

## Legal Reasoning and Logic

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*Abstract:*

This paper investigates the basis arguments of so-called legal logic and their relation to logic in its standard meaning. There is no doubt that legal arguments belong to logic in the wide sense (*sensu largo*), but their reduction to schemes of formal logic (*logica sensu stricto*) is a controversial issue. It can be demonstrated that only some legal arguments fall under explicit rules of formal logic, that is, having a deductive character. Most such reasoning is fallible, and its correctness depends on appealing to extra-logical principles taken from legal norms. For instance, if we say, “If it is permitted more, then it is permitted less” (*argumentum a maiori ad minus*), we assume that the concepts expressed by the words “more” and “less” are already defined.

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Relations between law and logic were discussed in antiquity and persist until today. The *Talmud* contains many examples of reasoning used in solving concrete legal problems (Schumann, 2017). Protagoras, the leading Sophist, had a student Euthalos. Both established by a contract that the student would pay the master for his teaching after Euthalos won his first court case. However, Euthalos decided not to perform legal practise but to enter politics. Protagoras decided to sue Euthalos for the payment. He argued that if he won the case, he would be paid on the basis of the sentence, but if Euthalos won the case, Protagoras would be paid according to the original contract, because Euthalos would have won his first case. Euthalos, however, answered that if he won, then he would not have to pay by the sentence, but if Protagoras won, then Euthalos would not be obliged to pay, because he lost the case. The ancient sources do not say how this controversy was solved, but even a provisional analysis shows that something is lacking as a premise in the argumentation in question. It seems that one must add a principle asserting what has legal priority — a court’s sentence or a contract in the case of a behaviour not occurring that activates an obligation. This example shows that so-called legal logic has two ingredients: schemes of reasoning applied in law and the general principles which instruct how to solve inconsistencies or ambiguities stemming from formulations occurring in legal texts. The status and scope of legal logic are central in legal theory and philosophy. Some authors (Sartor, 2005, p. XXV) say that fundamental oppositions in theoretical jurisprudence, such as those between natural law theory and legal positivism or legal functionalism and legal formalism, have their explicit reference to problems of

argumentation employed in law. Independently, whether this opinion is (fully) correct or not, the schemes of legal logic deserve attention. The extensive literature (Armgaradt, Canivez & Chassagnard-Pinet, 2015; Hage, 2005; Klug, 1966; Perelman, 1977; Prakken, 1977; Rahman, Armgaradt & Kvernenes, 2022; Weinberger, 1970) confirms this suggestion. In what follows, I will concentrate on logical aspects of legal logic.

How is legal logic related to logic in its standard understanding? Clearly, it depends on how logic is understood. Omitting details (a more extended analysis is found in (Woleński, 2007) we can distinguish two understandings: narrow (*sensu stricto*) and wide (*sensu largo*). The former identifies logic with formal logic, which is a collection of logical systems (e.g., standard propositional logic, predicate logic, modal logic), based on the concept of logical consequence. Logic *sensu largo* covers logic in the narrow, semantics (semiotics), and methodology of science. If someone decides to think about legal logic as formal logic, he or she immediately is confronted with a serious problem. Law consists of norms as linguistic items. Now, there is a big controversy whether norms are true or false, that is, whether they can function in premises or conclusions of correct inferences. More specifically, since the concept of logical consequence essentially employs truth and falsity (if  $B$  is a logical consequence of  $A$ , then if  $A$  is true, then  $B$  is true by logical necessity), what is a semantics foundation of normative logic, if norms are neither true nor false? In what follows, I will not discuss this question and adopt a simple (simplified, if you like) assumption that normative statements are reducible to deontic ones (i.e., forms) like “it is obligatory that  $A$ ”, “it is prohibited (forbidden) that  $A$ ”, “it is permitted that  $A$ ”, etc. However, the Protagoras–Euthalos case suggests that legal logic cannot be reduced to logic *sensu stricto* because it employs some other principle. Without entering into details, we can say that legal logic uses legal semantics and legal methodology. The adjective “legal” is important here because it refers to specific arguments (reasoning) performed by lawyers. For instance, most procedures in forensic science consist in drawing conclusions from empirical data and do not belong to legal logic, but they can still be considered as belonging to legal methodology in the wide sense. Perhaps one forgotten fact illuminates the conceptual situation. The term *deductio* had a special meaning in medieval Latin. It referred to the way of arguing by a party before a court. So, Protagoras performed a deduction, and Euthalos deduced his claim as well. This meaning later disappeared, and today we say that  $B$  is correctly deduced from  $A$  if and only if  $B$  is a logical consequence of  $A$ .

I do not suggest that standard logic has no application in law. Sometimes it helps in the process of legal interpretation. Consider the regulation

- (1) The candidate for a position  $P$  can be pointed by  $X$  and  $Y$ .

How do we understand this norm? The word “and” suggests that it is a conjunction, but legal understanding dictates the use of “or”. In fact, it was a controversy in Poland whether candidates for the Constitutional Tribunal are pointed out by the Presidium of Sejm and the group of 50 deputies or by the first or second subject. Defenders of the first solution understood “and” as a conjunction, but the latter view argued that we have to go with a disjunction, frequently expressed in the conjunctive form; they argued that a frequent legal stylistic custom uses “and” as a mark of disjunction. The controversy in question was resolved by a new regulation which explicitly employed “or”. Now consequences of both interpretations are far-reaching, because the former states much stronger conditions for the procedure of pointing out candidates for the Constitutional Tribunal than the latter. Of course, logic by itself does not solve the problem of interpretation of (1), but it helps in the evaluation of consequences of adopting a particular understanding of “and”.

*Argumentum a contrario* has a typical deductive structure. Consider article 127.3 of the Constitution of the Republic of Poland. It says “Only a Polish citizen who, no later than the day of the elections, has attained 35 years of age and has a full electoral franchise in elections to the Sejm, may be elected President of the Republic. Any such candidature shall be supported by the signatures of at least 100,000 citizens having the right to vote in elections to the Sejm”. The word “only” is important and immediately suggests that this regulation establishes necessary conditions for being a

candidate for the Polish presidency. Consequently, we can derive from this norm several consequences, for instance, “if a person is not a Polish citizen, he or she cannot be elected President of the Republic of Poland”. The formal structure of this argument is captured by the following scheme:

$$(2) (A \text{ (only)} \rightarrow B) \rightarrow (\neg A \rightarrow \neg B),$$

which is equivalent to

$$(3) (B \rightarrow A) \rightarrow (\neg A \rightarrow \neg B)$$

and

$$(4) A \leftarrow B \rightarrow (\neg A \rightarrow \neg B),$$

where the symbol  $\leftarrow$  refers to the reverse implication. A characteristic feature of the operation  $\leftarrow$  is that it allows simple transposition (Perelman, 1977), that is, without changing the order of arguments, contrary to  $\rightarrow$ , where we must move the antecedent to the position of consequent and reversely.

Another example of *argumentum a contrario* is displayed by the following:

- (5) An action  $D$  is a crime if and only if it is prohibited by the penal code during the period of its validity.

Perhaps it is interesting to observe that (5) expresses one of the most fundamental legal principles, namely *nulla poena sine lege* (no penalty without law). It decides that penal illegality is a sufficient and necessary condition of qualifying an action as a crime. Yet some additional comments are in order. Firstly, penal codes frequently state additional constraints for crimes, for example, that an action must be socially dangerous. In such a case, criminal illegality is a necessary condition of considering an action as criminal, but sufficient. A practical consequence of this assertion is that the necessary condition cannot be omitted. If someone says that extremely immoral deeds should be considered as legal crimes, he or she does not understand the principle *nulla poena sine lege*. It is interesting to observe that the prohibition of analogy against interests of accused persons is justified by the principle in question: assume that a penal norm  $A$  is extended by analogy. It can happen that an action formerly not qualified as a crime can be considered as legally penalised. Clearly, it might be at odds with *nulla poena sine lege* and *nullum crimen sine lege* (no crime without law). Secondly, (5) is not fully adequate because there are circumstances in which a given action is, so to speak, formally a crime, but it is not qualified as a crime because, for instance, the person who committed it acted in necessary self-defence, or he or she did not attain 18 years of age (or another age, depending on the code). Consequently, (5) should be rewritten as:

- (6) For every action  $D$  that does not hold circumstances excluding being penalised,  $D$  is a crime if and only if it is prohibited by the penal code during the period of its validity.

In (6), restricted universal quantification is used. There is a discussion whether excluding being penalised also eliminates criminality, but I skip this question as transgressing legal logic.

Logical analysis of *argumentum a contrario* is relatively easy, but things appear differently in the case of *argumentum a fortiori*. It has two forms, namely:

- (7) *argumentum a maiori ad minus*.

(8) *argumentum a minori ad maius*.

More specifically, both can be written, respectively, as:

(9) If it is permitted more, then it is permitted less.

(10) If it is forbidden more, then it is forbidden less.

Due to standard deontic logic and the definition  $FA =_{df} \neg PA$  ( $A$  is forbidden if and only if it is not true that  $A$  is permitted), (9) and (10) are logically equivalent and, thereby, they might be considered as two formulations of the same argument. However, this nice picture must be supplemented by a closer analysis. First of all, since the words “more” and “less” do not express logical constants, schemes (9) and (10) are not logical theorems. Consider the following cases:

(11) If it is permitted to vote, it is permitted to abstain from voting.

(12) If it is prohibited to drive at a speed of 100 km/h, it is prohibited to drive at a greater speed.

Clearly, if we define voting as something more than abstaining from participation in elections, (11) is a sound inference. Similarly, deciding that a greater speed is something more than a slower one, (12) is suitable. Supplementing (11) and (12), we obtain, respectively:

(13) If it is permitted to vote, it is permitted to abstain from voting, and since the second behaviour is less than the first, it is permitted to abstain from voting.

(14) If it is prohibited to drive at a speed of 100 km/h, it is prohibited to drive at a greater speed, and since driving at a speed of 150 km/h is more than driving at a speed of 100 km/h, then driving at 150 km/h is prohibited.

Yet, such arguments can be fallacious. Assume that rules for driving require that there must be a minimal speed, say 100 km/h. So, driving at 100km/h is permitted, but a speed less than 100 km/h is prohibited. Intuitively, killing is something more than injuring, but an executioner can kill doing an execution, yet he cannot injure (as a final effect) a convict. Thus, applying arguments *a fortiori* requires taking into account several regulations that contribute to the legal understanding of “less” and “more”.

The above considerations suggest that in the evaluation of schemes of legal logic, their formal structure is not the only criterion, perhaps with the exception of some simple cases of *argumentum a contrario*. In more advanced cases, like *argumentum a fortiori*, it is necessary to take into account more informal aspects. Note, however, that if we compare a legal argument with an ordinary one, the former refers to formal aspects constituted by legal norms. Thus, we should distinguish between formal in a logical sense and formal in a legal sense. For instance, that a reasoning is a deduction or not refers to its logical formality, but that legal norms prohibit analogy against interests of accused persons appeals to legal formality. Perhaps this fact is partially responsible for the opinion that legal logic is formal, because it appeals to forms established by legal prescriptions.

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