetry is not a matter of competency gap but rather "inadequacy" of utilized competences.

Authors Stenning and van Lambalgen [21] propose a different analysis of context effect on task results. They attack Wason's assumption that "if" in the rule has to be interpreted as a material conditional, which puts doubt on the assertion that there is only one correct answer. They propose to distinguish between the two forms of conditionals: one descriptive; other deontic. That may explain why two statements (Wason task and Wason task rephrased in familiar context), of supposedly the same logical form can lead to radically different outcomes. The task when rule is seen descriptively, is viewed by subject, as concerning determining if the rule is true or false for the given cards. With deontic interpretation of conditional truth of the rule is not an issue, only whether the rule is being followed or not. They notice that the original task may be interpreted as containing descriptive rule, increasing the cognitive burden on subjects. However, in the context of this paper, the more important aspect is the observation of the processing side nonmonotonic logic provides adequate model for analysis of subjects' reasoning. Presenting human reasoning in terms of nonmonotonic logic explains why reasoning in a system which could not be explained in terms of logic. More precisely it is cold but not in classical logic. This system can still be represented by a set of reasoning rules, just not build upon deductive inferences. In this view, deontic interpretation of the rule can be associated with classical logical conditional, when descriptive interpretation entails a different kind of conditional, nonmonotonic, that should be read "typically this X entails Y".

To answer what differentiates classical logic from the nonmonotonic one, let's consider the following property of deductive logic, one that holds for relation of classical consequence "$\models$":

Monotony: if $A \models B$ then $A \cup C \models B$.

Monotony states that if B is a logical consequence of A, then it is also a consequence of any set containing A as its subset. In other words, adding a new premise to inference cannot pre-empt earlier conclusions. Monotony follows straight from nature of logical consequence relation, $A \models B$ holds when B is true on every interpretation on with every sentence in A are true. Clearly, every day inferences do not conform to this requirement. Actually, not abiding to it is a defining property of so called defeasible reasoning, the kind of nonmonotonic inference that supposedly describes how every day reasoning works. Literature is rich in analyses of reasons why deductive reasoning is inadequate in describing the so called everyday inferences [4], [18].

There are many examples of nonmonotonic logics, but for our purpose semantic approach of Shoham [20] will be used. This theory is often referred to as *preferential logic*, it is a simple and elegant approach. Additionally it can be used to explain the MP-MT asymmetry and perceived system 1 - system 2 dichotomy.

Definition 4 *$L_\angle$ is a nonmonotonic preferential logic generated from L and $\angle$ when following demands are met:*

- *In a standard logic L that satisfy following demand: for all A, B and C in L, if $A \models B$, then also $A \land C \models B$.*
- *A strict partial order $\angle$ on the model of L is defined: $M_1 \angle M_2$, meaning that M_2 is preferred over M_1.*
- *Preferred model is one that: Model M preferentially satisfies A ($M \models_\angle A$); $M \models A$ and there is no other model M' such that $M \angle M'$. We call M preferred model of A.*

We can define a preferential consequence relation for that logic in the following fashion:

Definition 5 *Preferential consequence: A is a preferential consequence of B ($A \rightarrow_\angle B$) for any M, if $M \models_\angle A$, then $M \models B$.*

In other words $A \rightarrow_{\angle} B$ if all preferred models of A are models of B. This relation is nonmonotonic because it is possible that A and C have preferred models that are not preferred models of A alone. So with addition of C it may be that that B no longer holds in all preferred models of $A\&C$.

Now we can notice that preferential consequence relation easily explains MP-MT asymmetry. It refers to preferred models of A, but also to all models of B. Because of it, this consequence relation does not contrapose. For the relation to be contrapositive it would be required that all preferred models of not-B be models of not-A. It is quite possible there exist not-preferred models of A wich are also preferred models of not-B. Thus, the definition is not satisfied for not-$B \rightarrow_{\angle}$ not-A, and MP-MT asymmetry is explained.

4. Symbolic vs. Subsymbolic Paradigms

Classical view of human cognition is one analogous to symbolic computation in digital computers [23]. On this account information is represented as a string of symbols in memory of a computing unit or on a piece of paper. On the other hand connectionist claim that information storage have a non-symbolic character, information is stored in weights of connections between units of neural net. Connectionists perceive mental processes as dynamic and distributes evolution of activity in neural net. Each unit of this net activates depending on strength of connections and activity of neighboring units.

In late 20th century a heated debate ensued between proponents of symbolic and connectionist (subsymbolic) approach to cognitive science. One of most vocal opponents of connectionism were J. Fodor and Z. Pylyshyn [6]. They argued that no connectionist model of mind can have compositional semantics. That is the case because, as they argued, mental representations require systematicity and no neural network can exhibit this feature; therefore modeling of cognition have to be symbolic not connectionist. Systematicity is understood as a feature of representation that makes meaning of representation to correspond systematically to its structure. That means if we are able to represent expression “Peter killed Paul”, we must be able to represent expression “Paul killed Peter”. Putting details of this debate aside, prevailing view was that symbolic and subsymbolic approach are different and incompatible.

Concurrently, radical connectionists claimed inadequacy of symbolic processing as a model of mind. We discussed this in part 3 of this paper. To reiterate, they claimed that symbolic computing poorly explains holistic representation of data, spontaneous generalization, effect of context, and many other aspects of human cognition captured by their models. This failure to match the flexibility and efficiency of human cognition is in their eyes a symptom of the need for a new paradigm in cognitive science. This approach can be called *radical connectionism,* and it agenda can be described as eliminating symbolic processing as inadequate in cognitive science.

However, many connectionists do not view their paradigm as opposition to symbolic computation. So called implementation connectionists present an image in with mind is a neural net, but also a symbolic process on higher level of abstraction. In that view role of connectionist researcher is to find how a machine required to perform symbolic processes can be forged from neural network resources. Even more interestingly since 1990’s, models combining subsymbolic and symbolic paradigms appeared [1], [7], [25]. Unfortunately hybrid approach to problem fails address question about underlying difference between symbolic and distributed representation. Because of that it is proposed to inquire about possible equivalency between symbolic and subsymbolic models of computation.

The idea is that connection can come again from the side of logic, similarly to original McCulloch and Pitts proposition. Instead of classical logic we would turn to nonmonotonic one. This way we avoid problems with inadequacy of logical description to data collected during research on human cognition. The close relation between symbolic computation and logic is well known [10]. With neural nets it have to be shown that every logical model of a system is isomorphic to a member of distributed, or subsymbolic, subset. In fact, it is trivial to show that some nonmonotonic reasoning may be represented by neural networks. An example of neural net generatinc a nonmonotonic inference was shown as early as 1991 [2]. They propose to consider network consisting of four neurons $x_{1,}\ldots$, x4. They identify sets of active neurons with schemata. There are three schemata α, β, γ. Corresponding to following sets of active neurons: $\alpha = x_1, x_2$; $\beta = x_2, x_3$; $\gamma = x_4$. There are two excitatory connections, one between x_1 and $x_{2,}$ other between x_2 and x_3. Third connection between x_4 and x_3 is inhibitory connection. Assuming that inhibitory connection is stronger than excitatory one between x_2 and $x_{3.}$ Following situation is possible: giving α as input, the network will activate β ($\alpha \vDash \beta$); extending inputs to α and γ, effects in withdrawal of β ($\alpha \wedge \gamma \nvDash \beta$). That situation directly defy monotonicity, since including new premises (inputs) reduce set of conclusions.

However, this is just one specific case when neural network exhibits behavior equivalent to some nonmonotonic theory. Can we have an equivalency theorem? Theorem of that kind would show that for every nonmonotonic theory there exist a neural network able to compute that theory. Fortunately theorems of that kind has been proposed by logicians over the last few decades. Holldobren and Kalinke [9] gives a theorem of that kind. They show that for every logical program there exists a three layer feed forward network which computes it. Other example is presented by Leitgeb [13], [14], [15]. His proposition is especially interesting in context of this debate. He propose a way to represent propositional letters as a set of nodes in neural networks. At the same time Leitgeb shows that any dynamic system performing calculations over distributed representation can be interpreted as symbolic system performing nonmonotonic inferences. What can be interpreted as functional equivalence of reasoning representation between symbolic and subsymbolic processes.

5. Conclusions

The methodological position pursued in this article was one which looks for unification. In the case under discussions the point was to assume that symbols and symbol processing are a macro-level description of what is considered a connectionist system at the micro level. Hence, the idea is that the symbolic and the subsymbolic mode of computation can be integrated within a unified theory of cognition. We demonstrated that logical approach can be applied to model and describe processes of human reasoning, previously regarded as evading symbolic representation. Which leads to believe that, at least functionally, neural network activity is equivalent to nonmonotonic inferences.

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