MINISTERSTWO NAUKI I SZKOLNICTWA WYŻSZEGO

eFinanse
finansowy kwartalnik internetowy

USING THE VALUE AT RISK METHOD IN ESTIMATION OF INVESTMENT RISK IN THE METALLURGICAL SECTOR COMPANIES

Ewa Miłoś[1]

Abstract

The aim of this paper is to analyze the merits of using the Value At Risk method in estimating the risk associated with investments in metallurgical sector companies. The paper presents how to construct the model, various methods of its estimation and their advantages and disadvantages. In the research part of the paper, we analyze typical features of the returns distribution characteristic for metallurgical companies listed on the Polish stock exchange, and on their basis we select the method of the Value at Risk estimation. The analysis was made by comparing individual metallurgical companies to the Warsaw Stock Exchange Index (WIG). We also evaluated the usefulness of the variance-covariance method by examining the number of exceedances of the designated value exposed at the assumed levels of significance.

JEL classification: D8, C13, C22
Key words: risk, investment, metallurgical sector

Received: 30.10.2010 Accepted: 05.03.2011

Introduction

In the conditions of limited access to information and great volatility of the market situation, more and more attention is being paid to the risk of investing in financial instruments. The area of investment risk estimation, neglected in practice, is now gaining significance. Markowitz (1959), in his portfolio theory, emphasizes the importance of income and risk in investments. The use of the expected rate of return and the variance to choose the optimal portfolio highlighted the significance of not only profitability, but also investment risk. Recently the method of estimating Value at Risk has been gaining popularity. Its usefulness in analyzing market, credit and operation risk has been recognized by international finance institutions, but it can also be used in estimating the risk of investing in companies listed on regulated markets.

The essence of Value at Risk method

The VaR (Value at Risk) method, due to its simplicity and common use, allows us to compare different investments in a comprehensible way. It enables us to answer the question of what risk

[1] Ewa Miłoś, MSc, PhD student at Cracow University of Economics, ul. Rakowica 27, 31-510 Cracow, ewamilos@interia.pl.

the investor is facing in a given project. It is known to be a negative method, which focus on the negative effects of risks associated with the possibility of incurring losses in relation to the occurrence of adverse market conditions. This fact may be seen as its asset, as investors are often primarily interested in potential losses and only then in opportunities for spectacular, extremely positive situations.

VaR is a method of measuring risk through the estimates of the highest expected loss over a specified time period and at an assumed confidence level . As each statistical method, the VaR method of risk estimation is not perfect. Nevertheless, it provides an answer to the question of what capital resources should be put aside to cover potential losses connected with the investment.

The method is one of three methods recommended by the Bank for International Settlements (BIS) to estimate the risk connected with banking activities.

Bałamut (2002) points out that VaR is used in three activity areas of companies, especially financial institutions. Firstly, the method is used to determine the market position limits and to create diversified investment portfolios. In comparison to other methods, VaR allows us to compare the risks taken by enterprises in various classes of assets. Secondly, this method helps us to evaluate the activities of companies and it is not based solely on the profitability of the investment or the project, but also on the level of risk which a given capital involvement brings. The simplest indicator which can be determined in order to compare particular investment projects is the relation of the return rate to VaR, which is economic capital vital to secure the investment. The third significant use of the VaR indicator is its use as a measurement of capital requirements reported to supervisory bodies.

The construction of the Value at Risk model

The value at risk is defined as the loss of the portfolio market value calculated for a given time t, assuming that the probability of reaching or exceeding this loss in an analyzed period (t,t+1) is within a specified level of tolerance, which can be written by means of the following formula:

$$P(W_{t+1} \leq W_t - VaR_t) = \alpha, \qquad (1)$$

Where W_t = market value of the portfolio at a given time t,
W_{t+1} = market value of the portfolio at the time t+1,
α = tolerance level,
VaR_t = Value at Risk at a given time t.

K. Jajuga and T. Jajuga classify VaR as belonging to the group of measurements based on distribution quantiles, which results directly from the definition of this measurement. From the statistical point of view, VaR is defined as the p-quantile of the cumulative distribution function of losses. The basic formula for VaR, assuming normal distribution of losses is presented below:

$$VaR = \mu + \sigma\Phi^{-1}(p) \qquad (2)$$

where $\Phi^{-1}(p)$ = quantile of standardized distribution AT determined probability p,
μ = average return rate,
σ = standard deviation of return rates.

Generally, assuming periodical capitalization and a resulting normal return rate, Value at Risk can be represented in the following way:

$$VaR = -R_{\alpha} * W_0 \qquad (3)$$

$$R_{\alpha} = \frac{W_{\alpha} - W_0}{W_0} \qquad (4)$$

where R_{α} = return rate corresponding to the assigned distribution quantile.

The VaR method has seen many alternative approaches and improvements which took into account, among others, fat tails of financial data distributions.

Methods of Value at Risk estimation

By definition, VaR informs us what maximum loss we should be prepared for, in a defined period of time, at the assumed level of probability. We should be aware that we will never assume a 100% probability, because regardless of the width of trust range such estimation would be pointless. Analyzing series of financial data we should take into account the high degree of probability, close to 100%, but we will never obtain an unequivocal answer, adequate for all cases.

Orzeł (2005) differentiates several classes of calculating VaR:

1) parametric: estimation is limited to determination of unknown parameters for the assumed classes of distribution,
2) non-parametric: distribution is not determined a priori,
3) simulation: scenario methods using Monte Carlo simulation or basing on historical data, the values are averaged,
4) analytic: simplified assumptions are adopted and VaR is calculated on the basis of closed formulas.

The time horizon for which we estimate VaR is important. When we are interested in a short period of time, the main measure of risk is the volatility of prices, whereas in the long run we are usually interested in the risk of a long-term downward trend. Calculation of VaR for a short period, by calculating p-quantile of the cumulative distribution of losses, reflects a typical for the

market changeability of rates. To estimate the long-term market risk we take into account the trends in which prices move, determining the drift coefficient. An exemplary method based on estimation of p-quantile of distribution is the historical method, consisting in calculating the value at risk as a p-quantile from empirical cumulative distribution. In this way we can avoid a possible systematic error of specification. The estimation relies on the simulation of influence of historically observed returns from the risk factor, such as changes in share prices, on the value of the current position.
The method of variance - covariance is based on an assumption that in many cases the loss may be precisely or approximately presented as a linear function of risk factors' changes (Jajuga, 2001). With this assumption, VaR for a particular stock can be calculated using the following formula:

$$VaR(p)_i = \Phi^{-1}(p)\sigma_i \,|\, W_i| \quad (5)$$

where $\Phi^{-1}(p)$ = quantile of normal distribution at specified probability p,
σ_i = standard deviation of return rates,
Wi = initial capital.

Assuming that conditional distribution is normal distribution, quantile values, corresponding to standard levels of tolerance amounting to 0.05 and 0.01 are as follows:

$$\Phi_\eta^{-1}(0{,}05) = -1{,}65$$
$$\Phi_\eta^{-1}(0{,}01) = -2{,}33$$

Undoubtedly, simplicity of the method and easy access to historical data are among the greatest advantages of the method, but the assumption that changes in analyzed characteristics, for example changes in prices of listed shares, have approximately normal distribution, results in considerably lower precision of the models.
The Monte Carlo simulation is more complicated than historical or variance-covariance approaches. To obtain the quantile of distribution in this method, we should define essential variables for the project and estimate the model which, on the basis of available and expected data, will best describe the mechanism of particular variables. The use of this method consists in drawing many, usually several thousands, price observations of financial instruments, from which we create the distribution of the return rates of a particular financial instrument. Through determining the quantile of this distribution, we can directly calculate VaR. The process parameters, expected value and standard deviation are all determined on the basis of historical data.
Another approach to estimating value at risk, serving the purpose of determining discrete distribution of the analyzed variable value, which belongs to simulation methods, is the scenario

analysis. It consists in defining doable ways of shaping the variable value by means of usually at least three scenarios: pessimistic, optimistic and most probable ones.
The approach to defining the quantile of free distribution is seen as generalization of the variance-covariance method, in which we assume that the analyzed variables have normal distribution. In this approach we use stable distributions, being the generalization of a normal distribution, but they are characterized by fatter tails than normal distributions. Choosing the best distribution we should take into account that drawing statistical conclusions for some types of distributions, for example the stable ones, may prove problematic.
The approach basing on Extreme Value Theory is only indirectly used to define Value at Risk. Instead of directly defining the quantile of distribution of return rates, this method determines the extreme value of the distribution. The foundation of this theory is a theorem saying that the maximum of the set of random variables (for example return rates) has a limiting distribution belonging to the class of so-called generalized distributions of extreme values, whose form is known. They incorporate Frechet, Weibull and Gumbel distributions. It should be noted that Gumbel distribution is typical for distributions with standard tails, such as normal, log-normal and gamma distributions, whereas Frechet distribution is typical characteristic for fat tail distributions, and Weibull distribution is used for distributions without tails, determined on a limited range.
In this approach the quantile of distribution of maximum loss is defined by the following formula:

$$y = \mu - \frac{\sigma}{\xi}\left(1 - (-ln(1-\alpha))^{-\xi}\right) \qquad (6)$$

where y = quantile,
μ,σ,ξ = parameters of distribution.

The parameters of this distribution can be estimated using the method of the highest probability (Jajuga, 2001).

Advantages and disadvantages of particular approaches

In order to choose the best method of estimating VaR one should take a close look at their pros and cons. Comparing them with the characteristic features of financial time series will enable us to preselect the approaches which should be taken into account when estimating the value at risk during the investment in companies from the metallurgical sector. Such information will allow us to choose an optimal method of estimating VaR in the metallurgical sector, taking into account specific features of investments in such a sector and the principles governing this area of industry. Below we presented the most characteristic features of particular methods of estimation which influence the usefulness of presented approaches to estimating value at risk of the analyzed investment.

Undoubtedly, the method of variance-covariance is a widely used approach. This can be attributed to the simplicity of estimating parameters in this model. It is a method with wide use and therefore comprehensible to interested people and comparable to other estimations. Its drawback is the assumption of normal distribution, which does not always work well with financial time series. But in accordance with the relation saying that the more data subjected to analysis, the more normal the distribution becomes, this method may still be widely used.

Historical simulation brings great results if we assume that some behaviors of share prices are repeated over time and such parameters as volatility of share prices, typical for a given sector, will be, in principle, repeated. However, there are problems with obtaining comparable data, and the results are susceptible to the sets of data used in calculations. There are also some difficulties with determining the optimal length of the period from which data are used to estimate appropriate distribution.

The Monte Carlo simulation allows high precision, but it is time-consuming and complicated, so it is used only when other methods cannot be applied. In this approach the results depend greatly on the assumed price model.

When determining the quantile of free distribution, there is a possibility of using other than normal distributions, which can eliminate the imperfections of normal distribution during the estimation of financial time series. This approach, however, is connected with the problem of assessing the parameters of distribution on the basis of past data. The approach based on estimating the values coming from the distribution tail uses only extreme values. In this case, using classical methods of estimation may only lower the quality of estimations, and there is also considerable difficulty in defining the variables from the model.

The research carried out recently by outstanding financiers, econometrists and statisticians, often focused on the possibility of using models of conditional heteroskedasticity in forecasting behavior of financial time series. Their usefulness has been proved in neutralizing the effect of focusing the changeability and autocorrelation of return rates. However, improper use of the AR-GARCH model may lead to errors in estimation and consequently to taking decisions which are not optimal from the investor's point of view.

Methodology of the model testing

The Warsaw Stock Exchange listed 18 companies from the metallurgical sector on 31.12.2009 in the fixed quotation system. For our analysis we picked the companies which met both criteria listed below:

1) they were quoted in the period of 01.01.2000 – 31.12.2009,
2) they were characterized by a high level of liquidity, days in which their shares were sold and bought constituted at least 90% of all quotation days in the period of 01.01.2000-31.12.2009.

If the company did not meet the first requirement, the second one was not analyzed. The following companies met both criteria: Boryszew, Ferrum, Hutmen, Impexmetal, Kęty, KGHM, Mennica, Stalprodukt.

While choosing the optimal method, we took into account the universality of using particular methods, their usefulness in estimating value at risk for instruments of high changeability, and the possibility of estimating extreme situations, which happen on the metallurgical market more often than in other industries.

Metallurgical industry is a specific sector. Features, which influence the choice of VaR estimation method, are the high volatility of metallurgical companies' return rates, a significant impact of changes in economic situation on the valuation of companies and a quick industry reaction to changes of the macroeconomic situation.

VaR focuses on the possibility of losses. In the case of metallurgicalcompanies it could be observed more positive than negative extreme returns. Therefore, the estimation method focusing on the left tail of the distribution will not lead to a better estimation. Due to the large variability in the historical rates of return, the results of methods based on historical simulation and quantile distribution do not constitute a good base for predicting future price movements. There are also difficulties with the selection of an appropriate pricing model for Monte Carlo method due to the high volatility and positive skewness. The normal distribution used in the variance-covariance method does not significantly affect the quality of the results. At the same time, due to its simplicity and comparability, this method allows better results interpretation.

Considering the pros and cons of the VaR estimation method and the specificity of the metallurgical sector, it is legitimate to use the method of variance-covariance to estimate value at risk. The calculations presented in table 1 also support the use of this method.

Table 1: Data concerning average changes of share prices, their standard deviations, skewness and kurtosis

Company/index	Average	Deviation	Skewness	Kurtosis	Variance
WIG	0.000	0.000	-0.2	2.4	0.0002038
Sector	0.001	0.022	0.1	3.3	0.0004996
KGHM	0.001	0.029	0.0	4.1	0.0008486
Boryszew	0.002	0.034	1.2	11.6	0.0011547
Hutmen	0.001	0.044	0.4	28.0	0.001899
Impex	0.001	0.033	0.9	7.1	0.0010639
Stalprodukt	0.002	0.029	1.1	7.0	0.0008651
Mennica	0.001	0.023	1.4	12.9	0.0005274
Kęty	0.001	0.021	0.0	3.7	0.0004274

Source: own work on the basis of Warsaw Stock Exchange data

The skewness for WIG index is typical for typical financial time series,:it shows that there are more positive than negative return rates, but the negative ones have higher absolute values. As we can see in the table, the skewness for the companies from metallurgical sector, contrary to WIG, is positive. This situation means that although the negative return rates are a more frequent phenomenon, the positive ones have higher absolute values. It means that the use of distributions concentrated in the left tail is not as legitimate as their use in the calculations concerning the WIG index. For the investor it means that he should be aware that for these companies days with negative return rates are quite frequent and if they invest in the shares of the companies from metallurgical sector, they may quite often expect return rates of even over 10%.

Kurtosis for metallurgical companies is higher than for the data concerning the WIG index. It exceeds the value of 3, which is typical of normal distribution. The distributions of return rates of metallurgical companies are more slender and focused around the average than the normal distribution calculated on the basis of the average and standard deviation for the WIG index. While analyzing the legitimacy of using models of conditional heteroskedasticity all AR(1)-GARCH(1,1) models, estimated with the formula for linear regression showed very low, amounting to a couple of percentage points, coefficient R^2, which is the measurement of matching between the model and the distribution. Simultaneously the statistics for Durbin-Watson test amounted to around 1.8 for KGHM, Boryszew and Impexmetal companies and oscillated around 2 for other metallurgical companies. These results imply that there is no point in using the AR(1)-GARCH(1,1) model in estimating value at risk for tested companies.

Using the Extreme Value Theory for particular metallurgical companies, Gumbel and Frechet distributions were estimated and compared with the real values observed in the distribution tails. The Gumbel distribution is characteristic for distributions with standard tails, such as normal, whereas the Frechet distribution is typical of distributions with fat tails. From the analysis of GEV distributions for the WIG index and for metallurgical companies based on data from years 2000-2009 we can state that for extremely negative return rates for the WIG index the use of Frechet distribution makes sense, however, empirical distributions of extremely negative return rates for analyzed metallurgical companies did not show this relationship. For such companies as Kęty, Mennica or Stalprodukt , it seems more appropriate to use normal distribution than fat tail distribution.

The right-side skewness of distributions of return rates of metallurgical companies is an argument for using the variance-covariance method with normal distribution for estimating value at risk for metallurgical companies. Also better matching of data to the Gumbel distribution than the Frechet one implies no need to use the models of conditional heteroskedasticity. High kurtosis observed for distributions of return rates of metallurgical companies proves that empirical distribution of historical return rates, based with average value of return rate and standard deviation for particular companies of the sector is more slender and focused around the average than the diagram of normal distribution. However, taking into account the fact that while estimating value at risk one should focus especially on extreme values with particular attention paid to the left tail of distribution, high kurtosis is not a contraindication for the use of normal distribution while calculating value at risk in investments in a company dealing with processing metals. Moreover, wide use of normal distribution in estimations, the acceptance of this method in estimating value at risk of bank assets (though they have fat tails) points at the possibility of using normal distribution in calculating VaR. Additionally, the investors' practices show that in spite of considerable development of research on characteristic features of financial time series and modifications and developments of financial models, investors prefer the simplest, commonly comprehensible models in everyday decision making.

Estimating and verifying Value at Risk model

As a result of using the variance-covariance method in estimating value at risk we obtained the annual return rates and value at risk for a period of one day, which we presented in table 2. The value at risk was calculated for the confidence levels of 99% and 95% on the basis of formula (2), and the annual rate results from the calculation using the complex capitalization for annual periods.

As we can see in table 2 the annual return rates for metallurgical companies range from -10.4% to 52.4%. The negative return rate for the Hutmen company may be explained by the fact that it incurred considerable losses resulting from speculative investment in futures contracts. The highest average annual return rate, reached by Stalprodukt may be attributed to its expansive policy of acquisitions. For the purpose of comparison, the one-day VaR at the trust level of 99% equals 3.3% for the WIG index, and the annual return rate in years 2000-2009 was on average 7.7%.

Table 2: Annual return rate and VaR for metallurgical companies listed on Warsaw Stock Exchange on the basis of data from 2000-2009

Company	Annual return rate	1-day VaR (p=0.99)	1-day VaR (p=0.95)
Stalprodukt	52.4%	6.8%	4.8%
Boryszew	29.0%	7.9%	5.6%
Mennica	22.1%	5.3%	3.8%
KGHM	14.8%	6.8%	4.8%
Impexmetal	11.0%	7.6%	5.4%
WIG	7.7%	3.3%	3.3%
Kęty	7.6%	4.8%	3.4%
Hutmen	-10.4%	10.1%	7.2%

Source: own work on the basis of Warsaw Stock Exchange data

Comments: VaR was calculated for the confidence level of 99% on the basis of formula (2), whereas the annual return rate is calculated with use of complex capitalization for annual periods

VaR should theoretically be the highest for the most profitable companies. High income compensates high risk the investors have to take investing in the companies which take decisions burdened with a high dose of uncertainty. This relationship was disturbed by different nominal share price of the company. The shares of Stalprodukt, characterized by quite low level of VaR in comparison to other metallurgical companies, in spite of the highest profitability of the shares, are sold at the price of a couple of hundred zlotys each, whereas the shares of Impexmetal, whose changeability is quite high in comparison to their profitability, sell for a couple of zlotys. High prices of shares usually lead to lower changeability of return rates from these shares and less frequent use of such shares for speculative purposes, which explains their lower level of value at risk.

Table 3: VaR for metallurgical companies listed on Warsaw Stock Exchange, number of VaR exceedances for normal distribution and maximum daily loss on the basis of data from years 2000-2009

Company	1-day VaR (p=99%)	Number of VaR exceedances	Maximum daily loss
Stalprodukt	6.8%	1.4%	10.3%
Boryszew	7.9%	1.3%	26.2%
Mennica	5.3%	1.0%	11.8%
KGHM	6.8%	1.6%	21.0%
Impexmetal	7.6%	1.1%	17.5%
WIG	3.3%	1.6%	8.1%
Kęty	4.8%	2.0%	11.8%
Hutmen	10.1%	1.0%	61.9%

Source: own work on the basis of data from Warsaw Stock Exchange

In order to check the correctness of a chosen estimation method, Value at Risk was calculated for metallurgical companies covered by research on the basis of variance-covariance approach. VaR was calculated for the confidence level of 99% on the basis of formula (2). The results are presented in table 3, which illustrates the verification of the model based on normal distribution for estimation of value at risk. As we can see from the calculations, assuming the 99% level of confidence the number of exceedances is higher than 1% in all analyzed cases. The exceedances are all within the 1%-2% range. There is a possibility of using empirical historical distributions for particular companies, this will, however, result in significant complication of the model and the repetition of changeability of return rates, especially in case of metallurgical companies, is a pretty controversial issue. At the same time we should remember that, by definition, VaR shows us that with the assumed probability the loss should not be lower than a determined percentage value which, depending on the capital invested, can be calculated into sums of money. We still do not receive an answer to the question of how big the loss may be if the VaR level is exceeded. Investors in metallurgical companies must be prepared for daily losses amounting to some tens of percentage points. Assuming lower confidence level, for the probability of 95% in each case the number of exceedances is within the level of tolerance amounting to 5%, which was presented in table 4.

Table 4: VaR for metallurgical companies listed on Warsaw Stock Exchange, number of VaR exceedances for normal distribution and maximum daily loss on the basis of data from years 2000-2009

Company	1-day VaR (p=95%)	Number of VaR exceedances	Maximum daily loss
Stalprodukt	6.8%	1.4%	10.3%
Boryszew	7.9%	1.3%	26.2%
Mennica	5.3%	1.0%	11.8%
KGHM	6.8%	1.6%	21.0%
Impexmetal	7.6%	1.1%	17.5%
WIG	3.3%	1.6%	8.1%
Kęty	4.8%	2.0%	11.8%
Hutmen	10.1%	1.0%	61.9%

Source: own work on the basis of data from Warsaw Stock Exchange

Conclusions

Value at Risk model, enabling us to determine the value which is at risk, has already found its way to the list of needs investors and financial institutions created. Its simplicity and effectiveness and usefulness allow us to better protect against and prepare for possible losses connected with investments. The discussion presented in this paper allowed us to justify the usefulness and effectiveness of Value at Risk model, using the variance-covariance model to estimate risk connected with investment in metallurgical sector. The analysis presented in the paper proved that some additional assumptions, typical of financial time series, may be disproved due to the specificity of distribution of return rates on investments in metallurgical companies.
The specificity of the metallurgical sector also implies high changeability of share prices of companies operating in this sector. Therefore it is essential to realize what risks are involved in the investments in companies from this sector. Due to specific distribution of return rates on investment in shares of metallurgical companies, some assumptions and additional calculations made for financial time series may be omitted without affecting the preciseness of the model. At the same time using the most common and easyto-estimation and interpretation variance-covariance method used to calculate VaR is nota particular difficulty for the investor.
Studies have shown that the characteristics of the metallurgical companies' return rates distribution limit the usefulness of other than variance-covariance methods of estimation. At the same time simplifications in the use of variance-covariance method do not result in a significant number of VaR exceedances at the confidence level of 99% in the empirical verification, and the test for a 95% confidence level shows that all results fall within the tolerance level of 5%.
These results demonstrate the validity and usefulness of the Value at Risk method, especially varice-covariance approach, in the investment risk estimation in the metallurgical industry.

References

Bałamut, T. (2002). Metody estymacji Value at Risk. *Materiały i Studia, No 147*.
Jajuga, K., Jajuga, T. (2007). *Inwestycje: instrumenty finansowe, ryzyko finansowe, inżynieria finansowa*. Warszawa: PWN.
Jajuga, K. (2001). Value at Risk. *Rynek Terminowy*, No 13.
Kuziak, K. (2003). *Koncepcja wartości zagrożonej VaR (Value at Risk).* Wrocław: StatSoft Polska.
Markowitz, H. (1959). *Portfolio Selection. Efficient Diversification of Investments*. New York: John Wiley & Sons. Inc.
Orzeł, J. (2005). Ilościowe metody pomiaru ryzyka operacyjnego. *Bank i Kredyt*, No 7.