*Kielnarowa Review* (2025), **2**, 3–14
doi: https://doi.org/10.57656/kr-2025-0001

Kielnarowa
REVIEW

**ARTICLE**

# The Architecture of Relativistic Space-Time: A Critical Review

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**Abstract**

In this critical review, we try to extract from the rich space-time architecture those aspects that are responsible for time and causality. We start with a short review of some attempts to axiomatise the theory of relativity. We focus on the classical approach by Ehlers, Pirani and Schild. While not uncontroversial, it shows — from a quasi-operational perspective — the relations between various space-time substructures. Then we analyse temporal and causal threads in the fabric of the contemporary space-time theory. The so-called causal structure of space-time, nowadays elaborated in great detail, has entered into relativistic physics as a part of its theoretical tool-kit, and can be regarded as complementary with respect to the method of axiomatisation. We argue that a local time flow and an elementary concept of causality are a necessary minimum upon which further richer and richer space-time substructures are built. Our philosophical message is that the relativistic model supports neither the attempts à la Hume to reduce causality to time order, nor the endeavours à la Leibniz to derive time from causal order. Instead, the theory of relativity pictures space-time as a rich edifice which, when looked upon from various angles, displays different logical patterns of its superstructure.

**Keywords:** space-time, causality, time, general relativity

## 1. Introduction

In 1950, Cambridge University Press published a small book by Erwin Schrödinger entitled "Space-Time Structure" (Schrödinger 1985). It can be considered a forerunner of the later global approach to the geometry of space-time, which a decade later led to the famous theorems on the existence of singularities (Penrose 1965). According to Schrödinger, Einstein's theory of space-time is, to use today's phraseology, a "*theory of everything*". In the introduction to this book he wrote (Schrödinger 1985): "A four-dimensional continuum endowed with a certain intrinsic geometric structure, a structure that is subject to certain inherent purely geometrical laws, is to be an adequate model or picture of the 'real world around us in space and time' with all that it contains and including its total behaviour, the display of all events going on in it." Schrödinger expressed his belief that it would be possible to conceive other forces, besides gravity, as "purely geometrical restrictions on the structure of space-time". He himself explored only three levels or aspects of this structure: its general invariance, its connection and metric substructures.

Thanks to the research direction initiated by Schrödinger, we can now reconstruct the rich architecture of space-time, with the wealth of its substructures and their corresponding physical interpretations. Although a complete unification of quantum theory with the geometric structure of space-time is still missing, large areas of physics are indeed encoded in space-time geometry. The purpose of this critical review is to extract from this rich space-time architecture those aspects that are associated with the concepts of time and causality. It seems that the very possibility of doing physics as a science based on measurements is essentially related to causal relationships and measurements of time.

The advent of the theory of relativity has shed a completely new light on the role of time and causality in the structure of modern physic. Since the structural construction of a physical theory is best visible in its axiomatisation, we start, in Section 2, with a short review of some early attempts to axiomatise theory of relativity. Then, in Section 3, we focus on the classical axiomatisation proposed by Elhers, Pirani, and Schild (1972). We give a brief overview of this axiomatic system, which is now well-established, and thoroughly discussed in the philosophical literature. While not uncontroversial, it nicely discloses the structuring of space-time — i.e. shows mutual relations between its various substructures. We quote some critical remarks on the EPS axiomatics from (Linnemann and Read 2021; Adlam, Linnemann, and Read 2022a, 2022b) and summarise its current empirical status.

In Section 4 we give a selected account of what physicists call causal or cone structure of space-time. Our approach consists not so much in analysing separate problems involved in this structure, but rather in investigating temporal and causal threads in the fabric of contemporary space-time theory. The causal structure of space-time has been worked out in great detail — see, for instance, (Carter 1971; Hawking and Ellis 1973) or (Minguzzi and Sánchez 2008; Minguzzi 2019). In (Carter 1971, Sec. 12) one can find a (non-constructivist) axiomatisation of causal spaces, called there "etiological spaces". This approach entered into relativistic physics as a part of its theoretical tool-kit. The fact that this tool-kit was used by Woodhouse (1973) to improve the EPS axiomatisation, shows that these two approaches are, in a sense, complementary.

In Section 5, we argue that, from the geometric point of view, the notion of a local time flow and an elementary concept of causality are the minimum upon which a further hierarchy of stronger and stronger conditions are built. Finally, in Section 6 we revisit the traditional philosophical standpoints à la Leibniz (causality implies time) and à la Hume (time implies causality) in view of the contemporary relativistic model of space-time. We argue that neither of these ideas is reflected within theory of relativity. In contrast, the relativistic space-time is a rich holistic structure, in which the concepts of time and causality are irreducibly interwoven.

## 2. Some early attempts to axiomatise relativity theory

Perhaps the earliest axiomatisation of relativity theory is attributed to Robb (1914) who proposed an axiomatic system based on the concept of "conic order", and was able to derive both topological and metric properties of space-time from the "invariant succession of events". In the axiomatic systems of both Carnap (1925) and Reichenbach (1924) it is the temporal order that is reduced to the causal order. The same is true for the Mehlberg's system (Mehlberg 1980), which was elaborated within the broader setting of an interdisciplinary study of the causal theory of time[1].

The latter three authors were associated with logical empiricism and, in agreement with its philosophical ideology, aimed at clarifying logically the conceptual situation in the theory of relativity, which was extensively discussed at that time. In doing so, they *a priori* eliminated, with the help of their axioms, some "pathological situations". These included the logical paradoxes induced by causal loops. What they did not take into account was that the later development of physics might need such pathologies (see e.g. (Deutsch 1991; Lloyd et al. 2011)). In 1949 Gödel (1949) published his solution to Einstein's field equations with closed timelike curves (soon after more solutions with similar "time anomalies;' were found). A few years later the first general theorem concerning the global structure of space-time was proved stating that every compact space-time must contain closed timelike curves (Bass and Witten 1957).

1. Although Mehlberg's book appeared in 1980, it is based on his works dating back to 1935 and 1937.

A different strategy for the axiomatisation of space-time was proposed by Elhers, Pirani, and Schild (1972) (in the following, we refer to it as to the EPS axiomatisation). In contrast to the conceptual approaches of Reichenbach and Carnap, the EPS axiomatisation is based on mathematical structures inherent in general relativity and the operational character of physics.

It is instructive at this point to recall the distinction, in the spirit of Carnap (1967) and Reichenbach (1924), between constructive axiomatisation and deductive axiomatisation. The former "builds on a basis of empirically supposedly indubitable posits" (Linnemann and Read 2021), whereas the latter "proceeds semantically in a linear, non-circular fashion" (ibid.), i.e. its primary purpose is to establish the logical order of inference. For obvious reasons, we are interested in the constructivist axiomatisation of space-time; however, in the case of relativistic space-time, as we shall see, it is not fully achievable. So we have to be content with what physicists call a quasi-operational approach. The EPS axiomatisation belongs to this type of axiomatic systems.

## 3. Space-time architecture

From the mathematical point of view, a space-time in general relativity is a pair $(M, g)$, where $M$ is a four-dimensional differential manifold, and $g$ a Lorentzian metric defined on it. Furthermore, it is typically assumed that $(M, g)$ is connected and time-oriented (see (Bielińska and Read 2023) for a nice overview on the time-orientability issue).

We say that the manifold $M$ carries a Lorentzian structure. This structure not only contains several other mathematical substructures, which interact with each other creating a subtle hierarchical edifice, but also admits, on each of its levels, a physical interpretation, making out of the whole one of the most beautiful models of contemporary physics. We shall briefly describe the 'internal design' of this model. Our analysis will be based on the classical paper by Elhers, Pirani, and Schild (1972), in which the authors presented a quasi-operationistic axiomatic system for the Lorentz structure of space-time showing both its mathematical architecture and physical meaning.

The building blocks (primitive concepts) of this axiomatisation are: (1) a set $M = (p, q, \ldots)$, the elements of which are called events; and two collections of subsets of $M$: (2) $L = (L_1, L_2, \ldots)$, the elements of which are called histories of light rays or of photons (light rays or photons, for the sake of brevity); (3) $P = (P_1, P_2, \ldots)$, the elements of which are called histories of test particles or of observers (particles or observers, for brevity).

The axioms describe the following scenario: A light ray is sent from an event $p$, situated on a particle history $P_1$, and received at an event $q$, situated on a particle history $P_2$ (a message from $p$ to $q$). The message can be reflected at $q$ and sent to the event $p'$, situated on the particle history $P_1$ (an echo on $P_1$ from $P_2$). By a suitable combination of messages and echoes one can ascribe four coordinates to any event, and construct local coordinate systems. A bit of mathematical gymnastics, sanctioned by suitable axioms, allows one to organise the set of events, $M$, into a differential manifold (with the usual manifold topology).

The next set of axioms equips the manifold $M$ with a conformal metric. This is the usual metric (with Lorentzian signature) defined up to a multiplicative factor (the conformal factor). This metric allows one to distinguish timelike, null (lightlike) and spacelike histories (curves), and completely determines the geometry of null-curves.

To determine the geometry of timelike curves one needs additional axioms defining the so-called projective structure. Its task is to determine a distinguished class of timelike curves that are physically interpreted as representing histories of particles (or observers) moving with no acceleration (freely falling particles or observers).

Conformal and projective structures are, in principle, independent and they need to be synchronised. This is done with the help of suitable axioms, which enforce null geodesics passing through an event $p$ to form the light cone at $p$, and projective timelike curves to fill in the interior of this light cone. If this is the case, we speak of the Weyl structure.

In a Weyl space (a manifold equipped with the Weyl structure), there is a natural method to define length – the arc length – along any timelike curve. Such a length is interpreted as a time interval measured by a clock carried by the corresponding particle (proper time of the particle). However, proper times of different particles are unrelated; to synchronise them a suitable metric must be introduced on $M$. This can be done with the help of the following axiom: Let $p_1, p_2, \ldots$ be equidistant events on the history of a freely falling particle $P$, and let them be correspondingly simultaneous with events $q_1, q_2, \ldots$ on the history of a freely falling particle Q. Simultaneity is understood here in the Einstein sense: two events, $p$ and $q$, are simultaneous if an observer situated half-way between them sees the light signal emitted by $p$ and $q$ at the same instant as shown by the observer's clock. The metric structure is established if the events $q_1, q_2, \ldots$ on Q are also equidistant. The metric $g$, constructed in this way, contains in itself the Weyl structure; it is, therefore a Lorentz metric. This completes the construction of the relativistic model of space-time.

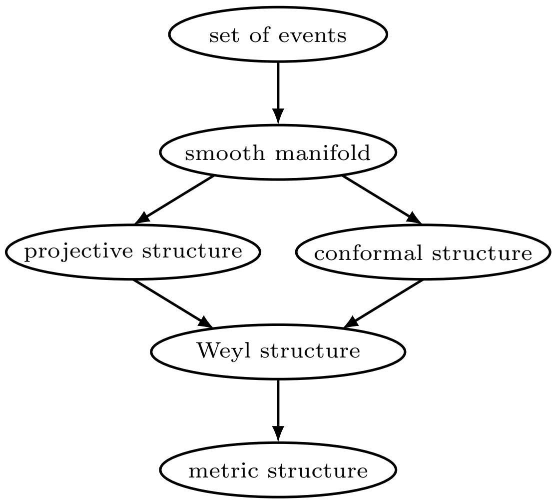


**Figure 1.** A diagram illustrating the subsequent layers of axiomatisation of general relativity following (Elhers, Pirani, and Schild 1972).

The EPS axiomatisation was recently thoroughly analysed in (Linnemann and Read 2021; Adlam, Linnemann, and Read 2022a, 2022b). The authors showed its limitations and gaps, and proposed some amendments. Here we quote only a few critical remarks, which seem relevant from the point of view of our purposes.

- In the EPS axiomatisation the topological Hausdorff condition is not explicitly postulated. However, when using a smooth manifold as a space-time model, the Hausdorff condition is usually assumed (see e.g. (Hawking and Ellis 1973, p. 13)). The consequences of dropping this condition are studied in detail in (Adlam, Linnemann, and Read 2022b) in the context of possible 'quantum' versions of the EPS axiomatics, in particular in relation to branching space-times (Luc 2020; Luc and Placek 2020; Belnap, Müller, and Placek 2021).
- The axiom assuring the existence of messages and echoes is not valid globally in strong gravitational fields (Linnemann and Read 2021; Pfister and King 2015).
- Lorentz metric is sufficient, but not necessary for the construction of a standard clock measuring the proper time. Indeed, Perlick (1987) has discussed such a clock, with the help of the message-and-echo method, just like in the EPS axiomatisation, entirely within the Weyl structure. This issue is discussed in detail in (Adlam, Linnemann, and Read 2022a, 2022b).

- The transition from Weyl metric to Lorentz metric can also be done by postulating the so-called "no second-clock effect". This effect occurs if a vector in a Weyl space, after being parallelly transported along two different curves to the same point, preserves its length, and if this is valid for all such configurations. This procedure, just like the procedure of simultaneity of two series of equidistant events, described above, requires the assumption that the corresponding space-time domain should be simply connected. This must be guaranteed by the axioms.

For more of constructive criticism on EPS approach, see for example: (Linnemann and Read 2021; Trautman 2012; Dewar, Linnemann, and Read 2022; Pfister and King 2015). A large amount of literature that has grown up around the EPS axiomatisation testifies to the pertinence of the issue.

It is important to stress that the axioms of general relativity, as formalised in (Elhers, Pirani, and Schild 1972), can be and have been tested against empirical data. For example, the axiom on the conformal structure of space-time would be spoiled if the velocity of photons would depend upon its energy (Amelino-Camelia et al. 1998; Amelino-Camelia 2013). No such effect has been observed to a high degree of accuracy (Abdo et al. 2009; Perlman et al. 2015; Pan et al. 2020). In the same vein, one could seek deviations from the universality of the projective structure by inspecting the dispersion relations of high-energy massive particles (Amelino-Camelia et al. 2016). Very recently, such possible effects were refuted basing on the scrutiny of astrophysical neutrinos (The IceCube Collaboration 2022).

The existence of the Lorentzian structure on top of the Weyl structure could be undermined through the observation of the "second clock effect" (Elhers, Pirani, and Schild 1972). The latter arises commonly in various modified-gravity theories (see e.g. (De Felice and Tsujikawa 2010)). Recently, the second clock effect was constrained using the CERN data on muon anomalous magnetic moment (Lobo and Romero 2018). For newer experimental searches for Lorentz structure violation coming possibly from torsion effects see: (Kostelecký, Russell, and Tasson 2008; Schettino et al. 2020; Delhom-Latorre, Olmo, and Ronco 2018).

Eventually, one can question the model of a space-time as a smooth manifold. This is typically done on the basis of some quantum gravity theory (Oriti 2009). Although the possible effects of these models are extremely hard to observe, the experimental efforts in this direction are progressing (Amelino-Camelia 2013; Addazi et al. 2022). In particular, some limits on space-time granularity have recently been established (Chou et al. 2016).

## 4. Causal structure and global time

Every text-book on relativity tells us that causality (the causal structure) is identical with the cone structure of space-time, that is to say with the Weyl structure of the above scheme (synchronised projective and conformal structures). Chronology relation[2] tells us how causal influences can be propagated along timelike curves, whereas causality relation[3] takes also into account causal influences propagating along null geodesics. Both determine channels through which causal influences can propagate rather than actual interactions between cause and effect. With the help of these relations we define the chronological future $I^+(p)$ and the chronological past $I^-(p)$ of an event $p$ as the set of all events which chronologically follow (resp. are followed by) $p$; and analogously, the causal future $J^+(p)$ and the causal past $J^-(p)$ of $p$.

If $M$ is a space-time manifold, at each of its points (events) $p$ there is a tangent space $T_pM$ equipped with the structure of the Minkowski space-time. Its causal structure is the familiar light cone structure of special relativity. This structure, essentially unchanged, is inherited by any local

2. An event $p$ is said to chronologically precede an event $q$, written $p \ll q$, if there is a future-directed timelike curve from $p$ to $q$; for the full account of chronology and causal relations see (Minguzzi and Sánchez 2008; Minguzzi 2019).

3. An event $p$ is said to causally precede an event $q$, written $p \preceq q$, if there is a future-directed timelike or null curve from $p$ to $q$, or $p = q$.

neighbourhood (called normal neighbourhood) of the space-time manifold $M$.[4] However, globally (outside normal neighbourhoods) causal structure can be very different from that of Minkowski space-time, sometimes extremely exotic and full of pathologies (see (Carter 1971; Minguzzi 2019)).

There exists an interaction between the causal structure of space-time and its topology. Sets $I^+(p)$ and $I^-(p)$ are always open, but sets $J^+(p)$ and $J^-(p)$ are not always closed. This allows one to define topology "innate" for causal spaces (i.e. defined entirely in terms of causal relations). It is the so-called Alexandrov topology[5]. This topology is weaker than the usual manifold topology[6].

This mathematical apparatus proved to be very efficient in disentangling various problems related to the structure of space-time. Engaging it into subtleties of space-time architecture has allowed Woodhause to improve the EPS axiomatisation (Woodhouse 1973). He was able to derive the differential and causal structures from more or less EPS axioms expressed in terms of chronology and causality relations. The main advantage of Woodhouse's approach is that no assumption has to be made concerning paths along which light signals propagate. They are deduced from statements regarding emission and absorption of light signals. In his approach, one has to assume that there is exactly one history of a particle through each point in each direction in space-time. Physically this means that it is possible to define the history of a freely falling particle through any point in space-time.

The above conceptual machinery provides a powerful tool for studying various global aspects of space-time. In what follows, we focus on those of them that elucidate mutual dependencies between time and causality.

The first condition that has to be imposed in order to have something resembling a temporal order is to guarantee the absence of closed timelike and causal curves. The corresponding conditions are called the chronology condition (the absence of closed timelike curves) and the causality condition (the absence of closed timelike or null curves[7]), respectively. The motivation is obvious: with such loops there is no clear distinction between the future and the past. Although the existence of relativistic world models with closed timelike curves (such as the famous Gödel's model 1949) shows that the idea of "closed time" is not a logical contradiction, yet the point is that the space-time structure strongly interacts with the rest of physics, what might be a source of many logical perplexities. Not only in a space-time with closed timelike curves one might kill one's ancestor to prevent one's birth, but also — more prosaically — in a space-time with causality violations no global Maxwell field could exist that would match a given local field configuration (Geroch and Horowitz 1979).

There is a rich hierarchy of stronger and stronger conditions[8] which improve temporal and causal properties (Carter 1971; Hawking and Ellis 1973; Minguzzi and Sánchez 2008; Minguzzi 2019). Among them there is an important condition, called strong causality condition, that excludes almost closed causal curves[9]. In such a space-time, the nonexistence of closed timelike curves is satisfied with a certain safety margin. Owing to this margin, the Alexandrov topology improves to the manifold topology.

However, this is not enough. The fact that any measurement can be done only within certain unavoidable error limits, prevents us from measuring the space-time metric exactly. Many "near-by metrics" are always within the measurement's "error bar". If measurements are to have any meaning at all — and the very existence of physics depends on this — we must postulate a certain stability of measurements. This postulate, as regarding relativistic causality, assumes the form of the stable causality condition; it precludes small perturbations of space-time metric to produce closed causal

4. Through the so-called exponential mapping, exp: $(T_pM) \to M$.
5. Sets are defined to be open in this topology if they are unions of the sets of the form $I^+(p) \cap I^-(p)$.
6. Alexandrov topology coincides with manifold topology if the strong causality condition is satisfied; see below.
7. Timelike and null curves are jointly called causal curves.
8. In fact, the hierarchy is nondenumerable.
9. More precisely, it states that each neighbourhood of any event in space-time contains a neighbourhood which no causal curve intersects more than once.

curves[10].

The condition of stable causality has therefore a certain philosophical significance. When it is not satisfied, measurements of physical quantities become essentially meaningless. As put by Hawking, "Thus the only properties of space-time that are physically significant are those that are stable in some appropriate topology" (Hawking 1971) [11].

It is a nice surprise that this condition of "physical reasonability" meets with another very "reasonable" property. To Hawking we owe the theorem: In a space-time $M$ there exists global time, measured by a global time function, if and only if $M$ is stably causal (Hawking 1969). The history of any clock in the universe (for instance, the history of a vibrating particle) is a timelike curve in space-time $M$. Indications of such a clock can mathematically be represented by a monotonically increasing function along this curve – the time function for this clock. If there exists a single function $\mathcal{T}$, which is a time function for a family of clocks filling the space-time $M$, then such a function $\mathcal{T}$ is called a global time function. It measures a global time in the universe. As we can see, there exists a deep connection between global time and the above mentioned measurement stability property. It shows that doing physics automatically requires both: global time and stable causality.

We should notice that the spacelike hypersurfaces $\mathcal{T}$ = const. give surfaces of simultaneity in the universe, but they are not unique. However, one can "synchronise the universe" by imposing a yet stronger causality condition. This is done in the following way. First, we define the Cauchy surface of space-time $M$ as a subset $S$ of $M$ such that no inextendible causal curve crosses $S$ more than once. This definition was first introduced in the theory of partial differential equations (Leray 1953). The initial data for a given equation are given on a Cauchy surface. A space-time $M$ is said to be globally hyperbolic if it can be presented as a Cartesian product $M \simeq T \times S$, where $T = \mathbb{R}$ is a global time and $S$ a Cauchy surface in $M$ (see (Geroch 1970; Bernal and Sánchez 2006)). The global time function $\mathcal{T}$, such that $\mathcal{T}^{-1}(t),\ t \in T$ is a Cauchy surface, is called the Cauchy time function.

A globally hyperbolic space-time is the closest to the Newtonian absolute space we can get within the framework of general relativity. It is "deterministic" in the sense that initial data given on a Cauchy surface determine, in principle, the entire history of the universe. However, the so-called Cauchy problem in general relativity, in all its mathematical details, is far from being simple and easy (see (Hawking and Ellis 1973, Chapter 7) or (Ringström 2009)).

## 5. Time and Causality as Primitive Notions

It is clear from the inspection of the axioms of general relativity that they presuppose some primitive notions of time and causality. Indeed, in order to make sense out of the "echos and messages" axioms one needs to assume that the signal is emitted *before* it is received. This statement finds its formal justification in the following reasoning. It is not an exaggeration to assume that each history of any test particle carries a $C^0$-structure, which assures the local homeomorphism with $\mathbb{R}$ . This can be interpreted as a local flow of time with no preferred time orientation. Local time orientation (local arrow of time) is a property that has to come "from outside". Time arrow is not a part of the space-time architecture.

In the EPS axiomatic system, echos and messages are operationally defined in terms of coincidences of clock readings and acts of emission or reception of light rays. However, intuition smuggles into this operationistic picture the idea that the received echo is actually *caused* by the radar signal. It seems, therefore, that some elementary notion of causality underlies the entire EPS system.

10. To define this condition precisely, one should consider the space Lor($M$) of all Lorentz metrics on a given space-time manifold $M$, and equip it with a suitable topology. Only then we are able to determine what a small perturbation of a metric means (Hawking 1969).

11. One could argue that in order to "do physics" it is enough for the stable causality condition to hold only locally. However, since the existence of closed causal curves is essentially a non-local condition, we would need some barrier protecting a given "locality" from causal anomalies.

Therefore, the notion of local time flow and some elementary concept of causality are the minimum upon which a further hierarchy of stronger and stronger conditions is built. As we have seen, the stable causality condition plays a special role in this hierarchy. It guarantees both the existence of global time and allows for stable measurement results.

All of the structures described above, suitably synchronised with each other, are contained in the Lorentz metric structure. It is a standard result that the Lorentz metric structure exists globally on a space-time manifold $M$ if and only if a non-vanishing direction field exists on $M$. Moreover, the Lorentz metric can always by chosen in such a way as to make this direction field timelike (Geroch 1971). Since such a field on a space-time manifold locally always exists, the same is true for the Lorentz metric. The existence of a Lorentz metric is strictly related to the possibility of performing space and time measurements; therefore, it is almost synonymous with the possibility of doing physics. Above, we have identified such a possibility with the existence a local "topological time" (a $C^0$-structure on each history of a test particle), here we have the same condition raised to the metric level[12].

A word of warning is to be made. Our conclusions are valid only within the conceptual framework of what we have called the relativistic model of space-time, and only within its reconstruction as it is presented above. Other axiomatic approaches to the geometry of space-time are possible (see, for, instance, (Andréka et al. 2013; Covarrubias 1993; Guts 1995)) and they can give rise to different interpretations[13]. However, we should take into account the fact that it is the general theory of relativity that is deeply rooted in this model, and since this theory is very well founded on empirical data it would be unwise to look for a different model (within the limits of its empirical verifications). We should also emphasise that the EPS axiomatic approach should not be easily replaced by other approaches since it renders justice, and does this very well, to both "theoretical practice" of mathematical physicists and operational demands of experimentalists (it has a strong (quasi-)operationistic flavour).

## 6. Message

It is perhaps commonplace to attribute the relational concept of time to Leibniz. Every philosopher knows that, according to Leibniz, time is but a relation ordering events one after the other. It is less known that at the end of his life Leibniz supplemented his relational conception of time with what later resulted in the causal theory of time[14]. His new idea attempted to identify the nature of relations constituting the time order. When we read Leibniz on ordering relations, we should not ascribe to him our present concept of formal order, since his own aim was to stress the difference between his own understanding and that of his English opponents (Newton and Clarke). In the "English theory", there exist two classes of entities: instances and events, and the "natural order" of instances[15] determines the order of events. Events happen at certain instances, but instances are independent of events. In Leibniz's approach, the events are the only class of entities, and time is a derived concept given by relations ordering events. The causal conception of time adds a new idea to Leibniz's philosophy. In his metaphysical-literary style: "the present is always pregnant with the future" (Leibniz 1969, p. 557), and as explained by Mehlberg: "…if one arranges phenomena in a series such that every term contains the reason for all those which come after it in the series, the

12. It is interesting to ask how this condition looks from the global point of view. The answer is that if $M$ is noncompact, such a nonvanishing direction field, and consequently a Lorentz metric, always exists, but if $M$ is compact it exists if and only if the Euler–Poincaré characteristic of $M$ vanishes (Geroch 1967).

13. Different – within certain limits. There is one important constraint: The mathematical structure of space-time must be preserved by all interpretations. One could say that the mathematical structure is "invariant" with respect to all admissible interpretations.

14. The note concerning this conception is found in Leibniz's essay entitled "The metaphysical foundations of mathematics" (Leibniz 1989) and published posthumously; see the extensive study by Henry Mehlberg devoted to the causal theory of time (Mehlberg 1980).

15. We would today say the order determined by the metric structure of time.

causal order of the phenomena so defined will coincide with their temporal order of succession." (Mehlberg 1980, p. 46). Leibniz's idea contrasts with that of Hume who attempted to reduce causal order to temporal order. Whitrow puts this in the following way: In Hume's view "the only possible test of cause and effect is their 'constant union', the invariable succession of the one after the other" (Whitrow 1980, p. 323).

The advent of the theory of relativity showed the role played by time and causality in the structure of modern physics in a radically new light and had a profound impact on our understanding of the world. "Time" was unified with "space" within the concept of space-time, which belongs to the basic formalism of general relativity, and the so-called causal structure of space-time is an indispensable tool for theoretical studies in gravitational physics. The confrontation of traditional disputes about the relationship between space and time with what the theory of relativity has to say about them is an important thread of foundational research. But when it comes to foundational research, the impact of quantum physics cannot be ignored. In the theory of relativity cause–effect interactions occur between definite events and the events themselves are identified with points in space-time. In quantum physics such a viewpoint on space-time points could change radically — this problem calls for a separate study.

The above analysis shows that, within the considered relativistic model, temporal and causal properties are strongly coupled with each other, and one cannot say which is logically (or ontologically) prior with respect to the other. They are unified in the Weyl structure to provide a basis for the fully-fledged concept of time and causality. In this sense, causal theory of time à la Leibniz (causality implies time) is not supported by the relativistic model considered here. The same should be said about an attempt, à la Hume, to reduce causal interactions to a merely temporal succession. It should be taken into account that axiomatic systems can be composed in various ways: various concepts can be selected as primitive concepts and various statements can be accepted as axioms of the system, depending on criteria one adopts. The EPS axiomatic system has the advantage over others that it is quasi-operational, i.e. its axioms describe some simple empirical procedures, although they do so in a highly idealised way. We could conclude that, from the philosophical point of view, space-time is a rich holistic structure, and the choice of a specific axiomatics corresponds to the choice of the angle at which we contemplate the whole.

**Acknowledgement**

M.E. acknowledges support from the National Science Centre in Poland under the research grant Sonata BIS *Beyond Quantum Gravity* (2023/50/E/ST2/00472).

Competing Interests None.

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