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A NOTE ON: AN EARLY WARNING SYSTEM FOR MARKET INEFFICIENCY

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Abstract

Violation of the efficient market hypothesis (EMH) in a specific market may lead to construction of bubbles which is a signal of inefficiencies. Although speculative bubbles soon decay, if they exist for a long time, they will lead to financial crises. Early warning systems (EWSs) are designed to quickly alert the market to crises. Under EMH, the logarithm of price is a martingale process. Thus, it is necessary to use a suitable EWS tool for violation of martingale properties of the logarithm of asset prices. In this paper, using the auto-regressive (ARTA) models, and assuming Markov structure between financial random variables, the conditional means are formulated as a simple regression. Then, using the recursive formula for least square estimates of regression parameters, the hypothesis of variables being martingale is tested. This approach leads to a probability index which serves as an EWS. Then, throughout two real data sets, it is seen that the results of the study are applicable to construct EWS for detecting stock market crashes as well as exchange rate market crises. A discussion section is proposed. Finally, based on these results, conclusions are given.

JEL classification: G21
Keywords: ARTA; EMH, EWS, Least square; Market inefficiencies, Martingale, Probability index; Regression model

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Introduction

An Early Warning System (EWS) represents the set of capacities needed to generate and disseminate timely and meaningful warning information that enables at-risk individuals, communities and organizations to prepare and act appropriately and in sufficient time to reduce harm or loss. Based on this definition, the International Federation of Red Cross and Red Crescent Societies have identified the four core components of an EWS, namely:

1) Risk knowledge to build the baseline understanding about the risk;
2) Monitoring to identify how risks evolve through time;
3) Response capability;
4) Warning communication which packages the monitoring information into actionable messages understood by those that need, and are prepared, to hear them.

This approach leads to an interesting view on the issue of crisis forecasting. Sometimes, violation of some equilibrium in an economy may lead to speculative bubbles and ultimately financial crises. One of the equilibrium conditions in most financial markets is the EMH.

Problem definition

Under the EMH, all necessary information is reflected in the price of assets. The EMH states that asset prices reflect true economic value because information is shared among market participants and rapidly incorporated into the stock price. Under the EMH, there are no other factors underlying price changes, such as irrationality or behavioral biases. In fact, EMH argues that economic bubbles don't really exist and there are no mispricing worries. Indeed, economic bubbles occur when asset prices rise far above their true economic value and then fall rapidly. In essence, then, the market price is an accurate reflection of value; no speculative bubble exists.

Fama (2013), one of the pioneers of EMH, argued that the financial crisis, in which credit markets freeze and asset prices drop precipitously, was a result of the onset of a recession rather than the burst of a credit bubble. All types of economic and business crises such as banking, financial, or currency crises have high economic costs. They inject negative effects throughout the whole of the economy. EWS prevent a crisis by sounding the alarm. Indeed, they present a forecast of bad events and are used primarily for detecting crises before damage has been done. For a bibliometric analysis of the EWS, see Klopotan, et al. (2018).

Under the EMH as well as rational expectations, the logarithm of an asset price has a random walk structure and therefore, it is a martingale, see Delcey and Sergi (2019).A martingale is the mathematical description of a fair game. Martingale theory is one of the cornerstones of modern mathematical probability theory with wide-ranging applications in stochastic analysis and mathematical finance. Thus, the aim of the current article is to provide an EWS for detecting possible future violation of martingale properties of prices which leads to inefficiency in financial markets, causes the mispricing of assets and finally converges to crisis. The main tool for checking the martingale properties is the stochastic approximation technique.

Stochastic approximation methods are a family of iterative methods typically used for root-finding problems or for optimization problems. The recursive update rules of stochastic approximation methods can be used, among other things, for solving linear systems when the collected data is corrupted by noise, or for approximating extreme values of functions which cannot be computed directly, but only estimated via noisy observations, see Borkar (2008).

Literature review

The present era of financial globalization has at least two important features. First, the high volatility of financial flows for emergent markets and frequency of financial crises; and second the fact that it has largely bypassed low-income developing countries (Leijonhufvud, 2007). Khatiwada and Mc Gir (2008) introduced a framework on how financial crises emerge and how to look at the problem. The indicators used to look at what stage of business cycle the economy is heading towards, are: employment, industry output, private consumption, GDP, etc. Thus, when these indicators worsen an economy downturn is bound to happen.

Shiller (2009) also tries to explain the financial crises through a psychological perspective called animal spirits. Shiller rejects the rational man through behavioral economics which is a conventional economic theory that is in favor of human motivation and capacities. The recent financial crisis was more widespread than the previous ones, as financial systems were more deeply connected with each other than before. The recent crisis began in the USA when the subprime mortgages defaulted. Financial institutions, which are the key intermediaries in the financial system, faced a systematic risk that froze and decreased the capital in the real economy. The subprime mortgages were designed with an interest payment, whereby the mortgagees were planning to refinance to avoid increased mortgage rates (Liapis et al., 2013, Thalassinos et al. 2015).

The rest of the paper is designed as follows. Theoretical results are given in Section 2. In Section 3, application of theoretical results in two real data sets are given and discussions are also proposed. Then, conclusions are given in Section 4.

Methodology

In this section, first using the ARTA models, and assuming a Markov Structure between risk factor random variables, a simple regression is made between future variable of risk factors and their current values. To test, the hypothesis of risk factors being martingale, using the recursive estimates of parameters of regression models, a probability index is derived for checking the martingale property. Finally, all theoretical results are summarized in a proposition.

To this end, let $\{X_t\}_{t\geq 1}$ (risk factors) be a mean corrected stationary in mean time series such that $E(X_t)=0, t\geq 1$. Let $\mathbb{F}_t$ be the σ-field (filtration) generated by $\{X_s; s\leq t\}$. By definition, X_t is a martingale with respect to filtration $\mathbb{F}_t$ if and only if $E(X_{t+1}|\mathbb{F}_t)=X_t$ for all t's. Equivalently, $E(X_{t+1}-X_t|\mathbb{F}_t)=0$. Assuming X_t is the first order Markov process, then

$E(X_{t+1}|\mathbb{F}_t)=E(X_{t+1}|\sigma(X_t))=E(X_{t+1}|X_t)=X_t$

Indeed, $X_t=X_{t-1}+u_t$ where u_t is a zero mean process such that $E(u_t|X_{t-1})=0$

a) *ARTA models*. According to the ARTA model arguments of Cairo and Nelson (1998), assume that $E(X_{t+1}|X_t)=\alpha+\beta X_t$, or, generally, from the regression perspective, assuming a linear function between and X_{t-1}; i.e.,

$$E(X_{t+1}|X_t)=\alpha+\beta X_t$$

Indeed, X_t is a martingale if and only if $\alpha=0$ and $\beta=1$ In our case, naturally, $\alpha=0$ is kept since $EX_t=0$ When, $EX_t\neq 0$, then, it is enough to replace X_t with $X_t-\bar{X}_t$, where $\bar{X}_t$ is the mean of $X_i, i=1,2,\dots,t$

Another useful practical solution for this problem is to replace EX_t with sample mean of L current observations of X_t (rolling estimate with window by length L , see Zivot and Wang, 2005).

Let $\hat{\beta}_t$ be the adaptive least square estimation of β, based on observation $X_i, 1\leq i\leq t$, i.e.,

$$\hat{\beta}_t=\frac{\sum_{i=2}^{t}X_{i-1}X_i}{\sum_{i=2}^{t}X_{i-1}^2}, t\geq 2.$$

Notice that $(\sum_{i=2}^{t}X_{i-1}^2)\hat{\beta}_t=(\sum_{i=2}^{t-1}X_{i-1}^2)\hat{\beta}_{t-1}+X_{t-1}^2\frac{X_t}{X_{t-1}}$. Thus,

$$\hat{\beta}_t=(1-\lambda_t)\hat{\beta}_{t-1}+\lambda_t\frac{X_t}{X_{t-1}}$$

constitutes a SA equation for $\hat{\beta}_t$ (See Borkar, 2008). Let

$$\hat{\gamma}_t:=\hat{\beta}_t-\beta \text{ and } \hat{\theta}_t:=\hat{\beta}_t-1.$$

Then, if $\beta=1$, then

$$\hat{\gamma}_t=(1-\lambda_t)\hat{\gamma}_{t-1}+\lambda_t r_t$$

and if $\beta\neq 1$, then

$$\hat{\gamma}_t=(1-\lambda_t)\hat{\gamma}_{t-1}+\lambda_t r_t+\lambda_t(1-\beta)$$

Also, when $\beta=1$, then

$$\hat{\theta}_t=(1-\lambda_t)\hat{\theta}_{t-1}+\lambda_t\frac{u_t}{X_{t-1}}.$$

Here, $\lambda_t=\frac{X_{t-1}^2}{\sum_{i=2}^{t-1}X_{i-1}^2}$ is referred as forgetting factor. By the way, in both cases,

$$\hat{\theta}_t=(1-\lambda_t)\hat{\theta}_{t-1}+\lambda_t R_t,$$

Where $R_t=\frac{X_t-X_{t-1}}{X_{t-1}}$ is the percentage change process of X_t. Here, it is assumed that R_t's are independent random variables with common mean μ and time varying volatility process σ_t. Indeed,

$$R_t=\mu+\sigma_t z_t$$

where z_t's are independent random variables with zero mean and unit standard deviation.

b) *Probability index*. Let $\varepsilon>0$ be a small positive number and if

$$|\hat{\theta}_t|<\varepsilon,$$

it is concluded that $\hat{\theta}_t=0$ and thus, have martingale property at time t. Assume that, up to time t-1, the X's have martingale properties and it is interesting to check if at time t, the martingale property is kept. To answer this question, the leading indicator

$$pr_t=P\left(|\hat{\theta}_t|<\varepsilon\middle|\hat{\theta}_{t-1}=l_{t-1}\right)$$

referred as probability index at which plays the role of early warning system for checking being a martingale at time . It is easy to see that

$$pr_t=P(|\lambda_t\sigma_t z_t+\gamma_{t-1}|<\varepsilon),$$

where $\gamma_{t-1}=\lambda_t\mu+(1-\lambda_t)l_{t-1}$.

As soon as, the empirical distribution of z_t is found, then the Monte Carlo simulation is run to compute p_t .The following proposition summarizes the above discussion.

Proposition 1. Let $\hat{\beta}_t$ be the adaptive least square estimation of β, based on observation $X_i, 1\leq i\leq t$, and $\hat{\theta}_{t-1}=l_{t-1}$. Suppose that $pr_t=P\left(|\hat{\theta}_t|<\varepsilon\middle|\hat{\theta}_{t-1}=l_{t-1}\right)$ Then, (*a*)-(*c*) are correct.

1) $\hat{\theta}_t=(1-\lambda_t)\hat{\theta}_{t-1}+\lambda_t R_t$, where $\lambda_t=\frac{X_{t-1}^2}{\sum_{i=2}^{t-1}X_{i-1}^2}$ and is the percentage change process of X_t.

2) Assume that t - 1, the X's have martingale proper-

ties and $\hat{\theta}_{t-1} = l_{t-1}$ where $|l_{t-1}| < \varepsilon$, the probability of X being martingale at time t, is $pr_t = P(|\lambda_t \sigma_t z_t + \gamma_{t-1}| < \varepsilon)$, at which $\gamma_{t-1} = \lambda_t \mu + (1 - \lambda_t) l_{t-1}$

3) The optimal value of λ_t is obtained by minimizing

$$\sum_{i=2}^{t} (\hat{\theta}_i - (1 - \lambda_t)\hat{\theta}_{i-1} - \lambda_t R_i)^2$$

with respect to λ_t. It is easy to see that

$$\hat{\lambda}_t = \frac{\sum_{i=2}^{t} (\hat{\theta}_i - \hat{\theta}_{i-1})(R_i - \hat{\theta}_{i-1})}{\sum_{i=2}^{t} (R_i - \hat{\theta}_{i-1})^2}.$$

As soon as $\hat{\lambda}_t$'s are derived, numerical methods such as Spline, response surface methodology, or regression analysis may be applied to find the functional form of $\hat{\lambda}_t$ as a function of t.

Remark 1.

Considering a prior π_t for $\hat{\lambda}_t$, then the Bayesian estimate of $\hat{\lambda}_t$ is the minimization of

$$\sum_{i=2}^{t} (\hat{\theta}_i - (1 - \lambda_t)\hat{\theta}_{i-1} - \lambda_t R_i)^2 + \log(\pi_t(\lambda_t)).$$

Remark 2.

Under EMH, the mean corrected version of logarithm of price of a financial asset (prc_t), i.e.,

$$X_t = \log(prc_t) - E(\log(prc_t))$$

is a martingale. Hence, it is enough, in the above discussion, to substitute X_t with $X_t = \log(prc_t) - E(\log(prc_t))$ and R_t with $\frac{r_t}{X_{t-1}}$, to use pr_t as EWS alarm of market inefficiencies. Here, r_t is the return of price prc_t.

Application

In this section, the above technical results are surveyed throughout real data sets. Then, discussions are proposed.

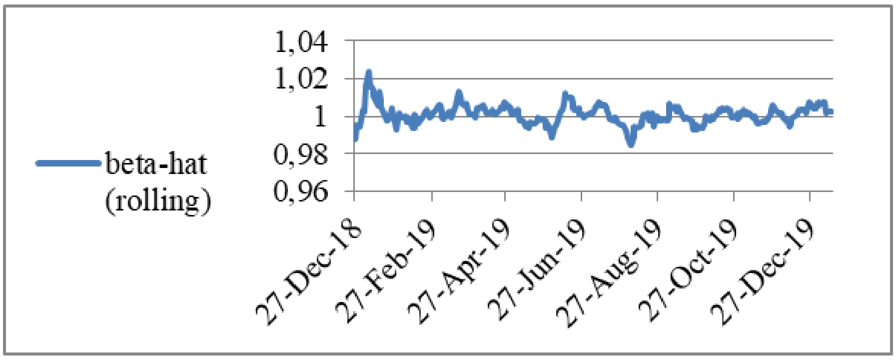
Real data sets

Here, two real data sets are analyzed and finally, the applicability of a probability index as an EWS is studied. These real data sets are Amazon Co. stock price and exchange rate of Great Britain Pound (GBP) against the US dollar (USD).

1) Real data set 1

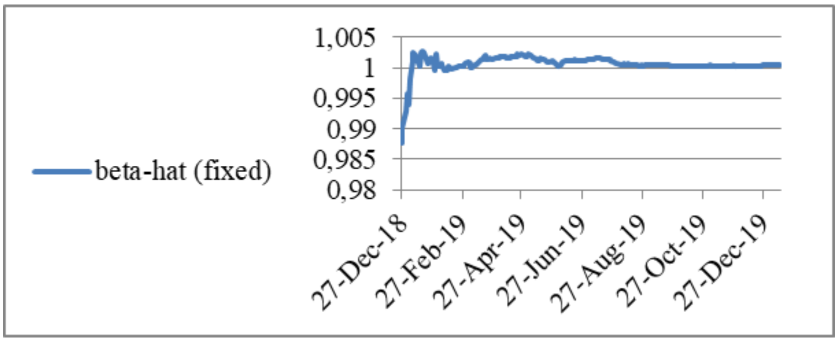
The daily stock price (prc_t) of *Amazon Co*. is studied for the period of 14-Dec-2018 to 13-Jan-2020 (271 observations). These data are taken from *www.Bloomberg.com* site. The log-return time series of prc_t; i.e., $u_t = \log(prc_t) - \log(prc_{t-1})$ is computed and it is seen it constitutes a white noise process. Thus, $\log(prc_t)$ is a random walk and therefore is a martingale. The overall mean of logarithm of 271 observations is 7.37 and the mean corrected series $X_t = \log(prc_t) - 7.37$ is derived. The rolling ($L = 10$) and fixed scheme $\hat{\beta}_t$'s are computed and plotted as follows. Clearly, from both figures, it is seen that $\beta = 1$.

Figure 1: Time series of rolling $\hat{\beta}_t$



Source: Own work

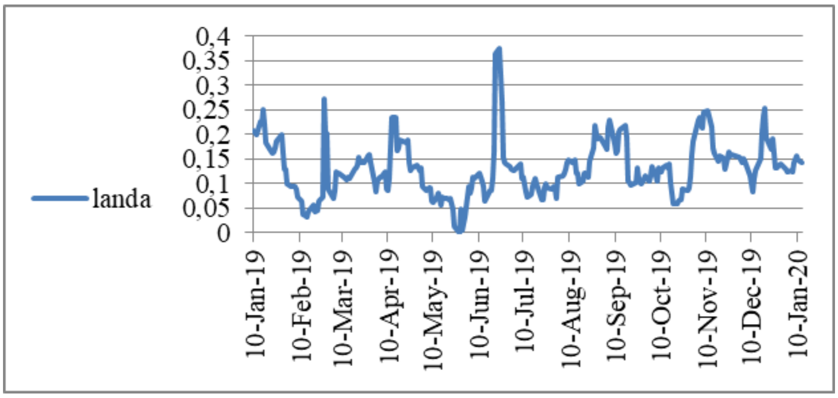
Figure 2: Time series of fixed $\hat{\beta}_t$



Source: Own work

The rolling optimal forgetting factor is plotted as follows

Figure 3: Time series of $\hat{\lambda}_t$



Source: Own work

To obtain pr_t, values of R_t's are computed. Its minimum (*min*), maximum (*max*) and range (*rng*) are –0.824, 0.7058, and 1.529, respectively. First, the transformation. $R_t' = \frac{R_t - min}{rng}$ is taken. It is seen that its mean (*mn*), variance (*v*), skew and kurtosis are 0.5421, 0.0165, -0.1548, and 4.162. It is seen that R_t' has symmetric light tail distribution. A beta distribution is fitted to R'_t. The first and second parameters of beta distribution, using method of moments, are estimated as

$$\begin{cases} c \times mn \\ c \times (1 - mn) \end{cases}, c = \frac{mn \times (1 - mn)}{v} - 1$$

respectively. These parameters are estimated at 7.61, and 6.42, respectively, which show the symmetric of distribution. Set $\varepsilon = 0.05$. Then,

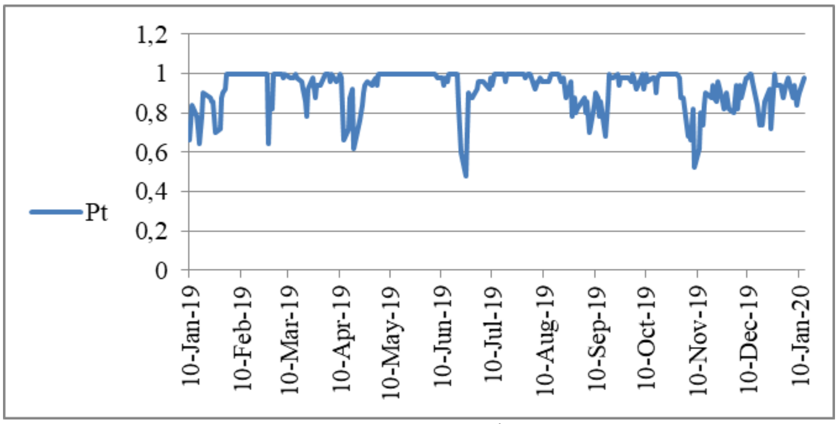
$$\sigma_t z_t = R_t - \mu = R_t - 0.0053 = 1.529R_t' - 0.824 + 0.0053 = 1.529R_t' - 0.8187$$

at which R'_t has beta distribution with parameters 7.61, 6.42, respectively. The time series plot of p_t is given. The repetition of Monte Carlo simulation is 50. Obviously, there is no inefficiency or bubble in the price of stock of *AmazonCo*. This probability is less than 0.5 at 26 Feb. 2019 and 21, 24 Jun 2019.

2) Real data set 2

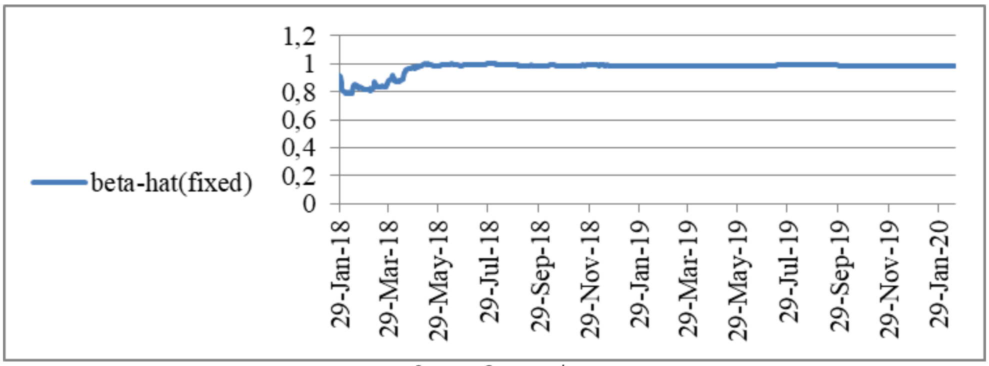
Here, the exchange rate $ex_t = \frac{GBP}{USD}$ is surveyed during 29 Jan. 2018 to 28 Feb. 2020 including 545 observations taken from *www.investing.com* site. Again, $X_t = \log(ex_t) - 0.264$ Time series of fixed $\hat{\beta}_t$ is plotted. It is seen that, in most cases, the market has efficiency. However, again, the plot of pr_t is plotted as follows. First, notice that $R_t' = \frac{R_t - 0.0039}{0.642}$ has beta distribution with parameters 0.255 and 0.561. Then, $\sigma_t z_t = R_t - \mu = R_t - 0.3122 = 0.642R_t' - 0.3083$. It is seen that after 30 May 2018, the market efficiency hypothesis is true with probability of one.

Figure 4: Time series of p_t



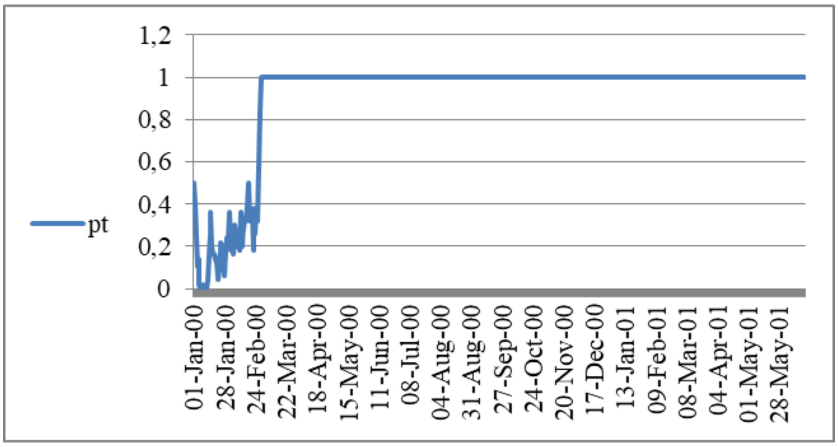
Source: Own work

Figure 5: Time series of fixed $\hat{\beta}_t$



Source: Own work

Figure 6: Time series of p_t



Source: Own work

Discussion

In the previous sub-section, the probability indicator shows the inefficiency of markets and then the possibility of a financial bubble and ultimately the financial crisis. It is interesting to check this out with the actual phenomena which happened.

1) Amazon Co.

Stock market shows an alarm at 26 Feb. 2019 and 21, 24 Jun 2019. By searching on the web, it is seen that Amazon will be the most important company of the 2020s, see https://www.cnbc.com/2019/12/13/amazon-will-be-the-most-important-company-of-the-2020s.html. Back then, Amazon was like eBay or Craigslist, an important website you went to for a specific purpose. It wasn't a daily habit, like Apple's iPhone, Google's search engine or even Facebook. Thus, really, there is no inefficiency and possibility of crashes in Amazon stock price.

2) *USD vs. GBP*.

The probability index of this financial index shows some inefficiency during 2018. To check this event, again, web searches show that this exchange rate has engaged some systemic risk during 2018, see European Central Bank report for 2018[2].

[2] Euro money market study 2018: Money market trends as observed through MMSR data

Conclusions

A financial crisis is a situation in which some financial assets suddenly lose a large part of their nominal value. Banking panics and stock market crashes, speculative attacks to a specified currency, the bursting of other financial bubbles and sovereign defaults are famous examples of financial crises. Financial crises directly result in significant changes in the real economy. Many economists have offered theories about how financial crises develop and how they could be prevented. There is no consensus, however, and financial crises continue to occur from time to time. The early warning system (EWS) plays important roles for predicting the financial crises.

The EMH plays an important role in noting unwanted events which lead to financial crises, ultimately. The EMH holds if the logarithms of all financial assets are martingale. To forecast financial crises, soon, before, they occur an EWS based on a probability index is constructed. This index uses the stochastic approximation of the slope of ARTA regression model. Two real data sets, including the daily stock price of *Amazon Co*. and $\frac{GBP}{USD}$ exchange rate are surveyed. Plotting the probability index shows when the specific market isn't efficient and when it is and with which degrees of belief. This index can be used online to provide useful information for all types of participants which exist in a given market, such as traders or hedgers.

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