Studia Humana
Volume 13:1 (2024), pp. 5–16
DOI: 10.2478/sh-2024-0002

From the History of Leśniewski's Mereology

Andrzej Pietruszczak

Nicolaus Copernicus University in Toruń
Moniuszki 16/20 Street
87-100 Toruń, Poland

e-mail: pietrusz@umk.pl
https://orcid.org/0000-0001-9133-5081

Abstract:
In this paper, we want to present the genesis of Stanisław Leśniewski's mereology. Although 'mereology' comes from the word 'part', mereology arose as a theory of collective classes. That is why we present the differences between the concepts of *being a distributive class* and *being a collective class*. Next, we present Leśniewski's original mereology from 1927, but with a modern approach. Leśniewski was inspired to create his concept of classes and their elements by Russell's antinomy. To face it, Leśniewski had to define the concept of *being an element of* based on the concept of *being part of*. Leśniewski showed that in his theory, there is no equivalent to Russell's antinomy. We will show that his solution has nothing to do with the original approach because, in both cases, we are talking about objects of a different kind. Russell's original antinomy concerned distributive classes, and Leśniewski's considerations concerned collective classes.
Keywords: mereology, Leśniewski, collective class, distributive class, set theory.

1. Introduction

Mereology arose as a theory of collective classes (sets). It was formulated by the Polish logician Stanisław Leśniewski. Collective sets are certain wholes composed of parts. In general, the concept of a *collective set* can be defined with the help of the relation *is a part of* and mereology may therefore be considered as a theory of "the relation of part to the whole" (from the Greek: $\mu\varepsilon\rho o\varsigma$, *meros*, 'part').

Leśniewski did not invent the concept of a *collective set*. It is discussed, for example, by Whitehead and Russell in comments in *Principia Mathematica* (1910) concerning the theory of classes developed in that work. Whitehead made use of such sets in his thoughts on the philosophy of space-time (Whitehead, 1929).

In mereology, as in everyday speech, the expression 'part' is usually understood as having the sense of 'fragment', 'bit', and so forth. Thus understood, the relation of part to the whole has two properties *irreflexivity* and *antisymmetry*, i.e.:

(irrP) *No object is its own part*.
(antisP) *There are not two objects such that the first could be a part of the second and the second is a part of the first*.

Thanks to condition (irrP), we have no difficulty interpreting the phrase "two objects" in condition (antisP). One can see that it concerns «two different» objects. Of course, the properties of *irreflexivity* and *antisymmetry* entail the property *asymmetry* which is difficult to express in natural language). Furthermore, *being part of* is assumed to be transitive:

(tP) *Every part of a part of a given object is part of it.*

Colloquially, we treat sets as "wholes consisting of some units" — disregarding the way they are created and whether the units they are composed of have a functional contribution to the whole. We will show that even with this approach, we must divide these wholes into two distinct kinds. Moreover, different wholes of both kinds can be formed from the same units. The so-called *distributive sets* forms the first type. Colloquially, we usually talk about the so-called *collective sets*, although there are also cases of distributive sets here. To understand what Leśniewski meant, we will explain what collective sets are supposed to be and how they differ from distributive sets.

The terminology used is artificial. On the one hand, the word 'set' is often a substitute for 'collection'. On the other hand, the basic meaning of the word 'distribution' is division. Thus, the return 'distributive set', i.e. «collected and distributed». However, this terminology is beneficial. The combination 'collective set' is to remind you that it is about the colloquial meaning of the word 'set'. The combination 'distributive set' suggests that the sets in question have little to do with collecting, accumulating, or combining. If anything, we are to associate them with these activities, understood in the abstract. For example, we unite voivodeships, but at the same time, we do not unite communes, thus not receiving the land territory of Poland. We collect cities, but we do not collect their streets, squares, etc. We understand this abstractly to such an extent that we also admit the existence of the empty set, which is nonsense in common sense. How can assemble «things that are not there»?

Just as the word 'set' is ambiguous, so is the term 'element of a set'. In other words, the word 'element' takes on a meaning that depends on the meaning of the word 'set'. Thus, when we talk about sets (resp. classes) and their elements, some misunderstandings can arise because of the multiplicity of meanings the terms 'set' and 'element of a set' possess. Let us quote an extensive excerpt from Ludwik Borkowski's book (1977, p. 146):

> The terms "set" and "element of a set" are used with two meanings. Understood with the first of these meanings, the term "set" signifies objects composed of parts, collections and conglomerations of a different kind. The elements of such type of set are to be understood as arbitrary parts of that set, where the term "part" is understood in its everyday sense, with which, for example, the leg of a table is a part of the table. A pile of stones is in this sense a set of those stones. The elements of that set are both individual stones along with the various parts of those stones, and thus, for example, the molecules or atoms of which those stones are composed. With this meaning, the set of given stones is identical to, for example, the set of all the atoms from which they are composed. Elements of a set so understood, such as the set of all tables, would be not only the individual tables but, for example, the legs of those tables or other of their parts. We shall say that we are using here the term "set" in its *collective* sense, as we are using it with that sense. A theory of sets and the relation *is a part of* understood in line with the above has been constructed by S. Leśniewski, who called it *mereology*.
>
> We use the terms "set" and "element of a set", with the second meaning in the following example: when talking about the et of European countries, we consider as elements of that set particular European countries, such as Poland, France and Italy, and we do not consider as elements the parts of those countries. With this meaning, the Tatra mountains or the Małopolska Upland are not elements of the set of European countries even though they are parts of certain European countries. We also use these terms with this meaning for example when, talking about the set of Polish towns, we consider as elements of that set towns such as Wrocław and Warsaw whilst not considering as elements of that set particular streets or squares or other parts of those cities. The terms "set" and "element of a set" have long been used with this meaning in logic, when speaking of extensions of names or concepts as certain sets of

> objects. In contrast to the first meaning, it is not possible to identify the concept of an element with the common concept of a part.

The second meaning of the term "set" has come to be called the *distributive* or *set-theoretic meaning*. Let us add further a section from the final paragraph of the book (Słupecki & Borkowski, 1967, p. 279) that makes some philosophical on sets.

> [...] the word "set" has two clearly distinct meanings in everyday speech, of which one is call the collective meaning and the second the distributive. With the collective meaning — a set of a certain objects is a whole composed of those objects in the same way that a chain is composed of links and a pile of a sand of grains of sand. With this meaning, a set of concrete, sensually perceptible objects is also a concrete and perceptually-available object. Using the term "set" with this meaning, we understand "x is an element of the set A" as having the same sense as the expression "x is a part of the set A" (with the word "part" having that meaning such that the leg of a table is a part of the table). A set theory understood in this way was built by S. Leśniewski under the name *mereology*. Using the term "set" with its distributive meaning, we consider the sentence "Mars is an element of the set of planets in our Solar System" as equivalent to the sentence "Mars is a planet in our Solar System". The difference in meaning is attested to by the fact that certain true sentences where "set" is understood with its first meaning are false when it is understood with its second meaning. For example, where the meaning is collective, it is true that a tenth part of Mars is an element of the set of planets in our Solar System, because it is a part of the whole arrangement; whereas that sentence is false if the meaning is distributive, because no tenth part of Mars is a planet in our Solar system.

It is evident from the above texts that the terms 'collective set' and 'distributive set' have different meanings. It would seem indeed that the single common characteristic is that, in both cases, it is possible to say that "a set of certain objects is a whole composed of those objects" (Murawski, 1984, p. 164). To put it another way, there may be a similar "way of creating sets" for both concepts. As Hao Wang (1994, p. 267) writes:

> There are two familiar and natural ways of construing sets [both conceptions of the creation of sets described here obviously concern distributive sets, *A.P.*]. On the one: hand, given a multiplicity of objects, some or all of these objects can be conceived together as forming a set; the process can be iterated indefinitely. This way may be called "the extensional conception of set." On the other hand, a set may be seen as the extension of a concept or a property in the sense that it consists of all and only the objects which have the property. This way may be called "the intensional conception of set." We tend to use both conceptions and expect no conflict between them. Yet in practice it makes a difference whether one takes the one or the other conception as basic.
>
> Roughly speaking, Frege begins with the intensional conception and Cantor begins with the extensional conception.

2. Distributive Sets – the Basic Principle

Used with their distributive senses, the terms 'set' and 'class' are often treated as synonyms. In certain versions of modern set theory a distinction is made between them. In such theories, *each set has to be a class, but not conversely*. Sets are a special kind of class: they are those classes which are elements of other classes.

In the case of distributive classes (sets), the collection — i.e., collecting of objects, regardless of their type — must be understood always in an abstract sense and not a spatio-temporal one. Quine (1981, p. 120) writes :

> The reassuring phrase 'mere aggregates' must be received warily as a description of classes. Aggregates, perhaps; but not in the sense of composite concrete objects or heaps. Continental United States is an extensive physical body (of arbitrary depth) having the several states as parts; at the same time it is

> a physical body having the several counties as parts. It is the same concrete object, regardless of the conceptual dissections imposed; the heap of states and the heap of counties are identical. The class of states, however, cannot be identified with the class of counties; for there is much that we want to affirm of the one class and deny of the other. We want to say e.g. that the one class has exactly 48 members, while the other has 3075. We want to say that Delaware is a member of the first class and not of the second, and that Nantucket is a member of the second class and not of the first. These classes, unlike the single concrete heaps which their members compose, must be accepted as two entities of a non-spatial and abstract kind.

With their distributive meaning, the terms 'class' ('set') and 'element' for any general name S, satisfy the BASIC PRINCIPLE given below in the form of a schema:

($\star$) The elements of the distributive set of Ss are all Ss and only Ss.

So when we talk about the distributive set of Ss, we mean the distributive set of all Ss, and composed only of Ss. For example, the elements of the distributive set of Polish voivodeships are all these and only them (and not communes, towns, villages, etc.).

The following principle of extensionality applies to distributive sets:

$$\text{if } X \text{ and } Y \text{ have the same elements, then } X = Y.$$

According to the above, all general names having the same referents determine one distributive set whose elements are their referents and which is their common extension. All empty names designate the same distributive set — null set, denoted by '$\varnothing$'.

The Podkarpackie is one of the Polish voivodships, but it is not a commune. It is the other way around with the Strzyżów commune. Therefore — under the basic principle ($\star$) — distributive sets of voivodships and communes differ in elements because the Podkarpackie voivodship is an element of the first one and not an element of the second one (similarly, the commune of Strzyżów is an element of the second one, but not an element of the first one). Applying the counterposition of the following principle of identity:

$$\text{if } X = Y, \text{ then } X \text{ and } Y \text{ have the same elements,}$$

we obtain that these sets are different:

$$\text{the distributive sets of voivodships} \neq \text{the distributive sets of communes.}$$

We do not need to apply the principle of extensionality, which is the inverse implication of the principle of identity. What we got proves that attests to the fact that the aforementioned distributive sets may not be identified with any spatiotemporal object. Indeed, the land territory of Poland is the only such object. However, with such an identification, we would get equality instead of inequality. So:

$$\begin{aligned} \text{the land territory of Poland} &\neq \text{the distributive sets of voivodships} \\ &\neq \text{the distributive sets of communes.} \end{aligned}$$

Similarly, Quine says in the previous passage is that if the class of the states of the USA occupied some 'place' in space, then it would the very same place that the USA occupies. The same would be true of the class of counties in the USA. We should therefore identify these distributive classes, contrary to condition ($\star$).

The presented analyses show that distributive sets are abstract objects. We may paraphrase the preceding considerations: it is possible «to collect abstractly» the communes whilst not collecting voivodeships and vice versa. Quine provides us with another example in support of the theory of the abstractness of distributive classes (sets) in an essay from (Quine, 1953, pp. 114–115):

> The fact that classes are universals, or abstract entities, is sometimes obscured by speaking of classes as mere aggregates or collections, thus likening a class of stones, say, to a heap of stones. The heap is

indeed a concrete object, as concrete as the stones that make it up; but the class of stones in the heap cannot properly be identified ,with the heap. For, if it could, then by the same token another class could be identified with the same heap, namely, the class of molecules of stones in the heap. But actually these classes have to be kept distinct; for we want to say that the one has just, say, a hundred members, while the other has trillions. Classes, therefore, are abstract entities; we may call them aggregates or collections if we like, but they are universals. That is, if there *are* classes.

As in the previous quoted passes, Quine is saying that if a class of stones occupied some «place» in space, then it would be a pile of stones. A similar thing would be said of the molecules in the stones. It is possible to «abstractly take» the stones «without moving» their molecules or vice versa.

Let us remind ourselves that in "the intensional conception of [distributive, *A.P.*] set", "a set may be seen as the extension of a concept or a property in the sense that it consists of all and only the objects which have the property" (Wang, 1994, p. 267). Thus, not as a property, but as its extension. A further excerpt from (Quine, 1981, pp. 120–121) will help clarify what is meant:

Once classes are freed thus of any deceptive hint of tangibility, there is little reason to distinguish them from properties. It matters little whether we read '$x \in y$' as 'x is a member of the class y' or 'x has the property y'. If there is any difference between classes and properties, it is merely this: classes are the same when their members are the same, whereas it is not universally conceded that properties are the same when possessed by the same objects. The class of all marine mammals living in 1940 is the same as the class of all whales and porpoises living in 1940, whereas the property of being a marine mammal alive in 1940 might be regarded as differing from the property of being a whale or porpoise alive in 1940. But classes may be thought of as properties if the latter notion is so qualified that properties become identical when their instances are identical. Classes may be thought of as properties in abstraction from any differences which are not reflected in differences of instances. For mathematics certainly, and perhaps for discourse generally, there is no need of countenancing properties in any other sense.

It is precisely what the two previously given principles of extensionality and identity providers are. It also shows that the notion of *distributive set* (or of *distributive class*) must be primitive, i.e. undefinable. After all, it is impossible — without falling into a "vicious circle" — to define sets as classes of abstractions in a set of properties.

3. Leśniewski's Views on Distributive Sets

One of the featured quotes from Quine on distributive classes ends with the words: "That is, if there *are* classes." Even such conditional «making the case» irritated Leśniewski, who categorically rejected the existence of distributive sets (classes). This is evidenced by his comments in the first part of his fundamental work "On the foundations of mathematics" (Leśniewski, 1927, 1928, 1929, 1930, 1931). The theory of types created by Russell and Whitehead and the theory of classes as the extensions of concepts created by Frege were both for Leśniewski objectless (1927, pp. 204–205) (the passages from Leśniewski's papers have been translated from the original and not taken from the English edition of this work (Leśniewski, 1991)):

I do not know what Russell and Whitehead understand in the commentaries on their system by class. The fact that, on their position, "class" is supposed to be the same as "extension" does not help me in the slightest, as I do not know what these authors mean by extension. I do not therefore know either, when they consider the matter of the existence or non-existence of objects as such whether their thoughts on the puzzle of existence and non-existence address those objects which are classes. [...] Not understanding the relevant terminology of Whitehead and Russell, I am not in particular aware where and to what degree their doubts as to the existence of objects, which are classes in their understanding of that term [The authors of the *Principia Mathematica* do in fact introduce as a problem the question of the existence of distributive sets, *A.P.*], may bear on particular positions I take in the theory of classes sketched earlier.

> In "*Principia Mathematica*", I did not find a single paragraph which I felt there was even the weakest presumption of calling into question the existence of classes as I understand them. Sensing in the "classes" of Whitehead and Russell, in a similar fashion as with the "extensions of concepts" of Frege, the scent of mythical paradigms from a rich gallery of "invented" objects, I cannot for my part divest myself of the inclination to sympathise "on credit" with the doubts of the authors on the matter of whether objects that are such "classes" exist in the world. — On the matter of the relation of my conception of class to the views represented in the commentaries of Whitehead and Russell on their system, a certain light may be here thrown by the views of Russell on "heaps". Russell writes in one of his works: "We cannot take classes in the *pure* extensional way as simply heaps or conglomerations. If we were to attempt to do that, we should find it impossible to understand how there can be such a class as the null-class, which has no members at all and cannot be regarded as a "heap"; we should also find it very hard to understand how it comes about that a class which has only one member is not identical with that one member. I do not mean to assert, or to deny, that there are such entities as "heaps". As a mathematical logician, I am not called upon to have an opinion on this point. All that I am maintaining is that, if there are such things as heaps, we cannot identity them with the classes composed of their constituents" [The passage Leśniewski refers to is to be found in (Russell, 1919, p. 183), *A.P.*]. If I understand the cited paragraph correctly, then the fact that a certain object P is a "heap" of some as, composed of all as, would still not be for Russell a sufficient basis on which to affirm that the object P is a "class" of objects a. Russell's terminology would remain most clearly in complete discord with my terminology; in accordance with his use of the expressions "class" and "set", and the use of the expression "heap" in our common, everyday language [. . .], I could always say of a "heap" of some as, that it is a set of objects a [of as, *A.P.*], but of a "heap" of objects a [of as, *A.P.*] composed of all as, that it is the class of objects a [of as, *A.P.*]. [. . .] The difficulty is in understanding in what consists the difference a "heap" of objects a [of as, *A.P.*] and "class" of objects a [of as, *A.P.*] from Russell's point of view, if both such things existed and if each of them were *composed* of all as and it is a difficulty which I do not know how to overcome.

As can see, the problem was that Leśniewski understood the word 'class' differently than Russell. Quite simply, Leśniewski categorically rejected the existence of sets (classes) in the distributive sense. Let us add that heaps are the same as corresponding collective classes. Leśniewski is right about this point. He could not, however, understand "on what rests the difference between" a heap of Ss (i.e. a collective class of Ss) and a distributive class of Ss, "if both such things existed and if each of them were composed of all" Ss. This led Leśniewski to claim that Cantor's set theory applies to — just like his mereology — of collective sets (Leśniewski, 1927, p. 190):

> My conception is, in this respect, on the one hand (as far as I have managed to observe) entirely consistent with the way the expressions "class" and "set" are used in the common, everyday language of people who have never held neither any "theory of classes" nor any "theory of multitudes". On the other hand, it is based on a strong academic tradition, running more or less continuously through countless past and present scholars, and in particular through George Cantor.

In Leśniewski's opinion therefore, mereology deserves the title of "The foundation of mathematics" in the same way as in Cantor's theory, since both theories are concerned with the same sets (classes). Leśniewski's main work, in which he presented his mereology, he thus called "On the foundations of mathematics" (1927; 1928; 1929; 1930; 1931). An earlier work pertaining to mereology carried the title "The foundations of the general theory of sets" (Leśniewski, 1916). Nowadays, it is undisputed that set theory deals with sets (classes) in the distributive sense.

However, Cantor's theory differs significantly from mereology. In the first one:

- there is the distributive empty set $\varnothing$, which excludes mereology (see section 4 below);
- the distributive set consisting of one object x is not x, i.e. $x \neq \{x\}$ (for example, $\varnothing \neq \{\varnothing\}$; $1 \neq \{1\}$, but in mereology we have $x = [\![x]\!]$, where $[\![x]\!]$ is the collective set consisting of one object x (see sections 4 and 5 below).

In Cantor's theory, the set consisting of the only object x, i.e. $\{x\}$, has one element. In mereology, the elements of a given set are all its parts and the set itself (see section 7 below). Thus, each part of x is an element of the collective set $[\![x]\!]$ because $x = [\![x]\!]$. It is only a single-element set if x has no parts. Moreover, since each part of an element of a given set is also an element of this set, it is impossible to determine the number of elements of a given collective set. Thus, collective sets have no use in mathematics.

4. A Definition of Collective Classes

The phrase 'is a class' does not appear in Leśniewski as a unary predicate. The term 'class' is always used in the context of 'class Ss'. Leśniewski connects the latter with the conjunction 'is', obtaining the phrase 'is a class of Ss'. We will not go into the details of the syntax of Leśniewski's mereology here, which is based on the syntax of his other theory — ontology. As mentioned, the concept of *being a collective class of Ss* is defined by the concept of *being a part of*. Leśniewski gave various definitions of this concept, equivalent in his mereology.

To get a relatively concise formulation of these definitions, let us use the artificial notion of *being an ingredient* introduced by Leśniewski (1928, p. 264, footnote 1 and definition I):

- an *ingrediens* of a given object is the object itself and each of its parts, where 'part' is understood with its ordinary sense.

By the above definition, the relation *is an ingrediens of* is reflexive, i.e.:

(rI) *Every object is an ingrediens of itself.*

By the above definition and (antisP), we obtain that *being an ingrediens of* is antisymmetric, i.e.:

(antisI) *There are not two objects such that the first could be an ingrediens of the second and the second is an ingrediens of the first.*

Moreover, by the above definition and (tP), we obtain that *being an ingrediens of* is transitive, i.e.:

(tI) *Every ingrediens of an ingrediens of a given object is an ingrediens of it.*

Using the technical notion *is an ingrediens of*, the first definition of a collective class Ss is as follows:

- it that an object x is a collective class of Ss means that the following two conditions hold:
  (a) every S is an ingrediens of x,
  (b) every ingrediens of x has a common ingrediens with some S.

Notice that from the given definition, we get the following three conclusions:

- If a name S is empty, then there is no collective class of Ss.

Assume for a contradiction that x is such a class. Since x is an ingrediens of itself, by condition (b), x has a common ingrediens with some S, but there is no S.

(i) Every object is a collective class of its ingredienses.
(ii) Every object having a part is a collective class of its parts.
(iii) Each object is a collective class of objects identical to it.

For any object x, we take the expression 'ingrediens of x' (resp. 'part of x', 'identical to x') instead of the letter 'S'. Then both conditions in the definition are tautological (we use the fact that each object is its ingredient).

Collective classes preserve the «nature of objects» from which they are built.

5. Axioms of Leśniewski's Mereology

In addition to the previously mentioned properties of the concept of *being a part of* (irreflexivity, antisymmetry, asymmetry, transitivity; see conditions (irrP)–(tP)), Leśniewski adopted two axioms regarding the defined concept of *being a class of Ss*. The first is uncontroversial. Namely, it says there can only be one class of Ss. In other words, for any objects x and y we put:

- *If x and y are collective classes of Ss, then $x = y$.*

Hence, (i)–(iii), we can write:

$$\begin{aligned} x &= \text{collective class of ingredienses of } x \\ &= \text{collective class of parts of } x\text{-a, if } x \text{ has a part} \\ &= [\![x]\!], \end{aligned}$$

where $[\![x]\!]$ is the collective set consisting of one object x.

The second of the axioms adopted by Leśniewski, however, is already controversial. Namely, it says that for any non-empty name S, there is a collective class of all Ss:

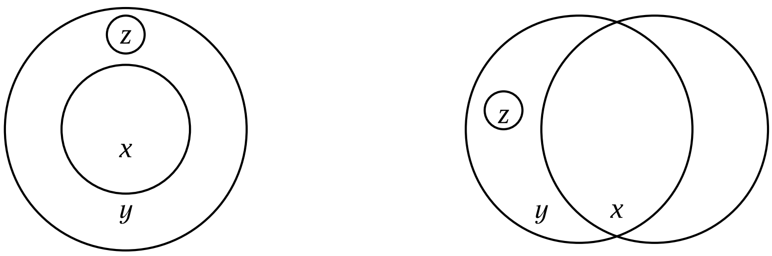
- *If there is an S, then there is a collective class of all Ss.*

This axiom is too strong, which limits its use. In modern terms, it only applies to point-free geometry and point-free topology (for details, see Pietruszczak, 2018).

Note that the given axioms entail the *polarization* of *being of ingrediens of*, also called the Strong Supplementation Principle, which is a fundamental property of the relations *is an ingrediens of* and *is a part of*. For any x i y we have:

(polI) *If y is not ingrediens of x, then there is an ingrediens of y having no common ingrediens with x.*

This principle is illustrated in the figure below:



Notice that from (rI), (tI) and (polI) for any x and y, we get:

- y is an ingrediens of x if and only if every object having a common ingrediens with y also has a common ingrediens with x.

6. Other Definition of Collective Classes

Other of Leśniewski's definition of a concept of *being a class of Ss* is as follows:

- it that an object x is a collective class of Ss means that for object y, the following condition holds:
  (c) y has no common ingrediens with x if and only if y has no common ingrediens with some S.

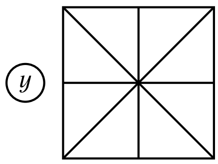
It is obvious that condition (c) is equivalent to the following:

(c′) y has a common ingrediens with x if and only if y has a common ingrediens with some S.

It is also easy to see from (c′) that if S is empty, then there is no collective class of Ss.
First, note that if an object x is a collective class of Ss, i.e., x satisfies conditions (a) and (b), then x satisfies condition (c′) (see Pietruszczak, 2018, p. 81). Indeed, suppose that x is a collective class of Ss. Firstly, assume that y has a common ingrediens, z, with x. Then, by (b), z has a common ingrediens, u, with some S. By transitivity, u is an ingrediens of y. So y has a common ingrediens with some S. Secondly, suppose that y has a common ingrediens, z, with some u, which is an S. Hence, by (a), u is an ingrediens of x. By transitivity, z is also an ingrediens of x. So x and y have some common ingrediens.

Second, in Leśniewski's mereology, both his definitions are equivalent. Namely, by (polI), if an abject x satisfies condition (c′), then x satisfies both conditions (a) and (b) (for details, see Pietruszczak, 2018, p. 142). For (a): By the ⇐-part of (c′), for any y we have: y has a common ingrediens with some S, then y has a common ingrediens with x. Hence for any z being an S and ant y, we have: if y has a common ingrediens with z, then y has a common ingrediens with x. Hence, by (rI), for any z being an S and any y we have: if y is an ingrediens of z, then y has a common ingrediens with x, i.e., every ingrediens of z has a common ingrediens with x. So, by (polI), we get: z is an ingrediens of x. Thus, we obtain that every S is an ingrediens of x. For (b): By the ⇒-part of (c′), for any y we have: if y has a common ingrediens with x, then y has a common ingrediens with some S. Hence, by (rI), if y is an ingrediens of x, then y has a common ingrediens with some S, i.e., every ingrediens of x has a common ingrediens with some S.

Let us see the operation of the given definitions in the figure below:



We see that x = the largest square in this figure satisfies the condition (c) both for S as 'triangle' and as 'square': y has no common ingrediens with x if and only if y has no common ingrediens with some triangle (resp. square) in the figure. So x is the collective class of triangles (resp. squares) in this figure:

the collective classes of triangles	=	the largest square
	=	the collective classes of squares

Similarly, we get:

the collective classes of voivodeships	=	the land territory of Poland
	=	the collective classes of communes

Once again, we see that distributive sets must be considered abstract objects. Namely, the distributive set of triangles in the figure above is different from the distributive set of squares in the figure because these sets have different elements, i.e.:

the distributive classes of triangles ≠ the distributive classes of squares

These classes cannot be drawn because they are abstract objects. Only their elements can be drawn. Notice that for the distributive version, we have the following:

the largest square	≠	the distributive classes of triangles
	≠	the distributive classes of squares

7. Elements of Collective Classes

Leśniewski was «inspired» to create his concept of classes (sets) and their elements by Russell's antinomy, which concerned the distributive class of all distributive classes that are not elements of each other. To face it, Leśniewski had to define the concept of *being an element of*. He meant *being an element of a given class*, but in his theory, all objects are classes and vice versa.

Leśniewski's definition the concept of *being an element of* is similar to the analogous definition adopted by Frege. Namely, Leśniewski assumed that for any objects x and z:

- x is an element of z if and only if for some meaning of 'S': z is a collective class of Ss and x is an S.

This definition and the definition of a collective set of Ss lead in Leśniewski's theory to the following conclusion already mentioned:

(\$) x is an element of z if and only if x is an ingrediens of z.

Thus, in Leśniewski's mereology, *being an element of* is the same as *being an ingredients of*, and not 15 the same as *being a part of*, as some authors claim (see, e.g., Borkowski, 1977; Słupecki & Borkowski, 1967). Indeed, firstly, suppose that x is an element of z, i.e., for some meaning of 'S': z is a collective class of Ss and x is an S. Then, by (a), we get that x is an ingrediens of z. Secondly, suppose that x is an ingrediens of z. Notice that we have (i): z is the collective class of its ingredienses. So we take S as 'ingrediens of z'. Then z is the collective class of Ss, and x is an S. Hence x is an element of z.

Thus, we see that collective classes and their members do not satisfy the previously mentioned fundamental principle for distributive sets:

($\star$) The elements of the distributive set of Ss are all Ss and only Ss.

Condition (a) from the definition of collective classes and (\$) say that we only have:

- All Ss ale elements of the collective class of Ss.

We see that there may be elements of the collective class of Ss which are not Ss.

8. Influence of Russell's Antinomy on the Creation of Mereology

As mentioned, Leśniewski was «inspired» by Russell's antinomy to create his concept of classes and their elements. To solve the problem of a class being composed of classes that are not members of themselves, Leśniewski developed his concept of classes (sets), which is very different from Cantor's. Leśniewski (1927, pp. 185–186) writes:

> Wishing "to conceive of something" and not knowing at the same time how to find any reasonable fault in any of the aforementioned assumptions on which the earlier "antinomy" rests, nor also in the reasoning leading to contradiction on the basis of those assumptions, I began to muse on examples of situations in which in practice I consider or do not consider such and such objects as classes or sets of such and such objects [. . .] and to submit for critical analysis my faith in the particular assumptions of the "antimony" in hand from that point of view (the puzzle of "empty classes" was not the theme of my considerations on that occasion because I treated the conception of "empty classes" from my first moment of contact with it as a "mythical" conception, taking without any hesitation the position that:
> (1) if any object is a class of objects a, then some object is an a.)

In his theory of (distributive) classes, Frege assumed that each concept determines the class of objects that fall under it. Russell noted that this assumption leads to a contradiction. Namely, he showed that the same assumption of the existence of a class of all classes not being their own elements leads to a contradiction. Classes not being their own elements are considered «normal». Thus Russell's antinomy says that the assumption of the existence of a class of all normal classes leads to a contradiction in Frege's theory.

As general name S, we take '(distributive) normal class'. According to Frege's theory, there is a distributive class of all normal distributive classes. Let us denote it by '$\mathcal{N}$'. According to ($\star$):

($\star_{\mathcal{N}}$) The elements of $\mathcal{N}$ are all normal classes and only them.

Hence for any distributive class X we obtain:

($\star_{\mathcal{N}}$) X is an element of $\mathcal{N}$ if and only if X is not an element of itself.

Since X was any distributive class, the above applies to $\mathcal{N}$. Hence we get a contradiction:

- $\mathcal{N}$ is an element of itself if and only if $\mathcal{N}$ is not an element of itself.

However, as Quine points out in (1987), this is based on the tautology of quantifier logic, in which 'R' represents any binary predicate:

($\top$) there is no x such that for any y: yRx if and only if it is not the case that yRy.

Indeed, assume for a contradiction that there is an x such that for any y: yRx if and only if it is not the case that yRy. Then, since y was any object, it applies to x. So we have a contradiction: xRx if and only if it is not the case that xRx.

Taking in ($\top$) R as *is an element of*, we get that there is no class from Russell's antinomy. Similarly, we get the well-known the barber paradox: no one shaves all those and those only who do not shave themselves. It is enough to take the predicate 'is shaved by' as R.

Let us return to the solution of Russell's antinomy in Leśniewski's version. In his theory, no class is normal. Indeed, every object is an ingrediens of itself, and *being an ingrediens of* is the same as *being an element of*, so every object is an element of itself. Hence the concept of *being a normal class* is empty. Furthermore, empty concepts do not designate collective classes. So there is no class of all normal classes.

Does the given solution have anything to do with Russell's original antinomy? Probably not, because in both cases, we are talking about objects of a different kind. Russell's original antinomy was about normal classes in the distributive sense. Leśniewski's considerations concerned normal classes in the collective sense. Finally, let us emphasize once again that for Leśniewski, there were only classes of the second kind. So for him, Russell's original antinomy was about nothing — is talking about «something» that does not exist. So it is a paradox only «apparently». Leśniewski wanted to show that there is no paradox of a class of all normal classes for his classes.

Since Leśniewski wanted to refer to Russell's original antinomy, he used the notion *being an element of*, not the notion *being a part of*. However, the question may arise: what if he used the latter term? Of course, we will not get a contradiction either. Due to the irreflexivity of *of being a part of*, the name 'class not being part of itself' applies to all classes, i.e., to all objects in Leśniewski's case. So if there are no objects (what Leśniewski did not exclude), we have only empty general names. If there is an object, then according to the axioms of mereology, the given name designates the object which is a collective class of all objects.

Acknowledgements

This research was funded by the National Science Centre, Poland, grant number 2021/43/B/HS1/03187.

References

Borkowski, L. (1977). *Logika formalna* (*Formal Logic*). 2nd Edition. Warszawa: PWN.

Leśniewski, S. (1916). Podstawy ogólnej teoryi mnogości I (Foundations of the general theory of sets I). Moskwa: Prace Polskiego Koła Naukowego.

Leśniewski, S. (1927). O podstawach matematyki 1 (On the foundations of mathematics 1). *Przegląd Filozoficzny*, *30*, 164–206.

Leśniewski, S. (1928). O podstawach matematyki 2 (On the foundations of mathematics 2). *Przegląd Filozoficzny*, *31*, 261–291.

Leśniewski, S. (1929). O podstawach matematyki 3 (On the foundations of mathematics 3). *Przegląd Filozoficzny*, *32*, 60–101.

Leśniewski, S. (1930). O podstawach matematyki 4 (On the foundations of mathematics 4). *Przegląd Filozoficzny*, *33*, 7–105.

Leśniewski, S. (1931). O podstawach matematyki 5 (On the foundations of mathematics 5). *Przegląd Filozoficzny*, *34*, 142–170.

Leśniewski, S. (1991). *Collected Works*. Vol. I. S. Surma, J. Srzednicki, D. Barnett, V. Rickey (eds.). Dordrecht, Boston, London: Nijhoff International Philosophy, Kluwer Academic Publishers.

Murawski, R. (1984). G. Cantora filozofia teorii mnogości (Cantor's philosophy of set theory). *Studia Filozoficzne*, *11–12*, 75–88.

Pietruszczak, A. (2018). *Metamereology*. Toruń: The Nicolaus Copernicus University Publishing House. http://doi.org/10.12775/3961-4

Quine, W. V. O., (1953). *From a Logical Point of View*. Cambridge Mass.: Harvard University Press.

Quine, W. V. O. (1981/1951). *Mathematical Logic*. Cambridge Mass.: Harvard University Press.

Quine, W. V. O. (1987). *Quiddities: An Intermittently Philosophical Dictionary*. Harvard University Press.

Russell, B. (1919). *Introduction to Mathematical Philosophy*. London: George Allen & Unwin, Ltd.

Słupecki, J., & Borkowski, L. (1967). *Elements of Mathematical Logic and Set Theory*. Oxford and Warsaw: Pergamon Press and PWN.

Wang, H. (1994). What is logic? *Monist*, *77*(3), 261–277.

Whitehead, A. N. (1929). *Process and Reality*. New York: Macmillan.

Whitehead, A. N., & Russell, B. (1910–1913). *Principia Mathematica*. Cambridge: Cambridge University Press.