Financial Internet Quarterly „e-Finanse" 2017, vol.13/ nr 4, s. 97-109
DOI: 10.1515/fiqf-2016-0039

THRESHOLD THEORY – MODELLING RISK ATTITUDE

Tomasz Kasprowicz[1], Andrzej Bednorz[2]

Abstract

In this paper we offer an alternative framework for examining why risk matters in the decisions of economic agents, and how the agent's risk attitude affects his decisions. This "Threshold Theory" framework is based on a real options approach and the observation that in many situations an agent faces one or more thresholds in the payoff function. These thresholds influence the agent's risk attitude. The theory's predictions help to explain many anomalies that the standard expected utility model cannot. Threshold Theory can also model behavior in contexts such as individual investor decisions, corporate governance and other agency problems. Further, we examine CEO decisions as a function of time to the CEO's retirement to test predictions of the Theory.

JEL classification: G31, M15, D01
Keywords: Decision-making under uncertainty, real options, agency theory, CEO behavior

Received: 14.07.2017 Accepted: 15.12.2017

1 The University of Dąbrowa Górnicza, e-mail: tkasprow@gmail.com.
2 e-mail:abednorz@gmail.com.

Introduction

One of the basic points of interest to economists is the behavior of people under uncertainty. Analysis of the problem dates back to Daniel Bernoulli (1738), but was developed in the state it is most widely known today by Von Neumann and Morgenstern (1947). The standard theory does not account for some anomalies (Kahneman & Tversky, 1979) and gambling, which are persistent and widespread. These anomalies were confirmed by laboratory experiments (Kahneman & Tversky, 1979; Grether & Plott, 1979) which laid the foundation for behavioral economics. Both of these approaches are based on assuming a certain attitude that humans exhibit toward risk. In particular the most common assumption under the standard model is the one of risk aversion that is even derived theoretically under certain assumptions (The St. Petersburg paradox (Bernoulli, 1738)). The behavioral approach on the other hand usually assumes loss aversion basing the assumption on observations.

The topic of actually why people should be risk/loss averse/neutral/seeking is however usually omitted under the silent assumption that it is derived from certain neurological or psychological phenomena that do not belong to economics. We have a different point of view and decided to develop the Threshold Theory that is designed to predict human attitude toward risk, or in another words, to predict whether people under given conditions should exhibit risk aversion or risk seeking. As the theory is based on a real options approach, we assume that the market for traded assets satisfies a no-arbitrage condition and that people are not satiated. The development of a theory seems difficult under such lax assumptions. However, we manage to derive an agent's attitude towards risk based on construction of the agent's portfolio. We define the agent's portfolio as the set of assets and contingent securities that may affect the agent's attitude toward risk given the decision he is facing. The assets in one's portfolio may be in the form of shares, currency, real estate, a job, etc. The payoff of contingent claims depends on the value of some underlying asset. It usually takes a monetary form; although in some cases may take another form (pleasure, satisfaction, etc.). Of special interest are cases when payoffs in utility are of a discontinuous nature as it allows us to observe and predict changes in human attitude towards risk. These cases are very frequent since assumption of infinite divisibility of financial assets may nearly hold in real life but is not realistic in terms of consumption.

The contribution of the theory developed in this paper is twofold. Firstly, on a theoretical level, it provides a framework in which the assumption of risk aversion is replaced with a more versatile and general modeling scheme. Secondly, Threshold Theory can be useful in all cases involving modeling the decisions of an individual or a group of homogenous individuals who cannot diversify the risk they are facing, including cases of entrepreneurs and financing of small businesses. In addition, the modeling scheme provided may be helpful in research in branches of corporate finance, such as agency theory and signaling theory.

Real options

It is widely agreed that the real options approach to economic phenomena gives excellent insights. Probably the oldest and the best known successful generalized option modeling is Black and Scholes' (1973) representation of equity as a call option on a company. Unfortunately, applications of the real options approach seem to be limited relative to its apparent potential. The main reason for this is the fact that volatility and other estimates for many underlying assets are not available ex ante due to infrequent trading or the absence of a market.

Threshold Theory avoids this common problem of the real options application because no valuation is made within the model as the marginal analysis is the main tool used. Because of this, the model may be applied to a wide range of assets including those that are not actively traded. The theory introduces a generalized real options approach to other fields of finance, in particular corporate governance, signaling and agency theories, and the behavior of undiversified investors. It seems that it is able to give a theoretical explanation for many phenomena observed in the business world, hence providing the discipline with a flexible modeling tool designed to provide predictions and not merely fit the data.

The model uses any tools available to describe an agent's portfolio. Mostly these are standard assets like stock, bonds, calls or puts. However, of special importance are two exotic options with non-continuous payoffs.

A Cash-or-Nothing (CoN) option is a binary option that pays a constant amount A at maturity only if the price of the underlying asset exceeds the exercise price. This option's price is just the discounted expected payoff. The closed formula for the value of such an option in Black and Scholes' (BS) world at time t=0 is as follows (Ingersoll, 2000):

$$CoN = Ae^{-rt}N(d2) \quad (1)$$

$$d1 = \frac{\ln\left(\frac{S}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (2)$$

$$d2 = d1 - \sigma\sqrt{T} \quad (3)$$

Where A is the amount paid in case the option finishes 'in- the-money',

N(x) is the value of standard normal cumulative distribution

N(d2) is the probability of an option finishing in-the-money (in risk neutral terms).

S is current value of the underlying asset

X is the strike price

σ is the underlying asset's volatility

T is the time to maturity

r is the risk-free rate

This is exactly the same interpretation as in the case of a vanilla call.

An Asset-or-Nothing (AoN) option is an option that pays the value of an asset if the price of an asset exceeds the strike price at maturity and nothing otherwise. The closed formula in the BS world for the value of such call option at time t=0 is as follows (Ingersoll, 2000):

$$AoN = SN(d1) \quad (4)$$

The relationship between various parameters and the value of an option can be found by taking derivatives of the value function with respect to the parameter of interest. It is known that for vanilla options, the value of an option rises as volatility increases. This is not necessarily true for a CoN option though, as the relationship changes with S. The reason for such dependence is that if S>X then there is no reward for finishing more "in-the-money", however there is a loss of A if the value of the underlying asset falls below X. Therefore, in such a case additional volatility brings no value from up-side potential and increases the risk of losses.

This observation is crucial for Threshold Theory. Figures 1 and 2 illustrate the impact of the change of volatility on the value of a vanilla call and CoN with respect to S for parameters: X=11, T=1, r=2%, A=10.

The model

Normally, the parameters used in option pricing are exogenously determined. Consider the alternative situation in which we own an option, but we have influence on some parameters that determine the value of the option. The influence is not absolute but is limited and can be exercised only at some points in time. In fact, these points when a decision is made are the points of interest for Threshold Theory.

Threshold Theory is interested in decisions

Figure 1: The impact of volatility on the value of a standard call option

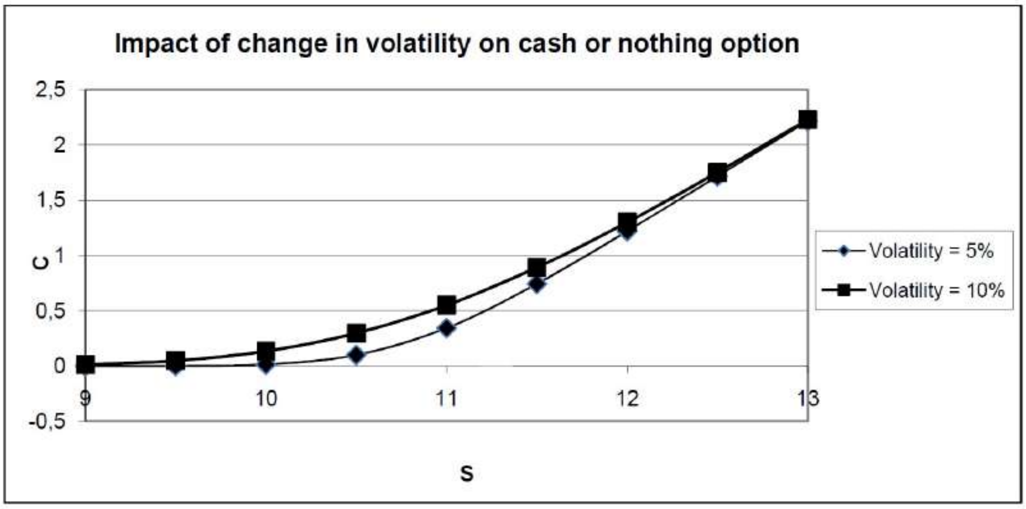
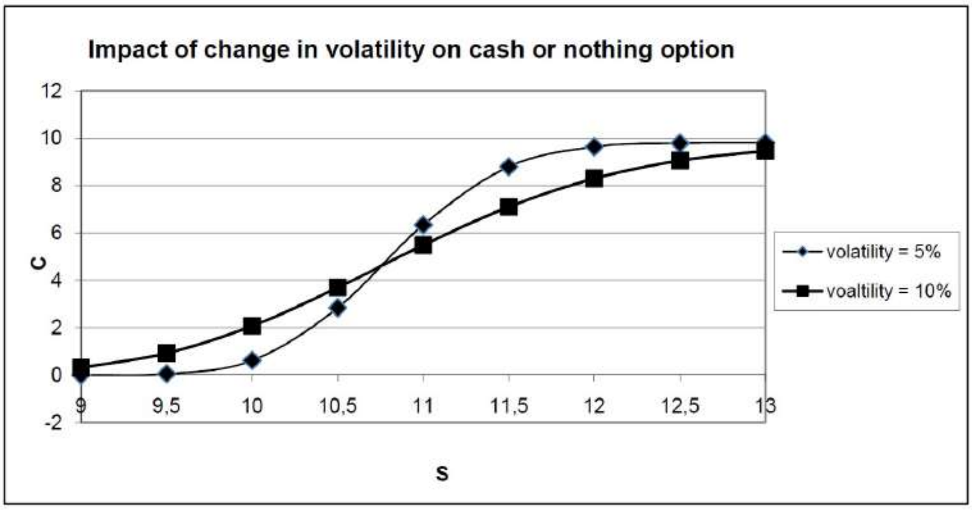


Figure 2: The impact of a change in volatility on the value of cash or nothing option



concerning two parameters of the underlying asset: the value of the underlying asset and its volatility. If we use our power to change the value of the portfolio we call it a pure wealth transfer, while a change of volatility we call pure risk change. Threshold Theory is mostly interested in predicting pure risk changes as these determine an agent's attitude towards risk.

Do examples exist in which power over these variables is feasible? The answer to this question is "yes" and we present two examples in a subsequent section.

Some examples

The case of financial options - we will start with the case of financial options. This familiar ground will allow us to proceed towards examples that are more exotic later.

Consider an agent holding in his portfolio two securities: a Cash-or-Nothing option written on IBM stock (paying $10 if IBM's stock price at maturity exceeds $11) and a share of stock in IBM. Note that the underlying asset of the option held is also a part of the portfolio. Figure 3 presents the payoff at maturity to the agent with respect to the value of the underlying asset at maturity.

Figure 3: Payoff at maturity from the portfolio consisting of the Cash-or-Nothing option and its underlying asset (C+S) versus value of option's underlying asset (S).

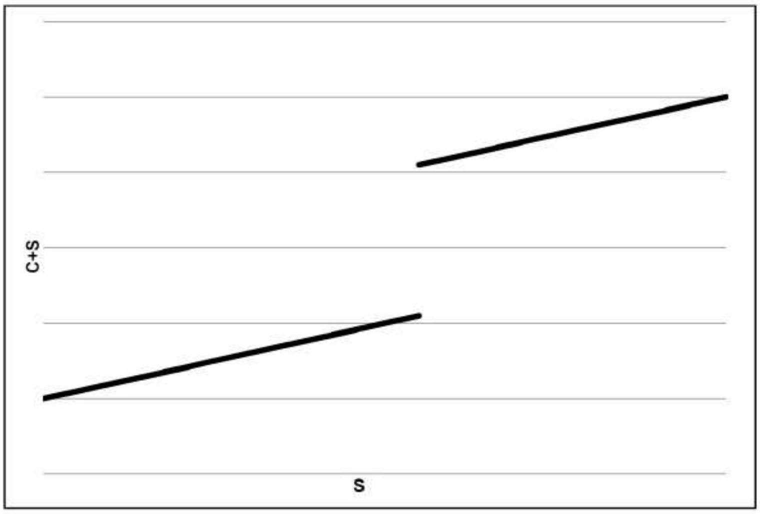
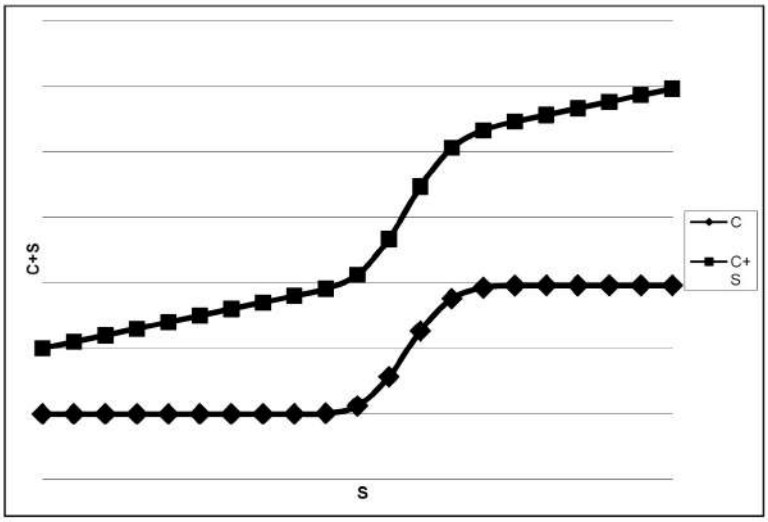


Figure 4: The value of a portfolio consisting of a cash-or-nothing option and its underlying asset (C+S) versus value of the underlying asset (S)



The Figure 4 presents the value of the portfolio before maturity (assumed parameters: volatility 5%, risk free rate 2%, time to maturity 1 year).

Now let us investigate what would be the reaction of the agent to changes in volatility and in the value of the underlying asset. This can be done by finding the derivatives of the value of the portfolio in relation to these two variables. The value of the portfolio is:

$$W = S + CoN = S + Ae^{-rT}N(d2) \quad (5)$$

Let us start by differentiating with respect to S

$$\frac{d}{dS}W = 1 + \frac{A}{S\sigma\sqrt{T2\pi}}e^{-rT}e^{-\left(\left(\frac{\ln(S)}{\sigma\sqrt{T}} + \frac{-\ln(X)+\left(r-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)^2/2\right)} \quad (6)$$

The derivative is always positive. This means that the agent will always benefit from an increase in the price of an underlying asset.

Now let us consider the influence of changes in volatility. Let us assume that a change in volatility has no impact on the value of an underlying asset. Then we can consider the derivative in terms of volatility:

$$\frac{d}{d\sigma}W = -\frac{A}{\sqrt{2\pi}}e^{-rT}\left(\frac{\ln\left(\frac{S}{X}\right)+rT}{\sigma^2\sqrt{T}} + \frac{\sqrt{T}}{2}\right)e^{-\left(\frac{\ln\left(\frac{S}{X}\right)+rT}{\sigma\sqrt{T}} + \frac{\sqrt{T}}{2}\right)/2} \quad (7)$$

The middle part of the equation $\left(\frac{\ln\left(\frac{S}{X}\right)+rT}{\sigma^2\sqrt{T}} + \frac{\sqrt{T}}{2}\right)$ determines the sign of the derivative.

We are interested in finding the values of the underlying asset for which the derivative is positive or negative; solving $\frac{dW}{d\sigma} = 0$ will tell us where the sign changes.

Therefore, whenever the value of an underlying asset is below $Xe^{-rT}e^{-\frac{\sigma^2 T}{2}}$ the agent benefits from an increase in volatility, and is harmed otherwise.

Generalized real options cases.

The marginal analysis performed above is interesting but seems to have limited importance. So far, we have been analyzing an example (presented at the beginning of this chapter) in which the agent is the recipient of market conditions. Therefore, the analysis reflects only the impact of some exogenous changes on the value of his portfolio. However, in the case of an agent influencing either the volatility or the value of the underlying asset, then such an analysis would be able to predict how the influence would be used. In other words, we could predict whether the influence would be used to decrease or increase value/volatility of the underlying asset.

In the example given, such an influence would be clearly used to increase the value of underlying asset in all cases (positive derivative with respect to S); increase volatility in all cases when the value of the underlying asset is below $Xe^{-rT}e^{-\frac{\sigma^2 T}{2}}$ and decrease volatility in cases where the value of the underlying asset is above $Xe^{-rT}e^{-\frac{\sigma^2 T}{2}}$. Cases in which the investor has an influence on IBM's stock returns (held by the agent in the example here presented) are exceedingly rare so the predictions seem to be irrelevant. Unless of course somebody wants to model the behavior of the CEO of IBM who both is in possession of a portfolio that is similar to the one presented and has direct influence over IBM's value and riskiness. This gives us an underlying idea for an example that leads directly to a testable hypothesis about human behavior.

Consider an agent who is the CEO of company XYZ. Her salary depends on the stock price of the company on January 1st each year. If the stock price is lower than X=$10 then she is fired and suffers loss of reputation. This means that the present value of her wages in her next job will be lower by $A compared to current conditions. Otherwise she is paid $1 for each dollar that the stock price S exceeds $10.

Her portfolio consists of two assets:

A cash or nothing option that pays $A in the event of the value of the underlying asset (stock of XYZ) exceeding X=$10

Call option on stock XYZ with a strike X=$10

Figure 5 presents the agent's payoff at maturity and the value of the portfolio before maturity.

The influence of the agent on the value of the underlying asset is pretty obvious as exercising this influence is in fact her job. By accepting various projects, she influences both the value and the volatility of the underlying asset.

Now, using again marginal analysis we will try to predict the behavior of the agent. We assume that A=$10 so these two options blend into an Asset-or-Nothing option.

The derivative of the value of the portfolio function (W) with respect to the value of the underlying asset is always positive. Therefore, the CEO will accept all positive NPV projects providing they do not alter the volatility of the company.

The derivative of the value function with respect to volatility gives us the known formula

$$S = Xe^{-rT}e^{-\frac{\sigma^2 T}{2}} \quad (8)$$

that determines the sign of the derivative.

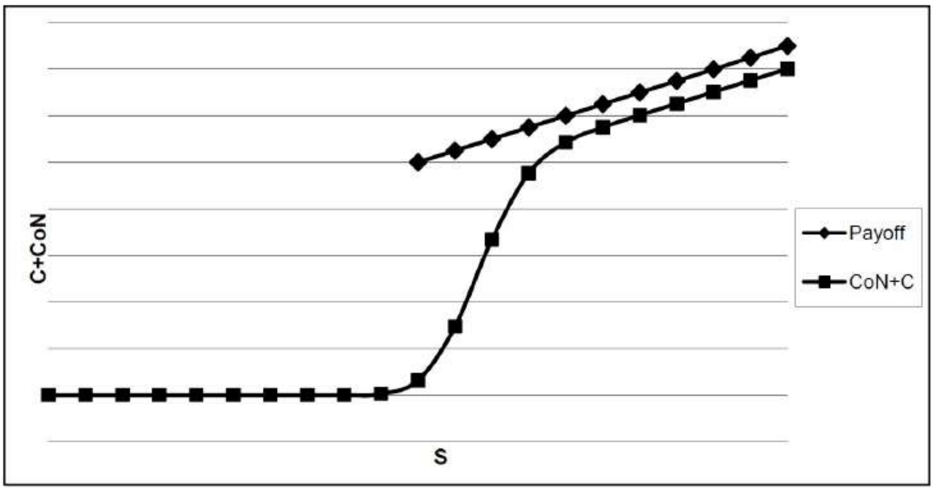
Therefore, whenever the stock price is below, then the CEO will gladly accept NPV=0 projects that increase the volatility of the firm and also some negative NPV projects, providing volatility increases sufficiently. On the other hand, if the stock price is above, then the CEO will accept 0 NPV projects if they lead to a decrease in volatility and if a project leads to an increase of volatility it must be rewarded with sufficient NPV.

Development of the model

We will assume that:

1) an agent is endowed with a contingent security which value depends on the value of the underlying assets (ST);
2) the agent is non-satiated
3) the agent has some influence on the underlying asset S, in particular the agent can:
   a) change the value of the underlying asset, providing an opportunity exists
   b) change the volatility of the asset, providing an opportunity exists
4) markets are arbitrage free

Figure 5: Payoff and the value of asset or nothing option



The influence is known by the market and this information is included in the value of the underlying asset. The nature of the influence as well as timing of the opportunities depend on the situation being modeled.

Let us review the notation used in this paper

1) T is time to maturity
2) S is the value of underlying assets
3) C is the value of the contingent claim
4) X is the exercise price of the contingent claim
5) A is the amount of money paid out by the CoN option in the event of it finishing in the money.

Let us also define:

1) The Threshold Point is the point at which the preference towards volatility changes
2) Positive risk attitude – all situations in which an agent tends to increase the volatility of his portfolio
3) Negative risk attitude - all situations in which an agent tends to decrease the volatility of his portfolio
4) Neutral risk attitude – all situations in which an agent is indifferent to either increasing or decreasing the volatility of his portfolio

The agent will act to maximize the value of his portfolio at each point in time. This follows from the assumption of the investor's non-satiation. Hence, we are able to predict the behavior of an agent when he faces a decision about the influence on the underlying asset. All we need to do is to perform marginal analysis, as described before. A positive derivative with respect to volatility means that the agent will exert his power over the asset in order to increase volatility. A positive derivative with respect to S means that the agent will exert his power over the asset so that the value of the asset increases. Interactions of these derivatives add a certain structure to the model. If the agent's portfolio consists of multiple assets, we may find the derivative of each part separately and add them up. In that way we are able to make predictions in more complicated cases. This is a very general framework that allows us to analyze many types of situations. Here we concentrate on those that include a jump in the payoff function as they allow for the Threshold Point.

It is worth noting that there is no need to calculate the exact value of the option and hence that there is no need for exact estimates of the parameters to infer the agent's decisions. Therefore, the main problem of applications of real options is alleviated. All we need to do is predict the changes in parameters (volatility, value of underlying asset) given the decision, which is considerably easier than finding the values of the parameters.

Some inferences arising from the model

Equity

Black and Scholes (1974) noticed that equity in a leveraged company is similar to a call option on a company as a whole. This observation is very insightful, but also somewhat confusing. The value of a call option increases with volatility, and therefore company owners should exhibit a positive RA regardless of the company's value (and become risk neutral only when the company's value goes to infinity). This implies that owners should accept some negative NPV projects providing they are sufficiently volatile. Furthermore, it would be beneficial for shareholders if companies just made fair bets among themselves, since this would increase volatility without any loss of value. In other words, engagement of a company in an infinite series of independent, fair coin flips of any bet size should significantly drive up the stock price.

This problem, a variation of which is also known as the "asset substitution problem", has been recognized before, but only in terms of imminent default. However, when the option model is treated literally, the asset substitution problem can be applicable to all companies. This implies that a Fortune 500 company would gamble all of its wealth on a bet with negative expected return (providing that the variance of returns is big enough); such an implication is very disturbing and does not find support in real life.

Moreover, if such a model is correct then dominant capital budgeting schemes are incomplete. Neither NPV nor traditional real options approaches take into consideration the possibilities mentioned above, which implies that practitioners are taught techniques that do not maximize investors' wealth. Furthermore, this framework shows that the investment policy of a company depends on leverage, violating one of MM's assumptions. [1]

Threshold Theory offers a solution to the problem illustrated in the "Fortune 500" example. Let's assume that S is the total value of a company, X the face value

1 This fact cannot be solved within the new framework and actually opens up interesting possibilities for further research.

of the debt, A the bankruptcy costs, and T the maturity of the debt. In such a situation, negative risk attitude is exhibited even before the value of the company reaches the threshold X and the problem vanishes. A positive RA, before the threshold is reached, is consistent with our standard understanding of an asset substitution problem.

Bond holders pay most of the bankruptcy costs as they are the ones that receive the assets of the company net of these costs. Therefore, it is not obvious why such costs would influence equity holders. One answer is related to diversification of investors. In such a case, equity holders will bear the costs of bankruptcy. As Haugen and Senbet (1978) noted in such a case, in line with Coase's (1960) theory, the parties should renegotiate the contract in order to avoid bankruptcy costs. However, if there are multiple parties involved then negotiation costs may be higher than bankruptcy costs.

The "underinvestment" (Myers, 1977) problem is also easy to comprehend within this framework. The problem is: in the case of high leverage not all positive NPV projects are accepted as most benefits accrue to debt holders. Threshold Theory stipulates even stronger implications. If the value of the company is much smaller than the Threshold, then many positive NPV projects will not be accepted because they are not risky enough. The reason is that in such cases there is a negative price of risk before threshold is crossed because, before the threshold is reached, there is a negative price on risk for the owners. If no threshold is included in the model, underinvestment would exist for any value of the company which again is inconsistent with empirics.

Agency theory and signaling

Agency theory is one of the basic ideas in modern finance and the assumption of self-interest is appealing to a rational decision-making paradigm. Threshold theory presents a modeling tool for this type of study. For the sake of illustration, let's assume that the CEO of a company would be fired at the Annual Meeting of Shareholders if the stock price is less than a pre-specified value (critical value). Furthermore, if the CEO is fired, the present value of income for the CEO drops (due to damaged reputation). In such a case, we are dealing with a cash–or-nothing option on stock S with the threshold at critical value X of the stock price, and the value of the decrease in the present value of salary equal to A. If we further assume that there exists some pay-performance sensitivity, we can construct such a portfolio wherein the contingent security becomes an asset-or-nothing option. This is very convenient, as closed valuation formulas for both types of options exist and hence risk attitude in various situations may be assessed.

Observation that this portfolio structure changes as the CEO nears retirement leads to interesting conclusions. As reputation is less and less valuable over time the AoN converges to a vanilla call. Hence CEOs should become more risk seeking over time.

In this framework, many other phenomena are easy to explain. For example, the hypothesis of managerialism states that managers derive some kind of utility from managing bigger assets. From a Threshold Theory perspective, however, this behavior is more obvious, as accumulation of assets (especially of cash or risk free assets) while the CEO has negative RA moves the stock price further from the threshold and decreases company-specific volatility if additional assets which are uncorrelated with existing business (cash, most securities, diversifying mergers). This maximizes the CEO's option value assuming that the threshold does not change. Similarly, value-destroying diversifying mergers will be performed in the event of a drop in S being more than offset by the increase in the manager's option value due to a decrease of volatility.[2] The impact of a merger on the CEO's portfolio is an interesting avenue of research opened up by Threshold Theory.

Other problems, such as CEO entrenchment and pay performance sensitivity, may be explained in natural and simple ways. More complicated phenomena, including dividend announcements and signaling effects, require more study and modeling. Several existing papers have used a logic similar to the one used in the development of Threshold Theory, such as Degeorge, Patel, Zeckhauser (1999). In this study, the authors found that managers manage earnings if earnings are not up to the expectations of the market (thresholds were set at: 0, recent years' earnings and analysts' consensus). In this case managers were trying to influence the value of the underlying asset (earnings) of their option to remain employed at the cost of increased volatility in the future.

2 In case we drop the assumption of constant threshold, we have to take into consideration how the threshold changes in relation to a company's assets size.

Application of Threshold Theory to corporate governance

Threshold Theory was designed to explain some anomalies unaccounted for by the Expected Utility Framework, but can it be used for empirical research? We believe that this is definitely the case. Here we would like to present one possible application of the Threshold Theory.[3]

Description of the model and development of a testable hypothesis

As noted earlier in this paper, a part of the human wealth of a manager may be modeled by a composition of a cash-or-nothing option and a vanilla call, each with stock of the managed company as an underlying asset. The strike of both options is set at the level of the stock price at which the majority of shareholders would become sufficiently unhappy and fire the manager. The call option reflects the performance-based part of salary, while the cash-or-nothing option reflects the reputational loss in case the manager is fired (which translates to lower future personal earnings).[4] According to Threshold Theory, the manager should exhibit a positive risk attitude if the stock price is below the threshold [5] and a negative risk attitude if the stock price is higher than the threshold.

However, if the manager decides to retire in the near future, then his portfolio changes as there is no loss due to damage to his reputation because reputation is no longer needed. Therefore, the cash or nothing option is no longer in the manager's portfolio and all that is left is a vanilla call option. In such a case the manager should exhibit positive risk attitude at all levels of the stock price. Hence, managers close to retirement are more likely to undertake actions that increase risks to the company. This inference is supported by Davidson, Xie, Xu, Ning (2005) who find that CEOs nearing voluntary retirement tend to engage in earnings management. This is a long-run volatility-increasing behavior, which also intends to influence value of the underlying asset. This sort of behavior is in line with the theory. In this study, we aim to check whether managers exhibit other types of risky behavior as retirement nears.

The overall riskiness of the company cannot be measured with beta alone as it omits the idiosyncratic risk. Therefore, as a proxy of total risk we will use the volatility of the stock price. According to the Threshold Theory, after the decision concerning retirement is made the manager should tend to increase the volatility of the company as a whole. Of course, the moment of such a decision is unobservable but managers usually are not able to adjust the volatility immediately; therefore, the process of volatility adjustment, as well as coming to the final decision, should take at least some time. To measure changes in volatility we compare volatility in years preceding the year of voluntary turnover (t -1 to t-3). Therefore, the testable hypothesis is:

H0: If a company's CEO is close to retirement then relative to the company's peers the volatility of the stock price should be increasing over time.

Methodology, data and results

This study focuses on a sample of companies in which the CEO voluntarily stepped down. The Executive Compensation Database was used to identify CEO turnovers between 1995 and 2007. The following screening criteria were used: Executive Compensation Database indicates that the reason for the turnover was retirement, the CEO was 60 or older, the CEO had at least 4 years tenure when leaving the office.[6] This screening yielded 98 observations. In order to further assure that the turnover was planned, or it was the last CEO position held by an individual, we screen whether the individual held another position after retiring at any company. Moreover, we screened press releases obtained from Lexis – Nexis concerning the turnover of CEOs aged 60-65 in order to eliminate forced turnovers disguised as retirements. In this way we seek to assure that there is little or no jump in the CEO's payoff function. These additional screens reduce the sample size to 96 observations.

The study investigates whether the total firm risk, proxied by volatility, changes over time. The time series nature of the test precludes effective use of regression analysis in such a study. There is no prediction concerning development of the process over time but only concerning its general direction (in particular increasing volatility). Therefore, it seems sufficient to compare volatility at

3 I would like to thank Wallace N. Davidson III for noticing this possible application.

4 For a detailed description of this case please refer to section II.3.4 of Chapter 1

5 The threshold point is discounted in a strike further modified depending on the type of diffusion process governing the behavior of the underlying asset. In the case of the Weiner process, the threshold is further adjusted by the expected value of lognormal distribution.

6 This also assures that there was no CEO turnover during the test period.

given points in time using a paired t-test.[7] We used yearly periods and the set under investigation consists of years t to t-3 where t is year of retirement. To have a basis for comparison, the changes in volatility will be compared against volatility of matched companies.

This comparison calls for a methodology of picking the match as well as control variables. In this study the inclusion of control variables is somewhat complicated. There are several types of variables influencing volatility that should be considered.

Variables represent tools that the CEO may use to change the company's volatility.

These include but are not limited to: leverage, R&D spending, earnings management, etc. Such variables should not be included in the test. This is because if we remove the influence of such variables then we would be unable to detect any meaningful relationship between CEO characteristics and volatility. By removing the influence of the tools that the CEO may use to change volatility, we also remove the link between the CEO and volatility.

Variables may determine the level of volatility that are not easily controlled by the CEO. These are the variables that should be controlled in the test as they may vary between companies and obscure the results. These variables are:

Industry

The industry life-cycle is known to be related to stock price volatility (Mazzucato & Tancioni, working paper; Mazzucato & Semmler, 2002)

CEO age and compensation

In general, risk appetite is believed to be related to the agent's age (MacCrimmon & Wehrung, 1990); at the same time, the current level of compensation may determine the potential loss in the case of being fired (A. Rashad Abdel-khalik, working paper). Such person specific variables should be controlled for as they may influence the behavior of the CEO. In particular retiring CEOs are typically older, and this fact may bias the results. Furthermore, compensation may be highly correlated with CEO wealth, which in turn may influence the risk attitude.

Variables that may affect the pace of changes in the volatility need to be included in the tests. As we are investigating the changes of the volatility over time we have to take into consideration whether CEOs of the compared companies are able to manipulate the volatility to a similar extent. The variables in question are firm specific:

It is much easier to manipulate characteristics of a smaller company. Therefore, when companies are compared they should be of a similar size.

Companies with comparably higher/lower volatility at the beginning of the comparison period may have a hard time increasing/decreasing its volatility even further. Therefore, compared companies should have comparable volatility to start with.

The screening variables from the third group are the ones best suited for specifying the match for each company in the original sample. We also include industry to be one of the variables used for picking a match as this is a categorical variable and therefore not well suited for regression analysis.

We picked a match in the following way. Each company in the original sample will be assigned a peer. The first screen will have two basic conditions:

the same industry (4-digit SIC code) – to accommodate potential temporal changes in volatility within industry

no CEO turnover in the investigated period.

The second screen will include screening variables from the group 3

similar size at the beginning of (t – 3) – to ensure that the pace of volatility change is comparable, assuming that it is harder to change the volatility of bigger companies. Size will be measured as market value of equity on the month of turnover as measured by Compustat.

similar volatility at the beginning of (t – 3) – to make sure that companies we are comparing are of similar risk at the start of the period under the investigation and also to eliminate influence of possible mean reversion in the volatility process. Volatility was measured as the standard deviation of the daily returns of each company's market value in a given year.

The similarity of volatility and size will be assessed using normalized Euclidean distance and Mahalanobis (1936) distance which resulted in identical matches in all but one case. The peer was selected as the one with the smallest distance to the data point in the turnover sample

7 Paired t-test is represented as $t_{n-1} = \frac{\hat{d} - \mu_0}{s_d / \sqrt{n}}$, where d is difference in values of variables between pairs, μ_0 is difference tested under null (in this paper always 0), s_d is standard deviation of observed differences and n is number of pairs.

Table 1: Summary statistics of retirement and matched sample

	t-3	t-2	t-1	t
Average retirement sample volatility	2.11%	2.15%	2.06%	2.20%
Average matched sample volatility	2.16%	2.33%	2.14%	2.13%
Median retirement sample CEO age				65
Median matching sample CEO age				59
Average retirement sample market size ($ mln)	4143			
Average matched sample market size ($ mln)	2234			
Average retirement sample CEO total annual compensation ($ ths)	3819	4457	4110	4780
Average matched sample CEO total annual compensation ($ ths)	2632	2893	3318	4377

out of all companies that were identified with screen 1.[8] Data concerning size were pulled from Compustat. Data on the stock performance was obtained from CRSP. Data on executive age and compensation were obtained from the Executive Compensation Database. Lack of suitable matches reduced sample size to 65 observations. Table 1 presents the main statistics of the samples.

The data provides two sets of information. Firstly, the matching procedure provided a close match with respect to average starting volatility; however, the retirement sample companies are almost twice as big as the matched companies. Finally, we observed a much higher median of CEO age in the retirement sample which also should be expected due to imposed constraints on CEO age, as well as the fact that retiring individuals are usually older.

After the match was identified we investigated the influence of the CEO's characteristics. To do so we ran a regression of observed volatility on CEO age and compensation.[9] The form of the regression is as follows:

$$Vol = \beta_0 + \beta_1 Age + \beta_2 Compensation + \beta_3 \mathrm{Re}\,tirement$$

Where Age denotes CEO age, Compensation is total compensation received in a given calendar year and Retirement is a dummy that takes a value of 1 if the observation comes from the retirement sample. The dummy was introduced because the event of retirement, which is hypothesized to have influence on volatility, is highly related to the age of the CEO. The regression sample consists of all observations from the matched sample (t-3 to t) for which all necessary data is available and all observations from the retirement sample for which all necessary data is available except for the year of retirement (t-3 to t-1). The results of the regression are presented in Table 2.

The regression shows that the CEO's age and compensation are not significant determinants of a firm's volatility. Therefore, further analysis will be performed on unadjusted variables.

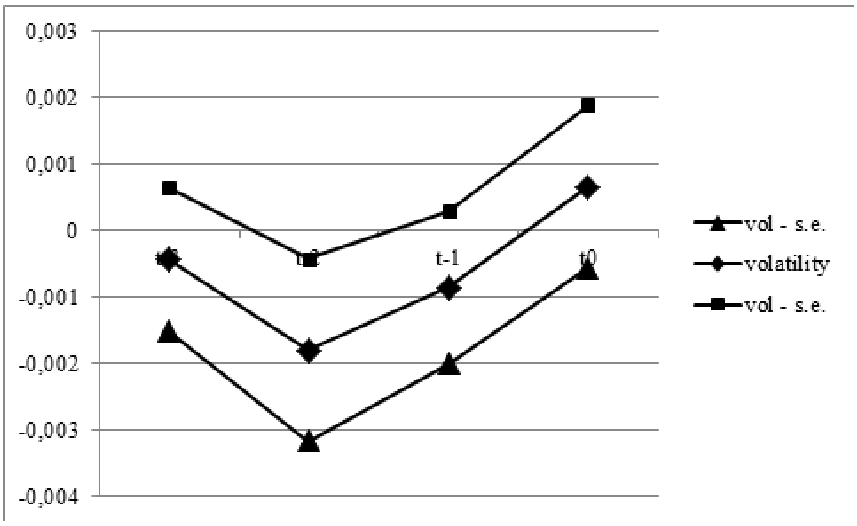
8 We have also used Euclidean distance on normalized variables, which is equivalent to Mahalanobis distance with assumed zero covariance between the variables. The matches assigned were the same for all but one company.

9 Total compensation is defined as Executive Compensation Database TDC1 variable consisting of (Salary + Bonus + Other Annual + Restricted Stock Grants + LTIP Payouts + All Other + Value of Option Grants)

Table 2: Results of the regression of volatility on age and compensation

Variable	Coefficient	p-value
Age	β1=-0.0001	19%
Compensation	β2=-8.87E-08	69%
Retirement	β3=0.0013	27%
R2=8.1%	Observations =403	p-value of F = 45%

Figure 6: Evolution of volatility differentia between retirement and matched sample



The difference between adjusted volatilities was computed. The evolution of average distance between volatilities is presented in Figure 6.

We can observe peculiar development of the process. The volatility differential increases for the last three years before CEO retirement supports Threshold theory. No year-to-year changes are statistically significant, but the difference between the peak at t and the trough at t-2 is significant at the 10% level for a two-sided test (p-value of 6.6%). The result implies that on average the retirement decision might be made two years prior to the retirement.

The final step is an investigation of changes in systematic risk. Betas were calculated using S&P500 as a proxy for the market portfolio. The changes in beta differentials are neither significant on a year to year basis nor in comparing extreme values.

Conclusions

In this paper we presented a novel approach to risk attitude modelling based on the real options approach. Contrary to the standard approach we do not assume risk preferences but instead we try to predict them. Despite somewhat involved mathematics, the application is quite straightforward. It is sufficient to identify discontinuity in the payoff function and check whether risk preference changes around the threshold.

This tool is valuable both as a source of insight into economic phenomena and as a research hypothesis generator. As an example, we provided an empirical study based on the Threshold Theory approach that yielded a not obvious hypothesis that was confirmed by the data. We encourage other researchers to apply the methodology to their area of study as it may be useful not only in economics and finance but also psychology, sociology and even zoology.

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