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Logical Ideas of Jan Łukasiewicz

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Abstract:
This paper discusses the main logical ideas put forward by Jan Łukasiewicz within their historical context and further development.
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Jan Łukasiewicz was born in Lviv (Lvov) in 1878 and died in Dublin in 1956. He studied philosophy in Lviv under Kazimierz Twardowski, obtained his Ph.D. in 1903 and Habilitation in 1906. In 1906, he became a *Privatdozent* at the University of Lviv, and in 1911, he was promoted to the position of extraordinary professor. Łukasiewicz moved to the University of Warsaw in 1915 and was appointed as the professor of philosophy at the Faculty of Mathematics and Natural Sciences. He formed, together with Stanisław Leśniewski, a powerful group of mathematical logicians (Warsaw School of Logic), including (there are mentioned only persons who began scientific career before 1939) Alfred Tarski, Adolf Lindenbaum, Mordechaj Wajsberg, Moses Presburger, Bolesław Sobociński, Jerzy Słupecki, Stanisław Jaskowski, and Andrzej Mostowski. Łukasiewicz organized the Polish Logical Society and essentially contributed in preparations to publishing *Collectanea Logica*, a specialized logical journal (unfortunately two first volumes printed in 1939 were destroyed). During World War II, Łukasiewicz taught at the Clandestine University in Warsaw. In 1944, Łukasiewicz obtained a permission (from the German authorities) to leave Poland. Finally, he settled in Dublin as the professor of mathematical logic at the Royal Irish Academy.

Scientific activity of Łukasiewicz can be divided into two periods. The first covers the years 1902-1915, and the second the years 1915-1956. Roughly speaking, he was occupied with various logico-philosophical problems in the first period. His Ph.D. thesis was devoted to the problem of induction. He considered induction as the inversion of deduction. Łukasiewicz's Habilitation concerned an analysis of causality. He treated the causal relation as necessary. Perhaps [1] is the most important early work written by Łukasiewicz. This book offers a very detailed analysis of the principle of contradiction in Aristotle. This book has two tasks: firstly, an interpretation of the principle of contradiction (PCon, for brevity) and, secondly, an evaluation of arguments for and against PCon. Three interpretations of this principle can and should be distinguished: logical (concerning sentences), ontological (concerning things), and psychological (concerning judgments in the psychological sense). Łukasiewicz argues that the last understanding is irrelevant for logic, because it is an empirical fact that people assert contradictory assertions. However, Łukasiewicz denies that the logical (as well as ontological) PCon has a logical justification. We cannot deduce it from more basic principles. Finally, according to Łukasiewicz, one might say that PCon in its logical (ontological) meaning is accepted for ethical reasons, that is, as an indispensable device to distinguish between truths and falsehoods. The reported book has an Appendix presenting rudiments of mathematical logic in the version of algebra of logic as developed by Boole, Schröder and Couturat – it was the first account of the subject in Polish. Łukasiewicz shows that PCon is not an axiom (so he denies that it is the so-called highest principle of thinking) and can be proved as a logical theorem.

Works on induction led Łukasiewicz to the foundations of probability theory. Firstly, he hoped to solve the problem of induction via probability theory, but he abandoned this idea in his later works. In particular, Łukasiewicz became sceptical about a logical value of induction. His general approach (see [2]) consisted in ascribing probability to open formulas (formulas with free variable, indefinite propositions), not to full sentences which are true or false. He defined probability in the following way. Let Fx be a formula with free variables and D a finite domain. Assume that n is the cardinality (the number of objects) of D and m is the number of those which satisfy Fx. Thus, the ratio m/n can be defined as the logical probability of Fx. Łukasiewicz argued that the mathematical theory of probability allows an extension of the mentioned definition to infinite domains. Łukasiewicz introduced a classification of reasoning, very popular in Poland. He distinguished two main kinds, namely deduction (premises are the logical reason, conclusions are the logical consequents) and reduction in which the conclusion acts as logical reason and the premises as consequent. Induction is a kind of reduction, but is has no great scientific value, particularly in justifications. According to Łukasiewicz, deductive procedures are at the heart of science. His views on induction can be considered as an anticipation of Karl Popper's anti-inductivism.

Many-valued logic became the most remarkable Łukasiewicz's achievement. His above-mentioned doubts concerning PCon (and the law of the excluded middle expressed in one of his lectures before the Polish Philosophical Society) resulted in rejection of the principle of bivalence (PBiv) saying that every sentence is either true or false. Łukasiewicz announced his discovery of a non-Aristotelian logic in 1918 and elaborated its various details in two lectures in Lviv in 1920 (I skip bibliographical references – all relevant paper are included in [4]; see also [6], [7], [8] for further information). The Łukasiewicz's first motivation for introducing many-valued (more precisely, three-valued) logic was more philosophical than formal. Firstly, he believed in human freedom, creativity, and responsibility. Secondly, he was convinced that these facts and values are not coherent with determinism as an ontological theory. Consequently, he came to the conclusion that we need a non-deterministic ontology and three-valued logic as a proper background for creativity, freedom, and responsibility. Łukasiewicz considered determinism as closely connected with PBiv. He immediately observed that the issue in question has affinities with the old question, already discussed by Aristotle, concerning future contingents. If A is a sentence about a future contingent event, for example, the sea battle tomorrow, is it true or false at the moment of issuing it, for instance, today. The Stagirite himself argued that although the sentence $A \vee \neg A$, expressing the law of excluded middle, is universally true, its constituents, that is, A and $\neg A$ are not, if concern future contingents. It can be also expressed in terms referring to properties of the concept of truth. Define that A is occasionally true provided that if A is true at t, then A is true at every moment t' earlier than t. Furthermore, A is eternally true provided that if A is true at t, it is also true at every moment t' later than t. Łukasiewicz rejected occasionality of truth, but agreed that truth is eternal. According to him, this position suffices for considering truth as absolute. Incidentally, the absoluteness of truth as defined by Twardowski and Leśniewski consisted in its occasionality and eternality.

Łukasiewicz realized very soon that his new logic should not be called "non-Aristotelian". Since it was based on rejection of bivalence, he began to use the label "three-valued logic". The status of PBiv became the crucial issue. According to Łukasiewicz, this principle is not a theorem of logic, but a metalogical rule, which can be accounted as the conjunction of the metalogical non-contradiction and the metalogical excluded middle. Its acceptance or not cannot be reduced to purely logical circumstances, but requires assuming of some extralogical decisions, for instance, ontological. Anyway, there is no logical force to accept PBiv. If we reject this principle, we can introduce more than two logical values. Łukasiewicz introduced the third logical value, usually denoted by the fraction ½. Its meaning is explained by rules related to traditional truth-tables. In particular, we have the logical value v: if $v(A) = ½$, then $v(\neg A) = ½$, if $v(A) = ½$, $v(B) = ½$, then $v(A \vee B) = ½$ and $v(A \wedge B) = ½$. According to these equalities, if $v(A) = ½$, then $v(A \vee \neg A) = ½$ and $v(A \wedge \neg A) = ½$. This means that the (logical) law of the excluded middle and the (logical) law of non-contradiction are not theorems (tautologies) of three-valued logic. In the inter-war period, Łukasiewicz generalized the three-valued logic (usually denoted by the symbol $\mathbf{Ł_3}$) to logics with finite and infinite number of values as well as formulated various axiomatizations of theses systems. Several results concerning many-valued logic were obtained by Łukasiewicz's students, namely, Lindenbaum, Słupecki, Sobociński, Tarski and Wajsberg.

The problem of interpretation of many-valued logic was essential. In his first works on three-valued logic, Łukasiewicz understood the third value as possibility. Later he abandoned this intuition and decided to speak about ½ as a logical value, which has the same status as other. Yet Łukasiewicz believed that one of the systems, two-valued or many valued, is satisfied in the reality – he conjectured that the logic with infinitely many values is "true" on the world. However, he gradually became more and more formalistic in his thinking about logic. According to him, logical systems are formal constructions, independent of their relations to the reality or applicability to concrete scientific or technical problems. Historically speaking, Łukasiewicz's work on many-valued logic was pioneering. Nicolai Vasiliev, a Russian logician had some ideas about many-valueness, but he did not elaborated them in a formal way. Emil Post, an American logician, constructed a many-valued logic, but it was rather a purely formal system without an intuitive interpretation. Today, study of many-valued logics (plural as justifying for a considerable plurality of such logics) is a branch of mathematical logic. Many-valued logic has also several technical and philosophical applications, for instance, offers a basis for studies on paraconsistency. In fact, $\mathbf{Ł_3}$ is sometimes considered as the first formalization of paraconsistency.

The first intuitive interpretation of the third value as possibility immediately led to the problem of the relation between $\mathbf{Ł_3}$ and modal logic. Łukasiewicz accepted the following principles: (a) if it is not possible that *A*, then not-*A*; (b) if not-*A*, then it is not possible, that *A*; (c) for some *A*, it is possible that *A* and it is possible that not-*A*. Łukasiewicz demonstrated that (a)–(c) cannot be proved in two-valued logic. Hence, implementing modalities into three-valued logic appeared as a possible solution. Tarski proposed to define "it is possible that *A*' as $\neg A \Rightarrow A$. This definition functions in $\mathbf{Ł_3}$. However, Łukasiewicz did not construct a system of modal logic before 1950, partially due to various critical remarks about his modal ideas. In particular, Ferdinand Gonseth observed that Łukasiewicz's assumptions entail that the formula $A \wedge \neg A$ is possible just in the case if $v(A) = ½$, contrary to the common claim that contradictions are impossible. Łukasiewicz tried to solve this problem and other difficulties by the modal system based on four-valued logic, but this proposal did not gain an acceptance. One of the main features of all Łukasiewicz's logical systems is that they are strictly extensional. It means that if $v(A) = v(B)$, then the formulas *A* and *B* are substitutable per *salva veritate*. On the other hand, modal operators, possibility and necessity, are not extensional in Lewis' systems. Consequently, if, for instance, if *A* is possible, *A* is true, *B* is true, this set of premises does not implies that *A* is possible. On the other hand, possibility and necessity as understood by Lewis are not definable in two-valued logic. Consequently, Lewis' modal logic is extension of two-valued logic and this circumstance generates intensionality. Defining modality by Tarski's proposal, admits embedding modalities into $\mathbf{Ł_3}$ (similar constructions are possible in systems with more than three values) and keeps extensionality. Incidentally, the principle of extensionality functioned as a fundamental dogma of Warsaw School of Logic (it was particularly stressed by Leśniewski) – this circumstance blocked formalizing intensional context by the Polish logicians.

Łukasiewicz extensively worked on propositional calculus (or rather calculi; see [3] for a summary). He invented a special logical notation (called Polish notation or the Łukasiewicz notation). This symbolism avoids punctuation signs (brackets, points) – the structure of a formula is determined by the succession of signs. Functors are represented by the capital letters: *N* (negation), *C* (implication), *K* (conjunction), *A* (disjunction), *D* (bi-negation) and *E* (equivalence). For instance, the formula (I employ small letters as propositional variables) $(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$ become *ECpqCNpNp*, the formula $p \vee \neg p$ is *ApNp*, the formula $\neg(p \wedge \neg p)$ is *NKpNp* and so on. Łukasiewicz built various axiomatizations of propositional calculi. He preferred the simplest constructions, for instance, with minimal number of possibly shortest axioms. Consequently, he was looking for single shortest axioms as the best. The most popular is his following axiomatization: *CCpqCCqrCpr* (in traditional setting: $(p \Rightarrow q) \Rightarrow ((q \Rightarrow r) \Rightarrow (p \Rightarrow r))$; the transitivity rule for implication), *CCNppp* (traditionally: $(\neg p \Rightarrow p) \Rightarrow p$; characterization of implication via falsity of the antecedent, *ex falso quolibdet*), *CpCNpq* (traditionally: $p \Rightarrow (\neg p \Rightarrow q)$; *ex falso quolibdet*). Of course, it is not the simplest one, because it consists of three axioms. Since *Dp* can be defined as *Dpp*, bi-negations suffices as the sole primitive concept of propositional logic (*C*, *K* and *A* must be supplemented by *N*). Consequently, the entire propositional calculus can be axiomatized by a formula consisting from *D*'s and propositional variables. Łukasiewicz also investigated partial propositional calculi, for instance, based on *E* as the sole functor and intuitionistic logic. After 1945, he introduced propositional calculus with the so-called variable functors. The idea is that this systems contains variables for functors (in standard version, propositional functors are constants). The resulting system is very powerful and allows proving that intuitionistic propositional calcuslus is more expressive than classical one.

Łukasiewicz had a deep interest in the history of logic. He proposed to look at historical logical doctrines as anticipations of modern formal logic. This research project required reading of older logic through glasses of modern tools. Łukasiewicz, guided by this methodology, achieved revolutionary discoveries. In particular, he showed that Stoic logic was another system than Aristotelian syllogistic. More specifically, the Stoics developed elements of propositional logic, but the Stagirite elaborated a logic of names. Aristotle was a favourite logician of Łukasiewicz. In fact, two books published by the latter during his lifetime concerned the ideas of the former (see [1] and [4]). Although Łukasiewicz did not agree with Aristotle in many important points, he was convinced that the Aristotelian logic requires a modern interpretation. It was offered in [4], where syllogistic was reconstructed as an axiomatic system assuming propositional calculus. The second edition of this book contains a detailed analysis of Aristotle's logic of modalities. Łukasiewicz's idea that old logic should be investigated as an earlier stage of contemporary logic became fairly revolutionary and essentially changed understanding of the history of logic.

Łukasiewicz was a philosopher by education. Although he maintained in the second period of his scientific activities that logic should be entirely purified from philosophical assumptions, he was continuously interested in philosophical problems of logic. He entirely rejected psychologism and protested against the use of the term "philosophical logic" as leading to conflating logic with psychology and epistemology. Mathematical logic is the only logic and must be separated from philosophy as well as mathematics. On the other hand, logic is a fundamental instrument of reasoning and rational thinking, the morality of speech and thought (Łukasiewicz's saying). In particular, philosophy should be axiomatized in order to be a science. In general philosophy, Łukasiewicz preferred ontology over epistemology. He argued that post-Cartesian philosophy with its epistemological orientation, culminating in Kant, poisoned logic by psychologism – Leibniz was the only exception. This assessment of the history explains Łukasiewicz's sympathies to Aristotle and the Schoolmen. Łukasiewicz defended logic against objections pointing out that it recommends the empty formalism, entirely inconsistent with needs of philosophizing. According to Łukasiewicz, logic as such does not privilege any concrete philosophy and can be reconciled with many philosophical positions. On the other hand, every philosopher should obey general logical principles as indispensable for rationality. Clearly, his philosophical views were more explicit in the years 1902-1903, but he did not lose philosophical interests until the end of his life.

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