Kielnarowa Review (2024), **1**, 3–11

Kielnarowa
REVIEW

ARTICLE

What is a particle?

Michał Eckstein[*][†]

[†]Institute of Theoretical Physics, Jagiellonian University, Łojasiewicza 11, 30-348 Kraków, Poland
[‡]International Centre for Theory of Quantum Technologies, University of Gdańsk, Wita Stwosza 63, 80-308 Gdańsk, Poland
[*]Corresponding author. Email: michal.eckstein@uj.edu.pl

Abstract
This is a short reflection on the notion of a 'particle' in particular and on the methodology of physics in general.

Keywords: particle, philosophy in science, quantum field theory

1. Introduction

According to the (*Oxford English Dictionary*) a *particle* is: "A minute fragment or quantity of matter; the smallest perceptible or discernible part of an aggregation or mass.". This agrees with the common-sense definition of particles as 'small things, out of which the bigger things are made of'.

Clearly, such a concept of particle is deeply rooted in the atomistic vision of the physical world, which — in turn — is supported by the development of physics in the past two centuries.

In philosophy of physics the 'particle' is often treated as a *primitive notion* (cf. (Eckstein and Heller 2022)), that is a concept which is "immediately understandable" and "employed without explaining its meaning" (Tarski 1994). The simplest example of such a primitive notion is that of a *point* in geometry.

Within an axiomatic approach, the primitive notions are utilised to spell out the axioms. Consequently, they are taken for granted, though they might be connected or restricted by the axioms. For instance, in geometry the axiom: "For every two points there exists a line that contains them both." links the primitive notions of a point and a line.

In this spirit, particles as primitive concepts appear, in parallel to "light rays" in the celebrated Ehlers–Pirani–Schild (EPS) axiomatisation of relativistic spacetime (Elhers, Pirani, and Schild 1972). More precisely, the authors take 'particle' to mean a "worldline of a freely falling particle". In either case, on the notions of particles and light rays a quasi-operational axiomatic system of a Lorentzian spacetime is established. As the authors admit, the particles are understood in the classical sense as "bodies whose extension and structure can, under suitable circumstances, be neglected". In fact, assuming that the particles are quantum or, more generally, non-classical, may lead to very different structures and axiomatics (Adlam, Linnemann, and Read 2022; Eckstein and Heller 2022).

2. Particles in classical and quantum physics

2.1 Classical theory

The notion of a particle adopted in the EPS axiomatics, which is in fact the intuitive one, stems from classical (*i.e.* 'before-quantum') physics. A *classical particle* is a pointlike object with certain inherent properties, such as mass, charge, position, momentum etc. Some of these properties are invariant, e.g. mass or charge, and some can change, e.g. position or energy, through free evolution or interactions with the environment. A single classical particle is treated as a passive 'test' object, the properties of which are influenced by the background forces (gravitational, electromagnetic, ...), but the background is *not* influenced by the particle. In other words, it is typically assumed that the backreaction of the particle on its environment is negligible, unless we consider a large number of particles, e.g. constituting a dust cloud.

Such understood classical particle is treated merely as an idealisation of *real* particles. On the fundamental level, the particles are assumed (still within classical physics) to act on whatever environment they are embedded in. This stems from a deep, though seldom phrased explicitly, methodological principle that *physical interactions are always mutual.*

The main point here is that the employment of the notion of a classical particle involves an ontological committent. While the formal concept is an unphysical idealisation, we do assume that there exist in Nature 'real particles', which are adequately modelled by classical particles. In other words, we assume that in a physical experiment we study the actual real particles and describe them formally as classical particles. Such a viewpoint is coherent with the common-sense intuition on empirical sciences — we observe real phenomena and attempt to model them mathematically with idealised concepts.

2.2 Nonrelativistic quantum theory

The quantum theory, however, drives us away from the common-sense viewpoint on particles. In non-relativistic quantum mechanics a 'particle' is still a primitive notion. Concretely, a *quantum particle* is a quantum system (the 'system' is itself a primitive notion) described by a noncommutative algebra of observables generated by position ($\hat{x}$), momentum ($\hat{p}$), and possibly some internal degrees of freedom (Strocchi 2008). It may have some objective (i.e. classical) properties, such as charge or mass, but it does not have a definite position nor momentum. The latter properties actualise (randomly!) only upon an active measurement[1].

A quantum particle is also an idealisation of a 'real' particle, but in a somewhat more convoluted sense. In the quantum world, a real particle is never completely isolated from its environment. Its (quantum) degrees of freedom get *entangled* with the degrees of freedom of the environment, hence, at the fundamental level, one must treat the particle and its environment (which is, actually, the entire physical Universe) as a single global system.

A quantum system is completely characterised by a *state*, that is a density operator on a Hilbert space $\mathcal{H}_P$. An isolated quantum particle is characterised by a pure state, that is a vector $|\psi\rangle \in \mathcal{H}_P$. This state is *intrinsic* — it pertains only to the particle itself and not to any external systems, such as 'environment', 'detector' or 'observer'. It could thus be seen as a quantum analogue of particle's properties. But whenever a particle interacts with its environment it becomes entangled with it, so that the total (pure) state of the system is a superposition of different states of the particle and the environment, $|\Psi\rangle_{P+E} = \sum_i c_i |\psi_i\rangle_P |\phi_i\rangle_E \in \mathcal{H}_P \otimes \mathcal{H}_E$. Furthermore, this decomposition is not unique — as the values of the coefficients c_i depend on the choice of the bases $\{|\psi_i\rangle\}$ and $\{|\phi_i\rangle\}$. Consequently, it is no longer possible to assign a unique vector in $\mathcal{H}_P$ to the particle itself. We thus see that, upon interaction with the environment, a quantum particle not only changes its state (i.e. 'quantum properties'), as it is the case for a classical particle, but actually it *looses its identity*.

At the operational level, a quantum particle is completely characterised by the algebra of observables that correspond to all possible measurements, which can be performed on it (Strocchi 2008). The possible outcomes of these measurements, along with the respective probabilities of occurrence are encoded in the particle's reduced density operator $\rho_P = \mathrm{Tr}_E |\Psi\rangle_{P+E} \langle\Psi| = \sum_i |c_i|^2 |\psi_i\rangle_P \langle\psi_i|$. Note

---

[1] One may attempt to save the common-sense viewpoint, for instance by adopting the Bohmian interpretation of quantum mechanics. Then, the particles do have intrinsic properties, in particular, trajectories, but the price to pay is the introduction of a fundamentally unobservable object – the 'pilot wave'. A serious drawback of Bohmian mechanics, which undermines its philosophical implications, is that it is not compatible with the theory of relativity (see, however, (Dürr et al. 2014)).

that while ρ_P is does not depend on the choice of the Hilbert space basis, its decomposition using $|\psi_i\rangle_P$ does. But the state ρ_P does not provide a complete description of the quantum particle, because it does not contain the information about the complex phases of the coefficients c_i. In a single measurement of a given observable A, with a spectral decomposition $A = \sum_a a\mathcal{P}_a$, we only register a single outcome a with the probability $\mathrm{Tr}\,\mathcal{P}_a\,\rho_P$. However, given a large collection of particles with the (effective) state ρ_P, or using weak measurements (Dressel et al. 2014), one can perform a *quantum state tomography* and reconstruct the state ρ_P to an arbitrary precision (Paris and Rehacek 2004; Wu 2013). One could thus say that the state ρ_P is a certain idealisation of a real quantum particle, which we can access empirically.

Finally, let us observe that a classical particle can also be seen as an idealisation of a quantum particle. Indeed, if the scales of the experimental setup are macroscopic, as it is surely the case for instance in astronomy, then one can safely assign an average classical trajectory to a particle, $x(t) = \mathrm{Tr}\,\rho_P(t)\hat{x} \approx \langle\psi(t)|\,\hat{x}\,|\psi(t)\rangle$ and we can neglect the particle's entanglement with its environment. In other words, if the resolution of the measuring device is much larger than the width of the quantum wave packet, then the particle at hand is effective classical.

2.3 Quantum field theory

The concept of a particle becomes even more cumbersome within the relativistic quantum. In fact, in quantum field theory a particle is no longer a primitive notion. It appears as a specific state of a quantum field — a *single particle state* —, which is a certain vector in the Fock space. The point is, however, that in such states exist only in a free, i.e. non-interacting, theory. For general interacting quantum fields states with a fixed number of particles appear only asymptotically — as “in” and “out” states for the S-matrix (Haag 1996).

Even at the operational level we cannot unequivocally identify a relativistic quantum particle. This is because any single-particle detector (and in fact any QFT detector at all) has a non-vanishing vacuum response (Reeh and Schlieder 1961; Peres and Terno 2004). In other words, a detector click may be induced by random fluctuation of the QFT vacuum[2].

We thus see that the realm of relativistic quantum theory is very different from the common-sense one: At the fundamental level there are no particles, only quantum fields[3]. In specific circumstances, for instance in detection events, one can (statistically) identify ‘single-particle’ phenomena. Furthermore, the properties of QFT particles are even more cumbersome than these of non-relativistic quantum particles. Firstly, a single-particle state has a non-vanishing probability of detection in any region of spacetime (Reeh and Schlieder 1961; Peres and Terno 2004). Secondly, the mass of a particle is a concept which depends on the energy scale[4]. On the other hand, the charge of an elementary QFT particle is a *classical* property, as the superselection rules forbid the existence of quantum superpositions of states with different charges.

3. An information-theoretic perspective

A rather different concept of a particle emerges from modern information theory, inspired by the device-independent approach to quantum information (Pironio, Scarani, and Vidick 2016). It focuses on the information-processing aspects of phenomena, while neglecting the physical details of the involved objects. In this context a ‘particle’ is a primitive notion and signifies merely a physical ‘*information carrier*’ (Brunner et al. 2014; Eckstein et al. 2020; Miller et al. 2021). This notion hinges upon another methodological principle that *information is physical*. It means that any information

---

[2] This phenomenon has led to some confusion concerning relativistic causality and signal propagation in quantum field theory (Hegerfeldt 1994; Buchholz and Yngvason 1994; Sabin et al. 2011).

[3] It is somewhat ironic that the Standard Model of Particle Physics is a theory, which says that in fact there are no particles in Nature.

[4] The effect of ‘running masses’ is a consequence of renormalization procedure (Weinberg 1995).

(classical, quantum or else) must be supported in *some* physical system. But the information itself is independent on the physical system, in which it is encoded. Indeed, in any actual communication protocol the information is faithfully transferred with the help of a number of different physical systems: antennas, cables etc.

Such an information-theoretic particle is a concept, which is independent of the physical theory. When describing an information processing protocol we are only concerned with the data (bit, qubit, etc.) and not with the physical carriers. The latter can be classical particles, quantum particles, field excitations or some other, possibly unknown, physical entities.

Nevertheless, information theory is not purely epistemic and it does involve an ontological commitment. Indeed, we must assume that the information carriers are available for the agents planning to execute a given protocol. In other words, when considering admissible information processing protocols we do assume that suitable resources exist in Nature. Furthermore, these resources exist independently of whether the agents decide to use them or not. Bringing the reasoning back to fundamental physics, we can say that quantum fields exist in Nature and free agents can use them (i.e. interact with them in a designed way) to process information.

The gist of the modern information-theoretic approach is that it is based on operational notions only. A particle in the information-theoretic sense in indeed operational, because it assumes that there must exist a physical device and an observer able to access the information carried by it. This might suggest a methodological guideline for the formulation of physical theories — they ought to be based solely on operational notions.

This guideline is often adopted in the context of quantum foundations because of the success of 'operationalising' the axioms of (non-relativistic!) quantum mechanics on the one hand (see (Chiribella and Spekkens 2016) and refs therein) and performing 'theory-independent' experiments on the other. The latter are variants of the famous EPR–Bell experiment, which indeed can be formulated purely operationally, without invoking the notions from any particular physical theory (see e.g. (Brunner et al. 2014)):

> Take two measuring devices A and B, each with two possible measurement settings ($x, y = 0, 1$) and two possible outcomes ($a, b = \pm$), and feed them with two particles coming from a common source. Arrange the setup so that the detection events at A and B are spacelike separated, the devices register at least 83% of particles and the measurement settings are random[5]. Gather a large detection statistics from many particles and different measurement settings and compare the correlations for a given pair of settings:
>
> $$\begin{aligned} &C(x,y) = P(a=b\,|\,x,y) - P(a=-b\,|\,x,y), && \text{for all} \quad x,y \in \{0,1\} \\ &S = C(x,y) + C(x',y) + C(x,y') - C(x',y'), && \text{for all} \quad x,x',y,y \in \{0,1\}. \end{aligned}$$
>
> If for some settings $S > 2$, then the experiment cannot be explained by *any* local hidden variable theory and if $S > 2\sqrt{2}$, then it cannot be explained by the quantum theory (relativistic or not).

We see that such an experiment provides a very powerful tool to test our mathematical theories against the empirical data. Its implementation requires no knowledge of any physical theory[6] and no a priori rules on how to model the 'particles' and 'detectors'.

But all that concerns an EPR–Bell experiment *in principle*. The actual EPR–Bell experiments are performed with sophisticated setups, the design of which is heavily based on the established physical theories, including quantum mechanics. What is more, physicists knew from the very beginning

---

[5] These are the famous "loopholes" in Bell-type experiments. The two former can be, and have been, closed (Aspect 2015), while the third one is in fact a methodological principle, and hence it cannot be closed (Eckstein and Horodecki 2020).

[6] Well, almost... We do need to know at least what "spacelike separated" means.

that in order to test quantum mechanics against local hidden variables one needs to engineer a specific quantum state, which is (as close as technically possible to) a maximally entangled state. In conclusion, while the EPR–Bell test, as described above, is formulated in a model-independent manner, its implementation must be done with concrete a physical system, of which we have a good understanding and control. But this of course requires a reliable and well-established theory describing the physical system at hand.

4. Conclusions and reflections

It is certainly instructive and valuable to formulate the descriptions of physical experiments in terms of purely operational notions — as it was done for the Bell test. However, one has to keep in mind that in actual experiments the physicists are bound to use complex theoretical schemes. The latter are based on a number of (typically not primitive) notions, which *only* make sense within these schemes. Consequently, the published results of experiments are hardly even understandable to non-experts.

Take, for instance, the following excerpt from the Higgs boson discovery paper (Aad et al. 2012), worth a Nobel prize:

> *Clear evidence for the production of a neutral boson with a measured mass of* 126 ± 0.4*(stat)* $\pm$ 0.4*(sys) GeV is presented. This observation, which has a significance of 5.9 standard deviations, corresponding to a background fluctuation probability of* $1.7 \cdot 10^{-9}$*, is compatible with the production and decay of the Standard Model Higgs boson.*

The "Higgs particle" is in fact a new quantum field with specific properties (charge, mass, spin) and coupling to other known quantum fields. The reported evidence for its existence came from the detection of its decay products of the type and rate "compatible with the Standard Model".

We thus see that the notion of a particle stemming from modern physics is rather far from the common-sense intuitive notion of a 'fundamental portion of matter'. What does it say about the fundamental level of the physical world?

If we take the Standard Model of Particle Physics at the face value we are pushed into the conclusion that what really exists are the quantum fields — ephemeral entities, which extend over the entire Universe. We have no direct access to them, even locally. In suitable experimental circumstances we can measure and identify the basic excitations of these fields, which we call the *elementary particles*. From these particles (that is, really, from quantum fields' excitations) atoms and molecules are made. This leads to a rather unsettling conclusion that all matter — trees, notebooks, stars and planets, this article and, indeed, ourselves — are eventually but quantum fields. On top of that, we face the notorious *measurement problem*[7]: We are soaringly missing an explanation on *why* and *how* we perceive a classical world, furnished with palpable objects, given that they do not exist at the fundamental level.

Despite the unquestionable success of quantum field theory, it seems unlikely that we have now reached the fundamental level of physics that would ever be accessible to humanity, although some of the most prominent physicists expressed such a viewpoint (Hawking 1988; Weinberg 1994). One can advance different arguments against such claims, for instance:

1. General Relativity is incompatible with quantum field theory (at least in the perturbative regime), so physics requires a unified 'quantum gravity' theory, which will surely change our views on the fundamental level of physics.
2. Any physical theory is based on mathematics and mathematics itself cannot be both complete and consistent — cf. (Hawking 2002)).

---

[7] See Chapter 11 in (Landsman 2017) for a nice interdisciplinary take on this problem.

3. Any physical experiment involves an ‘intervention’ of an observer, which affects the studied system. Furthermore, the source of this intervention (that is the ‘observer’) is not modelled within the theory, which the experiment was designed to test. Consequently, physics will never be complete because of the irremovable tension between the existence of universal laws of physics and the demand of their testability (Eckstein and Horodecki 2020).

In order to “save the phenomena” philosophers try to build sophisticated ‘interpretations’ of the quantum theory. It appears to me that these efforts (which seem completely hopeless if one attempts to ‘interpret’ in this way the full Standard Model of Particle Physics) miss the point. And the point is that modern physics is irreducibly tangled with abstract mathematics. The debate on the ontic status of mathematical structures persists in the philosophical discourse since at least 2500 years. Now we have strong reasons to claim that this philosophical problem expanded into physics — see (Heller 2021). The result is that such seemingly primitive and intuitive notions as a particle are in fact a highly mathematicised concept. The title question: “What is a particle?” cannot be answered on purely empirical grounds. The answer must take into account two aspects:

(1) What is a particle in a mathematicised theory T?
(2) Is theory T consistent with empirical data?

Note, however, that even if the answer to the second question is positive, it does not imply that ‘real’ particles are indeed particles in the sense of theory T, for there can be many inequivalent theories, $T_1, T_2, T_3, \ldots$, all consistent with the available theoretical data. For instance, the quantum field theory, and string theory, and loop quantum gravity, and many other, are all consistent with empirical data, while they entail radically different concepts of a particle.

This brings us to the question of how do we determine, which physical theory is actually adequate to model natural phenomena. A good account of how it works in practice was given by Thomas Kuhn in his famous book “The Structure of Scientific Revolutions” (Kuhn 2012): The development of Science is a cycle with two periods — stable and revolutionary. During the first one, there is a ruling paradigm of an accepted theory T_1. A revolution can be triggered if there is a critical amount of experimental data, which are not explained by T_1, but can be explained by a new theory T_2. In consequence of a scientific revolution is a paradigm shift from T_1 to T_2. From this viewpoint, we are now in a stable period with the paradigm of quantum field theory. String theory, loop quantum gravity, and other quantum gravity schemes, aim at starting a revolution, but so far without success.

It should be stressed, however, that the structure of scientific revolutions is not purely sociological, but reflects a deep property of the natural world (Heller 2009). It often happens that a new theory T_2 is proposed first, although there is no urgent need for it. Its rigorous mathematical structure married with operational concepts fosters predictions of some new, unforeseen, phenomenon, which is eventually observed in a dedicated experiment. A good example is Einstein’s General Relativity, which seemed to be an overkill to explain the tiny anomalous precession of Mercury’s perihelion. But, as it happened, GR turned out to have a huge predictive power. We had to wait an entire century to be able to register gravitational waves, predicted by Einstein in 1916. And we could not possibly get the idea that such a phenomenon might exist if we stayed within the Newtonian paradigm.

It is mathematics, which allows us to go beyond the “known unknowns” towards the “unknown unknowns”. In other words, a new theory allows us to formulate new operational questions, which could not have been formulated in the old theory. We often first need a new theory to propose a new experiment, which would put in question the old theory.

Does it mean that the old theory is falsified after a paradigm shift? Not at all! It simply means that we have reached the limits of its applicability. The new theory must reduce to the old one in a suitable formal limit (instance $\hbar \to 0$ or $c \to \infty$). But this does not mean that the new theory is better at explaining *all* experiments — the new ones and the old ones. Often, the new theory is quite useless for the description of phenomena well covered by the old theory. Indeed, it would not make much sense to use neither relativity nor quantum mechanics (let alone the Standard Model …) to explain the chaotic motion of a billiard ball — classical mechanics is much handier. In more philosophical terms, insisting on the reductionist picture leads us to the rather clumsy — and pretty useless — conclusion that the billiard ball is a very complicated bundle of quantum fields.

This suggests that scientific revolutions are in fact rather peaceful. The old theory is not guillotined from the physical world by the new one coming to power. It is simply put to a bastille, which limits its applicability, but does not prevent it from developing. Indeed, there was (and still is!) substantial development in classical mechanics, in particular in the domain of deterministic chaos, long after quantum mechanics and relativity became paradigmatic.

Consequently, the landscape of modern physics appears to be much more complex than the simplistic reductionistic picture — see Fig. 1. The latter implies an 'onion-like' structure of physical theories. A better analogy seems to be that of a manifold: Physical theories form an 'atlas' locally mapping different aspects of the physical world. For instance, quantum mechanics is the valid framework, roughly, when the considered length scales range from 10^{-13}m to 10^{-7}m. At much shorter distances, one crosses the energetic threshold for pair creation and hence quantum field theory is needed. At the other end, when objects become much larger in size than micrometers, then the quantum effects are negligible and classical theory provides a better description. The actual boarders between theories are usually misty and very interesting. In particular, the quantum-to-classical transition is being intensively studied, both theoretically and experimentally (Bassi et al. 2013; Carlesso et al. 2022). It is possible that it would bring a new theory, which does not belong neither to the classical nor to the quantum paradigm.

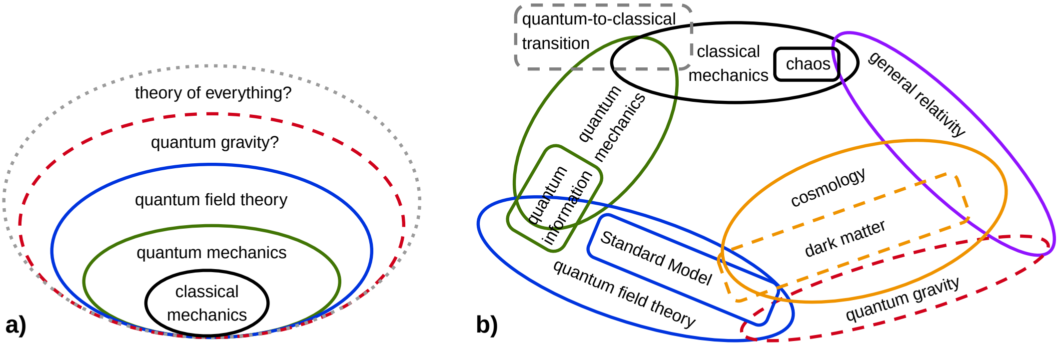

Figure 1. a) In the reductionistic vision physical theories form an 'onion-like' structure, in which every new theory generalises the old one, encompassing models of a larger class of natural phenomena. **b)** A better account of the actual status of physics is given by a 'manifold-like' structure, in which different physical theories agree (formally) at a certain overlap, but otherwise cover a different class of natural phenomena. The dashed lines signify domains, in which an empirically satisfactory theory is still missing.

Note that in the manifold picture the mythical 'quantum gravity' is reduced to a specialised theory encompassing a specific class of phenomena at the Planck scale. It might induce a revolution in cosmology (and, possibly, in particle physics), but it is not an all-embracing unified framework. Indeed, there will not be any use of quantum gravity, for instance, in deterministic chaos or atomic optics. On the other hand, these fields can and will develop independently of the status of quantum gravity. In the 'manifold' vision, there is no all-embracing 'theory of everything' (at least not in the usual sense of (Hawking 1988; Weinberg 1995)), but there are numerous uncharted regions marking the directions of possible future physical theories.

In this short note we have argued that the intuitive notion of a 'particle' is valid at macroscopic scales, but becomes more and more ephemeral when we ponder it at shorter scales and/or higher energies. We posit that this is actually the fate of *all* physical notions. There are no definite physical objects or phenomena, which can be named and described. They only exist within, and in the sense, of a specific theoretical framework, which has inevitable limits of applicability. The physical world must be contemplated from a holistic perspective, which takes into account a multi-layered irreducible structure of physical reality. And the latter is inevitably connected with mathematical structures modelling the different layers.

Acknowledgement

I am grateful to Michael Heller for numerous inspiring discussions and substantive comments on the manuscript.

Competing Interests None.

References

Aad, G., et al. 2012. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. *Physics Letters B* 716 (1): 1–29. ISSN: 0370-2693. https://doi.org/10.1016/j.physletb.2012.08.020.

Adlam, Emily, Niels Linnemann, and James Read. 2022. Constructive axiomatics in spacetime physics part III: a constructive axiomatic approach to quantum spacetime. *preprint arXiv:2208.07249.*

Aspect, Alain. 2015. Closing the door on Einstein and Bohr's quantum debate. *Physics* 8:123.

Bassi, Angelo, Kinjalk Lochan, Seema Satin, Tejinder P. Singh, and Hendrik Ulbricht. 2013. Models of wave-function collapse, underlying theories, and experimental tests. *Reviews of Modern Physics* 85 (2): 471–527. https://doi.org/10.1103/RevModPhys.85.471.

Brunner, Nicolas, Daniel Cavalcanti, Stefano Pironio, Valerio Scarani, and Stephanie Wehner. 2014. Bell nonlocality. *Reviews of Modern Physics* 86 (2): 419–478. https://doi.org/10.1103/RevModPhys.86.419.

Buchholz, Detlev, and Jakob Yngvason. 1994. There are no causality problems for Fermi's two-atom system. *Physical Review Letters* 73 (5): 613–616. https://doi.org/10.1103/PhysRevLett.73.613.

Carlesso, Matteo, Sandro Donadi, Luca Ferialdi, Mauro Paternostro, Hendrik Ulbricht, and Angelo Bassi. 2022. Present status and future challenges of non-interferometric tests of collapse models. *Nature Physics* 18 (3): 243–250. https://doi.org/10.1038/s41567-021-01489-5.

Chiribella, Giulio, and Robert W Spekkens, eds. 2016. *Quantum Theory: Informational Foundations and Foils.* Springer. https://doi.org/10.1007/978-94-017-7303-4.

Dressel, Justin, Mehul Malik, Filippo M. Miatto, Andrew N. Jordan, and Robert W. Boyd. 2014. *Colloquium*: Understanding quantum weak values: Basics and applications. *Reviews of Modern Physics* 86 (1): 307–316. https://doi.org/10.1103/RevModPhys.86.307. arXiv: 1305.7154.

Dürr, Detlef, Sheldon Goldstein, Travis Norsen, Ward Struyve, and Nino Zanghì. 2014. Can Bohmian mechanics be made relativistic? *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 470 (2162): 20130699. https://doi.org/10.1098/rspa.2013.0699.

Eckstein, Michał, and Michael Heller. 2022. Causality and time order–relativistic and probabilistic aspects. *preprint arXiv:2202.07302.*

Eckstein, Michał, and Paweł Horodecki. 2020. The experiment paradox in physics. *Foundations of Science,* 1–15. https://doi.org/10.1007/s10699-020-09711-y.

Eckstein, Michał, Paweł Horodecki, Tomasz Miller, and Ryszard Horodecki. 2020. Operational causality in spacetime. *Physical Review A* 101 (4): 042128. https://doi.org/10.1103/PhysRevA.101.042128.

Elhers, J, FAE Pirani, and A Schild. 1972. The geometry of free fall and light propagation. In *General Relativity, papers in honour of J. L. Synge,* edited by L O'Raifeartaigh. Republished in: *General Relativity and Gravitation* **44**, 1587 (2012). Clarendon Press, Oxford–London. https://doi.org/10.1007/s10714-012-1353-4.

Haag, Rudolf. 1996. *Local Quantum Physics: Fields, Particles, Algebras.* Theoretical and Mathematical Physics. Springer Berlin Heidelberg. ISBN: 9783540614517.

Hawking, Stephen. 1988. *A Brief History of Time.* Bantam Dell Publishing Group.

———. 2002. *Gödel and the end of physics.* Lecture, Dirac Centennial Celebration, July 20th 2002, University of Cambridge. http://www.damtp.cam.ac.uk/events/strings02/dirac/hawking/.

Hegerfeldt, Gerhard C. 1994. Causality problems for Fermi's two-atom system. *Physical Review Letters* 72 (5): 596.

Heller, Michael. 2009. *Filozofia nauki: wprowadzenie.* In Polish. Petrus.

———. 2021. How is the world mathematical? In *Philosophical approaches to the foundations of logic and mathematics,* 238–257. Brill. https://doi.org/10.1163/9789004445956_10.

Kuhn, Thomas S. 2012. *The Structure of Scientific Revolutions.* 50th Anniversary Edition. Chicago University Press.

Landsman, Klaas. 2017. *Foundations of Quantum Theory: From Classical Concepts to Operator Algebras.* Springer. https://doi.org/10.1007/978-3-319-51777-3.

Miller, Tomasz, Michał Eckstein, Paweł Horodecki, and Ryszard Horodecki. 2021. Generally covariant N-particle dynamics. *Journal of Geometry and Physics* 160:103990. ISSN: 0393-0440. https://doi.org/10.1016/j.geomphys.2020.103990.

Oxford English Dictionary. Accessed on 14.02.2024. https://www.oed.com/.

Paris, Matteo, and Jaroslav Rehacek, eds. 2004. *Quantum State Estimation.* Vol. 649. Lecture Notes in Physics. Springer Science & Business Media. https://doi.org/10.1007/b98673.

Peres, Asher, and Daniel R. Terno. 2004. Quantum information and relativity theory. *Reviews of Modern Physics* 76 (1): 93–123. https://doi.org/10.1103/RevModPhys.76.93.

Pironio, S, V Scarani, and T Vidick. 2016. Focus on device independent quantum information. *New Journal of Physics* 18, no. 10 (October): 100202. https://doi.org/10.1088/1367-2630/18/10/100202.

Reeh, H., and S. Schlieder. 1961. Bemerkungen zur Unitäräquivalenz von Lorentzinvarianten Feldern [in German]. *Il Nuovo Cimento* 22 (5): 1051–1068. https://doi.org/10.1007/BF02787889.

Sabin, Carlos, Marco del Rey, Juan José Garcia-Ripoll, and Juan León. 2011. Fermi problem with artificial atoms in circuit QED. *Physical Review Letters* 107 (15): 150402. https://doi.org/10.1103/PhysRevLett.107.150402.

Strocchi, Franco. 2008. *An Introduction to the Mathematical Structure of Quantum Mechanics.* World Scientific. https://doi.org/10.1142/5908.

Tarski, Alfred. 1994. *Introduction to Logic and to the Methodology of the Deductive Sciences.* Fourth. Vol. 24. P. 110. Oxford University Press.

Weinberg, Steven. 1994. *Dreams of a Final Theory.* Vintage.

———. 1995. *The Quantum Theory of Fields.* Cambridge University Press.

Wu, Shengjun. 2013. State tomography via weak measurements. *Scientific Reports* 3 (1): 1193. https://doi.org/10.1038/srep01193.