

ARIMA-GARCH MODELS IN ESTIMATING MARKET RISK USING VALUE AT RISK FOR THE WIG20 INDEX

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Abstract

This paper determines whether the VaR estimation is influenced by conditional distribution of return rates (normal, t-student, GED) and attempts to choose the model which best estimates VaR on a selected example. We considered logarithmic return rates for the WIG-20 index from 1999-2011. Then, on their basis we estimate various types of ARIMA-GARCH (1,1) models. Applying relevant models we calculated VaR for the long and short position. The differences between the models were settled on the basis of the Kupiec test.

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Introduction

Recent years have brought huge popularity of publications related to risk assessment (especially those concerning the Value at Risk method). Specialist literature offers many practical applications of the analyzed method, as, for example, using VaR as an element of the transaction system (Degiannakis, Angelidis, 2006; Ślepaczuk, Zakrzewski, Sakowski, 2011) evaluation of the risk of investment in raw materials (Pera, 2008) or the most popular application, namely banking risk evaluation (Jackson, Maude, Perraudin, 1998). Together with increasing popularity of Value at Risk we can observe noticeable development of the methodology of its assessment. The reason why empirical considerations are made is the fact that finding the method which would most precisely forecast risk would allow us to make accurate investment decisions, facilitate comparison of investments, and make the construction of investment portfolios more effective. A. Wilhelmsson (2009) emphasizes that realistic modeling of financial time series is vital for evaluation of assets and for risk management. He also notes, following the thoughts of T. Bollerslev (1987), that financial time series have some characteristics related to distribution of return rates, due to which the estimation of econometric models with the assumption of normal distribution of return rates may be less effective than in case of the models assuming other distributions. The significance of the precision of risk assessment was also pointed out by S. Manganelli and R. Engle (2001), who claim that if risk is not properly assessed, the institution exposes itself to not optimal allocation of capital and lack of financial stability. On the other hand, R. Engle (1982; 1995) emphasizes in his works how essential from the methodological point of view it is to state which part of model structure influences the accuracy of assessments. To find out what relations there are between model specifications and what model type was the best at assessing risk on Polish Warsaw Stock Exchange, we compared various types of ARIMA-GARCH models which were used to estimate Value at Risk of the WIG20 index.

Risk is one of the most important terms in modern finance and is, apart from return rate, the second basic feature of investment (Fieszder, 2009, p. 219). Risk measure is one of the fastest developing fields of contemporary financial econometrics and empirical finance. Risk is the basis for taking investment decisions and for developing transaction strategies. Scientists offer many proposals of measuring risk, however the Value at Risk (VaR) method is the most popular one among practitioners and theoreticians. This measure is recommended by many domestic and international institutions of banking supervision and financial market supervision. Undoubtedly, one of the biggest advantages of this offer is the fact that it presents risk in a quantitative and comprehensible way (Pipień, 2006, p. 134). This implies the possibility of comparing many types of investments in a way which is easy to communicate to investors or decision-makers. The VaR value is easy to interpret not only for economists or mathematicians, but also for people without any expert qualifications.

The essence and origins of the VaR method

The Value at Risk method dates back to late 1980s and was the consequence of publishing the models of financial option valuation (Black-Sholes model, 1973; Cox–Ross–Rubinstein model, 1979), which gave birth to the modern era of measuring and managing risk, not necessarily financial one, but generally understood (Pera, 2008, p. 274). The growth of the VaR method popularity could be observed since October 1994, when the American bank - J. P. Morgan offered free of charge detailed methodology of its estimating and a database containing quotations of variation and correlation ratios for the most significant parameters of the global market, necessary to apply it (Bałamut, 2002, p. 8). This has brought about growing interest in methods of risk estimations using the VaR both among individual investors and banking supervision institutions, which gradually implemented this method through numerous recommendations. In January 1996, the Basel Committee changed the principles of determining capital reserves from 1988 and introduced new principles of determining capital adequacy fully based on Value at Risk (Pipień, 2006, p. 135). Another Basel report concerning ways of estimating and measuring risk was written in 2004. It introduced amendments to evaluation of credit and operating risk, while in case of market risk, VaR is still the core of capital reserves estimation procedures. Quoting the definition by K. Jajuga [2000a, b] Value at Risk is the loss of market value whose probability equals the given tolerance level. The author of the definition promotes the following formula describing VaR:

$$P(P_t \leq P_{t-1} - \text{VaR}) = \alpha, \quad (1)$$

where:

P_t – value of the analyzed financial instrument at t moment,

α – level of significance.

VaR at the α level of significance is such value of loss that the probability that it will be incurred or exceeded at the t moment equals α (Doman, M, Doman, R, 2009, p. 198).

The above formula concerns an investor who has a long position. In this paper we also estimated VaR for the short position, described by the analogical formula:

$$P(P_t \geq P_{t-1} - \text{VaR}) = \alpha \quad (2)$$

This paper uses the modification of the formula, proposed by M. Doman, R. Doman (2009), which treats VaR as the percentage loss of the instrument value:

$$P(r_t \leq -\text{VaR}) = \alpha \quad (3)$$

$$r_t = 100(\ln P_t / \ln P_{t-1}) \quad (4)$$

We can distinguish the following methods of estimating VaR (Jajuga et al, 2000):

1. RiskMetrics,
2. Historical simulation,
3. Variance-covariance approach,
4. Monte Carlo simulation,
5. GARCH models,
6. Scenario analysis,
7. Estimating the quantile of a distribution,
8. Approach based on extreme value theory,
9. Approach based on using values from the distribution tail.

There are also methods which evolved from listed above for example:

1. CAViAR models (Engle and Manganelli, 2000),
2. Fraction-on-time (Leśkow and Napolitano, 2005).

Due to the features of financial time series (the return rates distribution is characterized by leptokurtosis) and having considered the level of calculation difficulties in relation to precision of estimates, this paper will analyze various types of GARCH models.

VaR modeling with various types of GARCH models

In our considerations we will analyze various specifications of the ARIMA GARCH (1,1) process.

The ARIMA model (Box and Jenkins (1976) consists of three elements: p, d, q.

The 'p' value defines the range of autoregression delays (AR). The autoregression model describes the following formula:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \alpha_2 r_{t-2} + \dots + \alpha_p r_{t-p} + \varepsilon_t \quad (5)$$

where:

r_t – value of time series at t moment (in this paper r_t – daily logarithmic return rate on the WIG20 index at the t moment).

r_{t-1}, \dots, r_{t-p} – delayed values of time series.

The 'q' value determines the range of delays of moving average (MA). The model of moving average is described by the following formula:

$$r_t = \theta_0 + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q} \quad (6)$$

where:

ε_t – residuals from r_t process.

The MA part may be treated as a random element with developed structure, reflecting autocorrelation of the residuals of the r_t process.

The 'd' value refers to the degree of process integration. If the r_t process is not stationary (or, to be more precise, integrated in a non-zero degree), it is usually transformed into stationariness by differentiating (the model estimation is done on the basis of r_t increments). This means that instead of one explained variable r_t we analyze Δr_t or, if Δr_t is still non-stationary, we consider the higher range increments, $\Delta^d r_t$.

The ARIMA process (p,d,q) can be presented by the following formula:

$$r_t = a_0 + \sum_{i=1}^p \alpha_i r_{t-i} + \varepsilon_t - \sum_{i=1}^q \theta_i \varepsilon_{t-i} \quad (7)$$

In practice, we usually omit the notation of the range of process integration as most non-stationary processes are integrated in the first degree.

The GARCH model is a generalization of the model of autoregressive conditional heteroskedasticity created by Engle in 1982. Its author, T. Bollerslev (1986) proposes the following formula modeling conditional heteroskedasticity (GARCH (1,1)):

$$\sigma_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + b_1 \sigma_{t-1}^2 \quad (8)$$

This formula can function in any type of ARIMA model. In case of ARIMA(p,q) model, the GARCH (1,1) process describes the following structure:

$$r_t = a_0 + \sum_{i=1}^p \alpha_i r_{t-i} + \varepsilon_t - \sum_{i=1}^q \theta_i \varepsilon_{t-i} \quad (9)$$

$$\varepsilon_t = Z_t \sigma_t \quad (10)$$

$$Z_t \sim i.i.d.f(0,1) \quad (11)$$

It must, though, meet the following requirements:

$$a_0 > 0, a_1 \geq 0, b_1 \geq 0$$

A certain exception, often used in VaR estimates, is the RiskMetrics method, which assumes that:

$$a_0 = 0, a_1 = 0,06, b_1 = 0,94$$

This case, however, will not be the subject of our further considerations.

The following models have been analyzed:

1. AR(p) GARCH (1,1) $Z_t \sim N(0,1)$,
2. AR(p) GARCH (1,1) $Z_t \sim t(0,1,v)$,
3. AR(p) GARCH (1,1) $Z_t \sim GED(0,1,v)$,
4. ARMA(p,q) GARCH (1,1) $Z_t \sim N(0,1)$,
5. ARMA(p,q) GARCH (1,1) $Z_t \sim t(0,1,v)$,
6. ARMA(p,q) GARCH (1,1) $Z_t \sim GED(0,1,v)$,
7. ARIMA(p,1,q) GARCH (1,1) $Z_t \sim N(0,1)$,
8. ARIMA(p,1,q) GARCH (1,1) $Z_t \sim t(0,1,v)$,
9. ARIMA(p,1,q) GARCH (1,1) $Z_t \sim GED(0,1,v)$.

$$p = 1,2,3,4 \quad q = 1,2,3,4$$

We used all p and q combinations. It gives us a total of 108 various models, and 36 different specifications with 3 different conditional distributions. They were estimated in two stages (consistent estimators), first the ARIMA part, then GARCH, both with the Maximum Likelihood Method. On the basis of each model we estimated VaR on the significance level of $\alpha = 0.01$ (recommended by BASEL), both for the long and short position, on smoothed values. VaR at the t moment was estimated on the basis of the model value at the t moment (ex post). Then we calculated the number of breakdowns. Each model was subjected to the Kupiec test (Kupiec, 1995) and the following results were obtained:

Table 1: Results of estimating selected models of ARIMA (p,q) GARCH (1,1)¹

Model	Conditional distribution	Position	Number of breakdowns	Value of LR statistics*
AR(1) GARCH (1,1)	Normal	Long	47	8.3*
		Short	41	3.67*
AR(4) GARCH(1,1)	Normal	Long	47	8.3*
		Short	39	2.5
AR(1) GARCH(1,1)	t-student	Long	38	1.99
		Short	26	0.56
AR(4) GARCH(1,1)	t-student	Long	38	1.99
		Short	26	0.56
AR(1) GARCH(1,1)	GED	Long	38	2
		Short	26	0.56
AR(4) GARCH(1,1)	GED	Long	38	2
		Short	26	0.56
ARMA(1,1) GARCH (1,1)	Normal	Long	46	7.42*
		Short	44	5.78*
ARMA(4,4) GARCH(1,1)	Normal	Long	48	9.24*
		Short	39	2.5
ARMA(1,1) GARCH(1,1)	t-student	Long	37	1.54
		Short	25	0.89
ARMA(4,4) GARCH(1,1)	t-student	Long	37	1.54
		Short	27	0.31
ARMA(1,1) GARCH(1,1)	GED	Long	37	1.54
		Short	25	0.89
ARMA(4,4) GARCH(1,1)	GED	Long	37	1.54
		Short	28	0.14
ARIMA(1,1) GARCH (1,1)	Normal	Long	49	10.22*
		Short	41	3.67*
ARIMA(4,4) GARCH(1,1)	Normal	Long	50	11.2*
		Short	39	2.5
ARIMA(1,1) GARCH(1,1)	t-student	Long	37	1.54
		Short	25	0.89
ARIMA(4,4) GARCH(1,1)	t-student	Long	37	1.54
		Short	27	0.31
ARIMA(1,1) GARCH(1,1)	GED	Long	37	1.54
		Short	25	0.89
ARIMA(4,4) GARCH(1,1)	GED	Long	37	1.54
		Short	27	0.31

* The asterisk marks valuations of statistics in which the null hypothesis was rejected at the significance level of 0.05.

Source: Own elaboration

¹ Due to a large number of models, we presented only some of them as examples.

LR is the Kupiec test statistics described with the following formula (Doman, M., Doman, R., 209, p. 204):

$$LR = 2 \left(\ln \left(\left(\frac{N}{T} \right)^N \left(1 - \frac{N}{T} \right)^{T-N} \right) - \ln \left(\alpha^N (1 - \alpha)^{T-N} \right) \right) \quad (12)$$

where:

N – number of VaR exceptions,

T – number of trials,

α – significance level.

H0: $f = \alpha$

H1: $f \neq \alpha$

f – the ratio of returns beyond $-VaR_t^l(\alpha)$ (in case of a short position $VaR_t^s(\alpha)$) to all analyzed returns.

The LR statistics has the distribution χ^2 with one degree of freedom. The critical value (CV) of the Kupiec test for the most frequently adopted level of significance 0.05 equals 3.8415. The null hypothesis is rejected if $LR > CV$ (Piontek and Papla, 2004, p. 9).

The null hypothesis of the test is rejected both in case of underestimating of potential loss and in case of overestimating VaR.

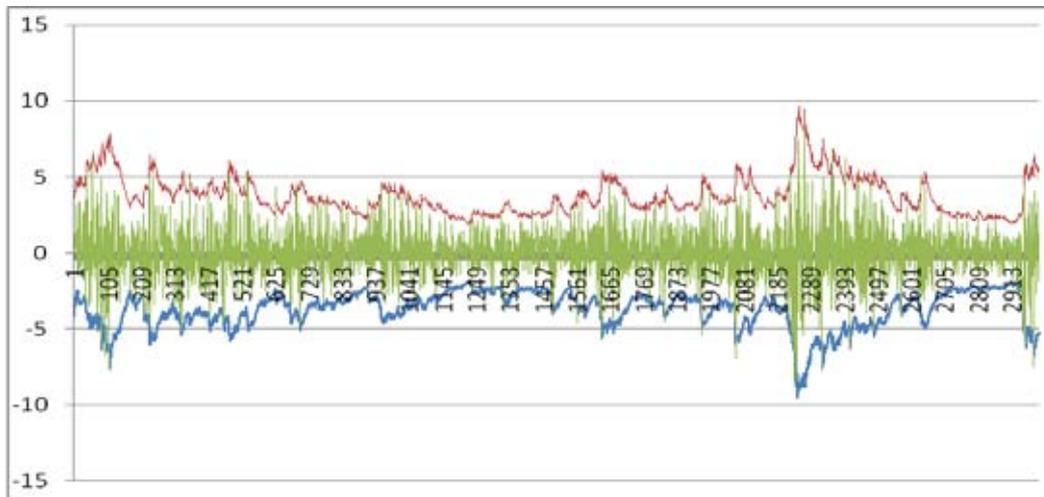
The estimation of parameters of 6 out of 108 models did not succeed, the problems resulted from the error made in calculating the hessian of the covariance matrix (models ARIMA(2,4) and ARIMA(3,3) with all analyzed conditional distributions). Therefore we compared the remaining 102 models (34 models for each type of conditional distribution).

After conducting the Kupiec test, the null hypothesis was accepted in 21 models for the short position in conditional normal distribution, while it was rejected for all models in case of the long position. For models with t-student conditional distribution, all null hypotheses were rejected, both for the long and the short positions, while in case of models with GED conditional distribution the Kupiec test null hypothesis was not rejected in any of 34 models, both for the long and the short position.

The differences resulting from the application of various conditional distributions can be presented on the example of the ARIMA (1,1) model:

1. Normal distribution:

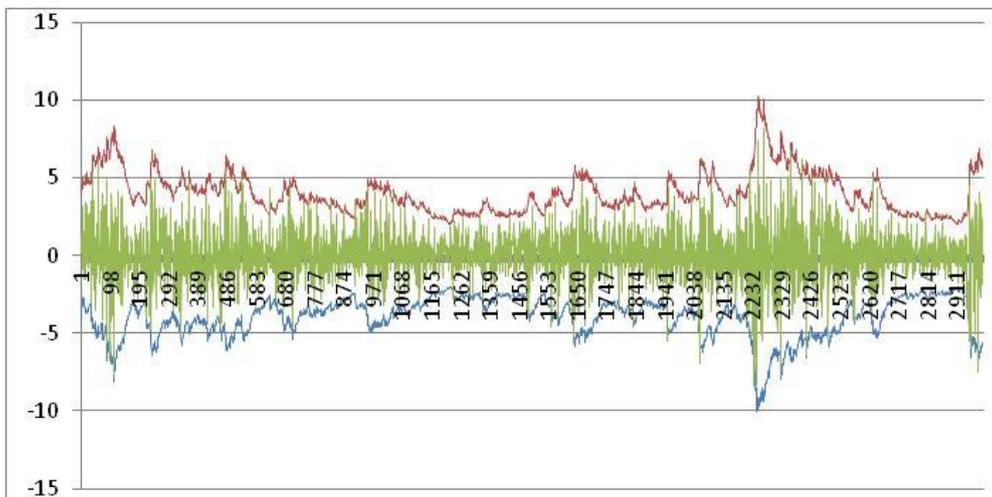
Figure 1: VaR for WIG20 1999-2011 Modeled ARIMA(1,1,1)GARCH(1,1) $Z_t \sim i.i.N(0,1)$



Source: Own elaboration

2. T-student distribution:

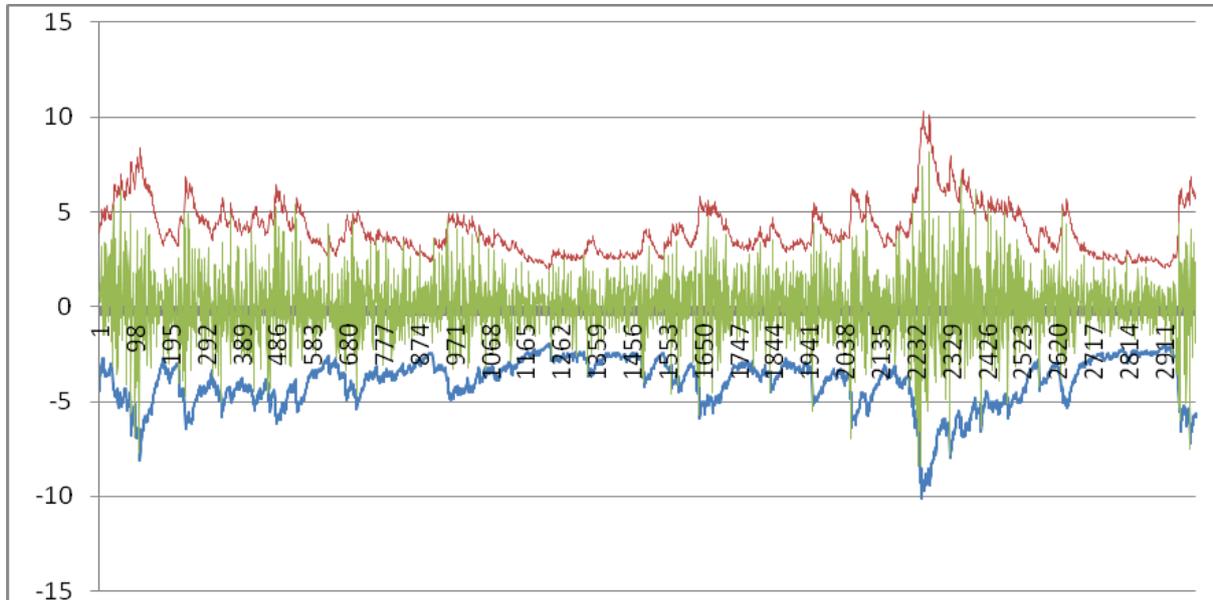
Figure 2: VaR for WIG20 1999-2011 Modeled ARIMA(1,1,1)GARCH(1,1) $Z_t \sim i.i.GED(0,1,\nu)$



Source: Own elaboration

3. GED distribution:

**Figure 3: VaR for WIG20 1999-2011 Modeled ARIMA(1,1,1)GARCH(1,1)
 $Z_t \sim i.i.GED(0,1,v)$**



Source: Own elaboration

Conclusions

We have considered 108 various types of ARIMA-GARCH(1,1) models. The parameters of six of them could not be estimated due to the error made while calculating the hessian of covariance matrix. For the remaining 102 models the VaR value was established for the long and short positions at the significance level of 0.01. After calculating the number of exceptions we conducted the Kupiec test, at the significance level of 0.05. The results have shown that there are significant differences between models with different conditional distributions. However, there are no discernible differences in case of ARIMA models with the same conditional distribution. In case of GED and t-student distributions, in none of the models, regardless of the position adopted, the null hypothesis was rejected, therefore we can claim that both distributions were very precise in modeling VaR in the WIG20 index in 1999-2011 on smoothed values. In case of models with normal distribution, the number of null hypothesis rejections depended on the position; models with this distribution were better at describing VaR for the investor adopting the short position (for 21 models the null hypothesis was accepted). On the other hand, for the long position, all models rejected the null hypothesis. The VaR results then were not only affected by the conditional distribution of the models but also by the type of the analyzed position. It means that when investor decide to measure risk using Value at Risk by GARCH models, is much more effective to calculate values based on the models with other than normal conditional distribution.

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