FUZZY NUMBERS IN VALUATION OF REAL INVESTMENT PROFITABILITY

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Abstract

In response to the weakness of traditional efficiency assessment methods taking risk into account, the modification of the Certainty Equivalent method is proposed in this paper. The possibility of connecting solutions from different fields provides for the elaboration of a more effective tool to illustrate and indicate the accurate level of risk in the investment efficiency calculus, which is the matter under consideration in the paper. The authors propose to use the modified method of a Certainty Equivalent that is based on fuzzy numbers. The aim of the method is to make decisions that are less incorrect. The work should be treated as an introduction to proposed further research on the subject.

INTRODUCTION

In making decisions concerning investment, most investors take their profitability into account. The methods used to estimate investment profitability can be arranged i.a. considering the time factor. This allows us to take into account the change of money value over time. In this case, the division of methods into static and discount (dynamic) methods is discussed. The Net Present Value (NPV) method and the Internal Rate of Return (IRR) method are the most popular. NPV and IRR popularity is the result of the advantages attributed to these methods i.a. the way of expressing net profits as net cash flows, when taking into consideration the entire investment period, variable money value over time and simple computing calculus. The ease of result interpretation, as well as the possibility of conducting analyses of investment risk, can be also considered (Wrzosek, 2008). Unfortunately, it should be emphasized that the NPV and IRR methods are not ideal. The issues connected with the discount rate, establishing cash flows or risk reflection are the weaknesses of those methods. The risk issue is essential in assessing tangible investment profitability and the problem, to some extent, is considered in the method of certainty equivalence, risk-adjusted Net Present Value (rNPV) as well as the RADRA method. This paper proposes another position in this area, using previous experience and the knowledge of fuzzy sets.

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The discount methods are accused of considering a total discount rate in the entire computing period. Such a situation is true in a stable economy, where inflation is always at the same level. Unfortunately, in the present economic reality, most economies cannot be called stable. It can be impossible for the constant discount rate to reflect the real money value change over time. Furthermore, it should be added that the discount rate, considered in that way, does not take risk influence on the rate into account. The variety of methods determining the discount rate is another problem (Rogowski, 2004).

A further problematic issue connected with real investment profitability valuation, is the matter and dissimilarity of Net Cash Flow (NCF) calculation.

These issues, both with reference to the setting of the discount rate and predicting NCF, put the problem of considering investment risk in question.

In the present work, fuzzy numbers are proposed to be used in valuation of investment profitability. The areas where fuzzy numbers could restrict the weaknesses of the applied methods so far are described. The way to take investment profitability valuation risk into account, via fuzzy numbers, is also shown.

THE DYNAMIC METHODS RESTRICTION OF REAL INVESTMENT PROFITABILITY VALUATION

THE DISCOUNT RATE

The discount rate is a measure of the applied interest rate which must be gained to pay the credit interest or to balance the interest of an alternative deposit that was closed to invest the funds and cover the premium of risk (Michalak, 2007, p. 88). It is assumed that the discount rate in dynamic methods, used to bring the cash flows to the present moment, means the cost of capital (Pluta, 2009). The cost of capital is defined as the invested capital return rate expected by investors, essential to investment realization. Because the invested capital can come from the equity - as well as financial capital, the financial capital and equity - should be priced during capital cost valuation (Duliniec, 2007). The optimum combination of different sources of capital, both equity and financial capital together with the costs of the individual elements, is the base to set the Weighted Average Cost of Capital (WACC) (Ostaszewski, 2001).

WACC is the amount of costs of the separate capital components weighted by sharing those elements in the total capital (Pawłowski, 2007; Walica, 2007). An especially problematic issue is to establish the equity, as well as the weight of participation of the given element in the total capital. In spite of the access to different techniques of equity cost valuation, the problem is to calculate it. In some cases, it is even impossible. Nevertheless, the establishment itself is not a problematic issue; the real problem is to accept the constant discount rate condition, which means that one discount rate is determined. In this case, weights of participation of the capital cost are constant in the period of analysis. Setting the weight in WACC, we can choose one among the techniques in setting a weight:

1) at the basis of historical structure (according to the book or market value).

The structure established in that way is perceived as the best one. Because of that, owners "want to save" it for the future. So, to make the structure constant, they choose the funding source very carefully.

2) at the basis of target structure.

It is recognized as the best capital structure. It is the established structure of proportions between "equity and debt" and a company will try to achieve this.

3) at the basis of marginal structure.

The weights are set not at the basis of the total capital structure that a company has at the present moment, but at the basis of the capital structure of a new investment, which is perceived as the ultimate (the last). In that way, the given capital and assets resources are enlarged.

The flaw that connects all those techniques is in accepting the condition that the capital structure does not change in time. This can be seen as a simplification.

CASH FLOW CALCULATION

Net Cash Flow (NCF) valuation, connected with the investment under consideration, can cause a crucial problem. Cash flows are predominantly predicted over the perspective of several years – over a dozen years usually. It is charged with uncertainty to the future business conditions. As the time horizon of prognosis rises, the level of uncertainty rises as well. So, it must be emphasized that the problem is not in the methods of establishing cash flows, but only in whether they come true in the future. Pęksyk

(Pęksyk, 2009) dealt with methods of selection problems during risk factor assessment, under the circumstances of an unstable environment.

The risk of real investment profitability valuation

Every investment is connected with risk. It is a huge challenge to consider at profitability valuation. With reference to risk, many aspects can be important, from the issue of cash flow valuation to the evaluation of the development of the market situation.

In valuation of cash flows with every technique the flows are described for every base year. This period for some investment can be even over a dozen years long. The risk in the determined flows is connected with the deviation from the expected, predicted level. The future is foreseen very rarely. The more distant (the longer period) it is, the smaller the prediction effectiveness is. "The deviation from the expected/ assumed level" is one of the generally used definitions of risk. But the deviation can be a better situation than the assumed one and can be a chance to gain extra benefits (Sierpińska, 2005).

The aim of minimizing the mistakes that can happen in cash flow prediction is to consider the investment risk which accompanies the project. It is possible by using methods which take this element into account during the calculating procedure.

Two ways of approaching a methods division that consider investment risk can be found. The first one divides methods into two groups (Wrzosek 2008; Zachorowska 2006):

1) methods of direct risk consideration,

2) methods of indirect risk consideration.

The second approach is connected with the criteria of using technique and divide methods into four groups:

1) methods of effectiveness correction,

2) methods of sensitivity analysis,

3) probabilistic and statistic methods,

4) methods of operational research.

Methods of effectiveness correction are based on the chosen parameter corrections of cost-effectiveness calculation. The second group – the sensitivity analysis – allows us to determine critical values and safety margins. Owing to these methods, we can analyze the influence of changes on efficiency measures. The group of probabilistic and statistic methods uses the calculus of probability and statistics. Strictly,

expected values are determined in the calculus. These methods are used when the probability distribution of particular variables is known. The last group of methods is based on simulation. Owing to these methods, it is possible to examine many variables as well as to consider the combination of uncertain factors.

The feature of methods included in the group of direct risk consideration is a risk mapping of the absolute criteria that is based on the method. It concerns an algorithm of the chosen method. The risk of investment efficiency valuation, owing to direct methods use, can be considered for three elements, which are made up in the algorithm of the method that values the efficiency.

In the case of the most popular method of investment efficiency valuation, which is the NPV method, the risk can be directly taken into consideration in the following way:

1) in the net cash flows – using the Certainty Equivalent (CE) method,

2) in the discount rate – by the discount rate with risk,

3) in the economic life cycle length of an investment – the method of limited Payback Period (Rogowski 2004).

The method of limited Payback Period "argues with" the crucial assumption of considering the entire investment life cycle, in spite of simple interpretation and many rationales inclining towards the use of this method. This is because the method of limited Payback Period lies in cutting the project economic life cycle down and checking to see if Net Present Value is still positive, supposing such conditions. The basis of such calculations is the assumption that if NPV for the shorter project life cycle is greater than zero, it is also positive for a longer project life cycle. The checked NPV value for the shorter payback period makes sense because the benefits in the nearer future are more certain than those which can be found in the distant future. Not only the exclusion of the total life cycle of the investment can be regarded as controversial, but the subjectivism of determining the limited payback period may also be suspect. Thus a perfect tool cannot be found to support the risk minimization.

Taking risk in the discount rate into account, it can be examined from two points of view. The first refers to the negation of a constant discount rate in the total period of the investment. In this approach, the limitation or minimization of the risk is considered using a variable discount rate, which is shown in Formula 1.

Formula 1: NPV with the variable discount rate

$$NPV = \sum_{t=0}^{n} \frac{NCF_t}{\prod\limits_{i=0}^{t}(1+r_i)} \text{ , (1)}$$

Source: Wilimowska, 2001

tage of such a net present value presentation is the fact that the company operates in a dynamic environment and there are a number of factors, variable in time, that influence the discount rate. This instability, as well as the lack of certainty that the factors will not significantly fluctuate, are the basis for agreement with the opinion that the discount rate should have a dynamic nature. However, the proposal of the variable discount rate for every year itself is still too little, because in one period the rate is still constant. The solution could be the use of techniques or tools which refer to continuous processes (e.g. differential calculus in discrete mathematics).

The second approach, adjusting risk in the discount rate as well, is the method of the Risk Adjusted Discount Rate Approach (RADRA). In this method, risk is adjusted by the rising discount rate value, because of an added premium for risk. Owing to that, risky decisions are made with a definite safety margin. The margin is the extra income determined by the value of premium for risk. It guarantees that the venture will be cost-effective, even if the expected income from the investment is not entirely gained. The initial way of determining the risk adjusted discount rate, which means in the RADRA method, is open to question because of the subjectivism in determining the premium for risk. Thus in the published literature, the proposed way to determine the value of a risk adjusted discount rate is the CAPM model. It shows the relation between income and risk in a balanced

financial market. However, using the model in practice is rather difficult because the knowledge of the project beta factor is needed. The beta factor determines the variety of a project return rate with relation to changes of investment payback in the market portfolio. Additionally, the weak point of the technique is the limited possibility of use in the case of companies that are not listed on the stock exchange (Sierpińska, 2005; Wilimowska, 2001; Schierenbeck, 2003). The difficulty of considering continuous processes, as well as the problem of determining the premium for risk, discourage common use of the risk-adjusted discount rate method.

The last method that adjusts risk directly is the CE method which comes from the expected utility hypothesis. This is connected with the decisionmaker's relation to risk. The CE is defined as the value gained with certainty that has the same utility as the expected value of the uncertain decision (Sierpińska, 2005, p. 391). Thus, the CE of a cash flow is the part of cash flow value gained with certainty, which is priced by the investor in the same way as the expected flow value (Spremann, 2006; Zhang, 2010). The flows called CE are discounted with a risk-free discount rate. The CE of net cash flows is set as the ratio of those flows and the rate in the subsequent calculating periods (Robichek & Myers, 1966; as well as in Polish literature: Sierpińska, 2005; Wrzosek, 2008). The rates are determined by the following formula:**Formula 2: CE rate**

$$e_t = \frac{\left(1 + r_f\right)^t}{\left(1 + r_f + r_{pr}\right)^t} \;, (2)$$

Source: Sierpińska, 2005, p.391

where:
e_t – CE rate in the following year t,
rf - risk-free return rate,
r_{pr} - premium for risk,
t=0,1,…n – the following year of calculating period.

The dependence between risk-adjusted net cash flows and the CE can be described by a formula:

Formula 3: CE rate II

$$e_t = \frac{NCF_{tPEW}}{NCF_{tNIEP}} \;, (3)$$

Source: Wrzosek,2008, p. 224

where:
e_t – CE rate,
NCF_{tPEW} - risk-free flows called CE,
NCF_{tNPEW} - uncertain cash flows.

The CE factor can reach values from the range and the value level of the factor decreases with the risk increase (Ostrowska, 2001). Thus, the corrected NPV value is lower than the expected one.
Using the CE, the corrected net present value is:

Formula 4: A corrected NPV value

$$NPVc = \sum_{t=0}^{n} e_t NCF_{tNIEP} a_t = \sum_{t=0}^{n} NCF_{tPEW} a_t \;, (4)$$

Source: Wrzosek, 2008, p. 224

where:
NPVc – a corrected NPV value,
 – CE factor in year t,
 – discount factor counted for risk-free discount rate,
NCF_{tPEW} - risk-free flows so-called CEs,
NCF_{tNPEW} - uncertain cash flows

The risk in the CE method is separately ascribed to each cash flow. This means, no assumptions, connected with dependence between risk level and money value change in time, are directly imposed. Such a dependence is characteristic for a risk-adjusted discount rate (it connects money value change over time with risk represented by a premium for risk). Such a separation gives the CE method a theoretical advantage over the RADR method (Rogowski 2004).

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Fuzzy numbers

The generally used uncertainty description with practical use of differential calculus causes a lot of problems and turns out to be insufficient. The use of differential calculus needs precision and huge knowledge of the studied phenomenon. Unfortunately, such a knowledge is rarely acquired by a decisionmaker. The need to devise an effective mathematic system to carry out operations on uncertain data, as well as familiarity with the method's limits of differential theory, have lead to the development of new scientific disciplines – including the theory of fuzzy sets. As Dolata (2010) states, fuzzy numbers arithmetic in many situations turns out to be a very effective way of uncertainty representation.

It is obviously an alternative in relation to the probability distribution of a random variable. The use of this arithmetic allows us to show the state of our knowledge of the considered phenomenon (Dolata, 2010, p. 14).

Fuzzy models are based on fuzzy sets. The basis of the use of fuzzy sets in risk representation is the need to describe phenomena and conceptions which are ambiguous and imprecise just as the conception of risk is. The model aims to determine the area of consideration called space. Next, using a membership function, a membership degree in a fuzzy set is attributed to each element (Minasowicz, 2009).

According to the definition (Kosiński, 2004; Sánchez, 2003), a fuzzy set A in space X is an ordered set of pairs:

$$A = \{(x; \mu_A(x)): x \in X\}, \quad (5)$$

where $\mu_A: X \to [0,1]$ is called a membership function of the fuzzy set A. The function imputes the membership degree of the fuzzy set A to each element $x \in X$, where "1" means the full membership degree to the fuzzy set A ($x \in A$), "0" means no membership degree to the set ($x \notin A$) and values between "0" and "1" mean a partial membership to the fuzzy set A. Whereas the fuzzy numbers are a special case of the fuzzy sets defined on $X=\mathbb{R}$ (the real number set). Making a definition – a fuzzy number is a fuzzy set A, defined on the real number set $A \subseteq \mathbb{R}$, where the membership function $\mu_A: \mathbb{R} \to [0,1]$, simultaneously fulfilling the conditions:

1) normality – the fuzzy set A is normal, so there is an argument for which the function reaches the value 1->$\sup_{x \in} \mu_A(x) = 1$,

2) convexity – the fuzzy set A is convex,
   ->$\mu_A[\lambda x_1 + (1 - \lambda)x_2] \geq \min\{\mu_A(x_1), {}_A(x_2)\}$,

3) continuity – a function μ_A is is continuous intervals.

The problem met during fuzzy numbers determination is the choice of the membership function. The standard solution (the one generally used) is a trapezium function, which includes the specific case – a triangular function (Kuchta, 2001, p. 16-18):

1) a trapezium fuzzy number:

When $a_1, a_2, a_3, a_4 \in \mathbb{R}$ such that

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{for } x \in [a_1, a_2] \\ \frac{a_4-x}{a_4-a_3} & \text{for } x \in [a_3, a_4] \\ 1 & \text{for } x \in [a_2, a_3] \\ \text{otherwise } 0 \end{cases}$$

2) *Figure 1:* **Presentation of the trapezoidal fuzzy numbers. A triangular fuzzy number is when $a_2=a_3$**
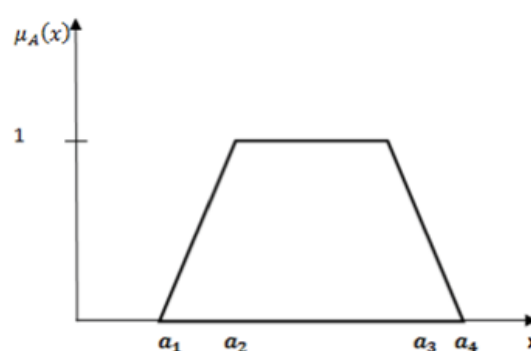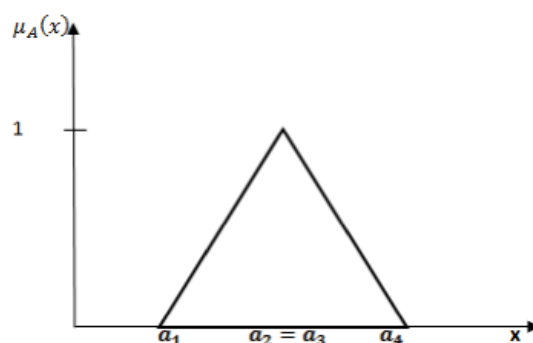
*Figure 2:* **Presentation of the triangular fuzzy numbers**

The extension of trapezium and triangular expression is determination of left-handed and right-handed fuzzy numbers (in fuzzy sets theory it is related to level L and γ of membership function).

3) A left-handed fuzzy number is determined by the following membership function:

$$\mu_A(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & \text{for } x \in [a_1, a_2] \\ 1 & \text{for } x \in [a_2, \infty] \end{cases}$$
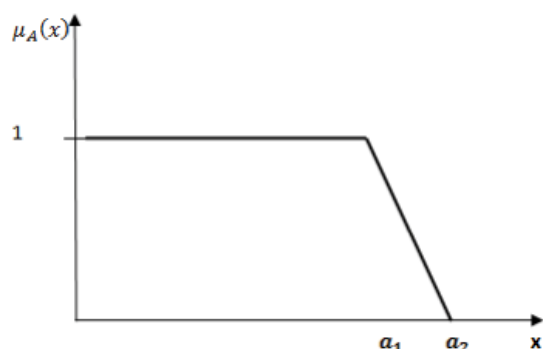
Figure 3: **Presentation of the left-handed fuzzy numbers**
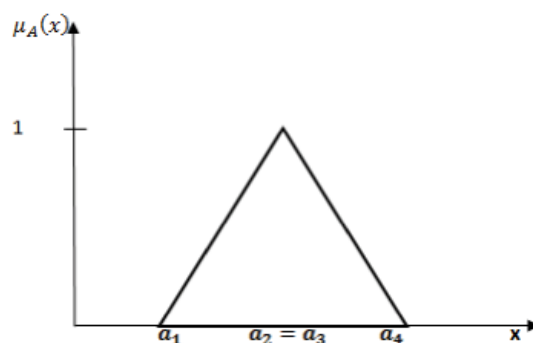
4) According to the presented record, when a right-handed fuzzy number is defined, the function is:

$$\mu_A(x) = \begin{cases} 1 & \text{for } x < a_1 \\ \dfrac{a_2 - x}{a_2 - a_1} & \text{for } x \in [a_1, a_2] \\ 0 & \text{for } x \in [a_2, \infty] \end{cases}$$

Figure 4: **Presentation of the right-handed fuzzy numbers**

## A MODIFIED CE METHOD THAT USES FUZZY NUMBERS

The knowledge connection of fuzzy numbers and proposed methods of investment valuation establishes an area of research. The effect could be a proposal of a method that rejects the weaknesses or the limitations in generally used methods. In the presented material, the fuzzy number-based modified CE method is used to calculate the investment valuation. The method, in the original form, includes future uncertainty by correcting the predicted cash flows with CE factors set a priori. The cash flow CE was a part of cash flow value, received with "certainty", that an investor priced the same way as the expected flow value. The proposed solution should be found valid. The crucial value of the method is the assumption that the risk is adjusted separately for each cash flow. There is no permanency, contrary to discount rates in NPV method. Additionally, no assumptions, connected with the dependence of risk level and money value change in time, are directly imposed. Such a dependence is characteristic for a risk-adjusted discount rate – it connects money value change in time with risk represented by premium for risk. The problem of the use of the CE method is surely the set of CE factors in the following calculating periods, which is et (CE factor in the following year t), that can have values from the range $0 \leq et \leq 1$. It is proposed to make the CE factor equivalent to a fuzzy number function in time:

$$e_{t} = \mu_{A}(t)$$

The fuzzy number will be a modified right-handed fuzzy number with double break-points in a1 and a2 period, which is shown in the following figure:

Figure 4: **Modified right-handed fuzzy number with double break in a1 and a2 periods**
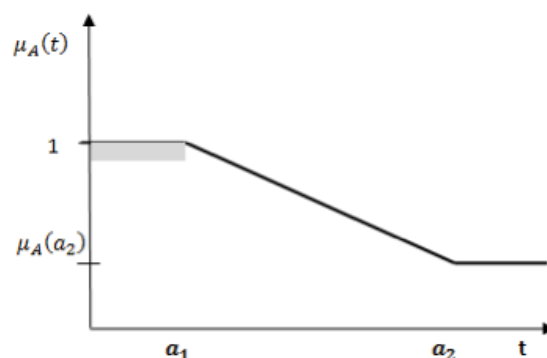
Predicted cash flows in the nearest future are "more certain" than those ones predicted in further periods. It can be assumed that after a defined period, the level of uncertainty stabilizes on a defined level, and certainly it is not zero. In the use of this method, the following must be determined:

1) the length of the "certain" forecast, not corrected with the CE factor or the length of the period for which

the factor is assumed on the high and constant level, it was graphically determined on the level 1. A grey area can also be introduced and it can be assumed that the factor is in range [1;0,95] and its assumption is dependent on the level of risk valuation,

2) the period of CE factor decrease – a period between a1 and a2,

3) the second break-point, which is a2 period after which we assume the constant CE factor value, on the minimum level for the investment under analysis.

The reductive assumption in the proposed solution would be an assumption of a linear CE factor decrease in the period between a1 and a2, and the decrease would assign the previously assumed indicated assumptions.

The proposed solution of fuzzy number use, in valuation of investments, needs to have an elaborated methodology of key parameters established,

which would be dependent on the risk level of the investment. The higher the risk level of the investment, the lower the CE factors. This means that the starting and constant, in the first period, CE factor should have the lowest value of the accepted ones; the first break-point – which is the period a1 – should be the fastest one and the decrease in the depth of the CE factor–which is $\mu_A (a_2)$ – should be the biggest one. The proposed solution of investment valuation, based on fuzzy numbers, would need to determine the characteristics conditioning the level of the proposed venture.

In Table No. 1, an example of a hypothetic investment, with break-points in third and seventh years, is shown. There are presented the characteristics, essential to be used in the solution that is based on the idea of using fuzzy numbers in the valuation of an investment.

Table 1: the example illustrates the hypothetical investment venture with the break points in the third and seventh years

Characteristics	$\mu_A ([0;a_1])$	a₁	a₂	$\mu_A (a_2)$
Value	0,95	3	7	0,40

Source: Own work

The values of the particular characteristic, given in Table 1, are random and their accurate determination in the aspect of valuation of the risk of an investment is key. To avoid subjectivism in selection of the values which are needed to set the characteristics the proposed methodology needs to be elaborated.

SUMMARY

The lack of certainty that accompanies investment decisions urges us to search for new or additional tools, techniques, or methods that allow us to make

choices with a lower level of error. Of course, no solution connected with future valuation will bet able to eliminate unsuccessful decisions. Nevertheless, the proposed solution, in the range of valuation investment efficiency, enriches significantly the previously proposed methods. It allows us, at the same time, to consider the risk level by subjective selection of values with the required characteristics. The authors are going to do further research to elaborate the directives that simplify the selection of the indicated characteristics, referring to the individual investment under analysis.

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