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FUZZY REPRESENTATION OF A LIMIT ORDER BOOK AS A MEASURE OF STOCK LIQUIDITY

Szymon Stereńczak[1], Adam Marszałek[2]

Abstract

In this paper, we seek to ascertain whether the recently developed ordered fuzzy number (OFN) representation of a limit order book (LOB) by Marszałek and Burczyński (2024) may serve as a measure of stock liquidity. In particular, we aim to test whether this measure contains similar or distinct information than other stock liquidity measures, particularly relying only on best buy and sell orders. To this end, we have investigated the data on 259 companies in total over an eight-year period from 2014 to 2021. In total, we have compared the tested measure with eight different measures derived from the LOB data, all of which are computed on a weekly basis. Our results indicate that the OFN representation of a LOB correlates with other liquidity measures in the time series but is less correlated with them in the cross-section. Furthermore, it contains a distinct piece of information compared to the other measures, suggesting that it may capture a different liquidity dimension. In summary, the findings of the study suggest that the OFN representation of a LOB could serve as a measure of stock liquidity, particularly for large-volume trades and stocks with shallow or fragmented order books.

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[1] Poznań University of Economics and Business, Poznań, Poland, e-mail: szymon.sterenczak@ue.poznan.pl, ORCID: https://orcid.org/0000-0003-4825-3229.
[2] Cracow University of Technology, Cracow, Poland, e-mail: adam.marszalek@pk.edu.pl, ORCID: https://orcid.org/0000-0002-9973-982X.

Introduction

The question of how to measure stock liquidity in a way that reflects it properly is still being discussed in the academic community. The answer to this question is crucial, as the use of a specific measure may sometimes lead to different outcomes (Brennan et al., 2012; Marshall & Young, 2003). Given that liquidity has serious implications for a range of areas, including asset pricing, portfolio management, trading strategies, and risk management, it is imperative for investors to engage measures that accurately and precisely gauge stock liquidity. Stock liquidity is an elusive concept. It is defined as the ability to trade (buy or sell) large quantities of shares at any time, at no cost, and without causing an unfavourable price impact (Stereńczak et al., 2020). It is widely accepted in the literature that stock liquidity is a multifaceted concept, encompassing several transactional properties of the market. Sarr and Lybek (2002) distinguished five distinct dimensions of stock liquidity: tightness (trading cost), immediacy (trading speed), depth (trading volume), breadth (number of orders) and resiliency (price impact), with each dimension measured using different metrics. This multidimensionality of stock liquidity (Sarr & Lybek, 2002) poses a significant challenge in addressing the question of how liquidity should be measured.

Researchers and investors can apply various metrics to measure stock liquidity, which can be grouped into three categories, depending on the data demanded for calculations. The most accessible liquidity measures rely on daily transaction data, such as open, close, high and low prices, volumes, etc. These are termed low-frequency liquidity proxies, and as the extant literature (e.g. Amihud, 2002; Corwin & Schultz, 2012; Fong et al., 2017) demonstrates, numerous liquidity proxies have been developed that require only readily available daily data. However, it should be noted that these proxies are based on certain assumptions, the fulfilment of which may be challenging in markets experiencing significant infrequent trading problems (Bleaney & Li, 2015; Chelley-Steeley et al., 2015), like the Polish one. Moreover, the capacity of these measures to gauge liquidity only ex post restricts their applicability, for instance, in circumstances of sudden declines in stock liquidity or the implementation of high-frequency trading strategies. Furthermore, technological advancements and the subsequent increased accessibility of high-frequency financial data (Hussain et al., 2023) render the utilisation of low-frequency liquidity proxies increasingly untenable. Consequently, investors are likely to place greater reliance on high-frequency liquidity measurement, which can reflect stock liquidity ex-ante (Gao et al., 2019).

The second group of liquidity measures encompasses proxies that rely on intra-daily data but require both order- and transaction data for computation, and as such are termed transaction-based measures (Aitken & Comerton-Forde, 2003). Examples of this group include Kyle's λ (1985), Glosten's and Harris's (1988) fixed and transitory spread components, or Hasbrouck's (2009) Bayesian estimator with Gibb's sampling. While these measures are more precise and accurate in capturing liquidity, especially for asset pricing purposes (Gao et al., 2019), they still can be computed only ex-post, which limits their applicability in situations where ex-ante liquidity is required. These shortcomings are alleviated by the third group of liquidity measures, which require the data on order flows to be computed, thus being termed order-based measures (Aitken & Comerton-Forde, 2003). These metrics are particularly well-suited for analysing ex-ante liquidity (Gao et al., 2019), which is derived from the inherent dynamics of buy- and sell orders entering the market. The application of these metrics enables investors to assess the level of stock liquidity with greater precision. However, it should be noted that these measures are not without their shortcomings.

The most widely applied stock liquidity measure from this group, the bid-ask spread, is based exclusively on the best buy- and sell orders' prices, irrespective of their volumes (depth). However, given that the ordered quantity may be very low, the bid-ask spread may not accurately reflect liquidity for trades of volumes exceeding the best buy- and sell orders' volumes. In response to this shortcoming, one can utilise effective spread, however, it is uninformative about the size of the trade and also relies on the last trading price. The measures of depth, in particular volume and dollar volume of the best sell- and buy-orders, are not related to transaction costs. Large-volume orders can be present in the limit order book, but their prices may deviate significantly from the mid-price, last trading price or intrinsic value. Consequently, the execution of these trades can incur substantial costs. Some measures have been devised to address this issue by integrating cost with depth. Examples include the quote slope (Hasbrouck & Seppi, 2001), composite liquidity (Chordia et al., 2001), and the order ratio (Ranaldo, 2001). However, these measures only reflect transaction costs relative to the quantities of orders, and they rely exclusively on the best buy- and sell-offers. This renders them unstable and vulnerable to manipulation, which is of particular importance for less liquid stocks.

To address the aforementioned limitations, it is recommended to consider the entirety of the limit order book (LOB) data when assessing liquidity. The simplest method of including the entire LOB data is to calculate the volume-weighted bid-ask spread, which is simply the bid-ask spread averaged across all orders with the orders' volumes being the weights for each order's price. This approach, however, does not take into account the volume an investor wishes to trade. This shortcoming can be mitigated e.g. by presenting the trading cost as a function of the size of a trade, in which case the function's slope may reflect the price impact of a trade, integrating trading cost with trading volume. Notable examples of such approaches include the elasticity of demand and supply as demonstrated by Kalay et al. (2004) and the LOB slope as proposed by Næs and Skjeltorp's (2006). However, it should be noted that these measures assume a linear relationship between trading cost and volume, consequently resulting in a constant price concession per unit of trade. While voluminous orders can be concentrated and have limit prices close to the mid-price, these measures can underestimate trading costs for large trades.

One such measure that relies on the data from the entire LOB, is an ordered fuzzy numbers (OFNs) representation of a LOB (Marszałek & Burczyński, 2024). In contrast to the measures mentioned above, the OFN measure of liquidity offers a robust approach to the handling of the dynamic and irregular nature of the limit order book data, as it allows for the capture of non-linearities and asymmetries of the LOB structure at the moment. This distinguishes the OFN measure from conventional liquidity measures, such as volume or spreads, as it not only quantifies the number of orders available but also their distribution relative to the reference price. Moreover, the utilisation of ordered fuzzy numbers (Kosiński et al., 2003, 2022) renders this representation less susceptible to minor perturbations and noise (e.g. from market manipulations), thereby providing a more stable and precise mathematical framework for capturing the complexities of LOB data.

The objective of this paper is to provide an answer to the following research question: can the OFN representation of a LOB be an effective measure of stock liquidity, providing information that is complementary to conventional liquidity metrics? To achieve the study's main aim, the comparison is made between the OFN liquidity measure and conventional liquidity measures that rely solely on data from the LOB, with particular attention paid to those measures that rely exclusively on best buy- and sell orders. The methods employed in this study are consistent with those utilised in 'horse races' of liquidity measures (e.g. Ahn et al., 2018; Fong et al., 2017; Goyenko et al., 2009; Lesmond, 2005; Marshall et al., 2013). The measures are compared in several aspects, including their distributional properties and correlations among them. The exploratory nature of the correlation analysis is further supported by the analysis of the information content as depicted by the principal component analysis (PCA), complemented by the factor analysis with the use of varimax rotation. The study encompasses companies that are constituents of the WIG20, mWIG40 and sWIG80 indices over the eight-year period from 2014 to 2021.

The present paper makes several contributions to the extant literature. Firstly, it adds to the ongoing debate on liquidity measurement. Although several 'horse races' of liquidity measures have been undertaken, e.g. by Goyenko et al. (2009) for the US market, Fong et al. (2017) for global international markets, Ahn et al. (2018) for emerging markets, and Marshall et al. (2013) for frontier markets, the outcomes of these studies are not consistent, suggesting that the most suitable measure of liquidity varies across different markets. This finding lends support to the pursuit of developing novel measures that are likely to capture stock liquidity with greater precision. The OFN representation of a LOB, as proposed by Marszałek and Burczyński (2024) is put forward as a measure for stock liquidity, and it is demonstrated that it can be superior in capturing stock liquidity as it is able to capture more than just one liquidity dimension. Secondly, we make a contribution to the extant knowledge by offering several original insights about the measurement of stock liquidity within the Polish market. Primarily, the analyses conducted herein encompass exclusively high-frequency liquidity measures derived from the order book data, most of which have hitherto not yet been applied within the Polish context. Moreover, in addition to the correlation among the measures, the information content of the aforementioned measures is also given full consideration. In the context of the Polish stock market, a number of studies have been conducted with the objective of comparing liquidity measures. The majority of these studies employ correlation analysis as a means of comparing low-frequency measures. To date, however, no studies have focused exclusively on high-frequency liquidity measures derived from LOB data, nor have any of them sought to evaluate a newly developed liquidity indicator. For instance, Będowska-Sójka (2017) and Stereńczak (2016) compared the indications of various low-frequency measures, while Będowska-Sójka and Garsztka's (2019) study in turn was directed towards a comparison of both low-frequency liquidity and various spread measures. Subsequent studies by Będowska-Sójka (2018) and Stereńczak (2019) sought to ascertain the performance of low-frequency liquidity proxies in reflecting the bid-ask spread calculated from order data, while Będowska-Sójka and Kliber (2018, 2021) examined the information content and transfer between low-frequency liquidity and volatility measures. Only Olbryś (2017, 2018) as well as Olbryś and Mursztyn (2018) comparatively analysed high-frequency liquidity measures, though their analysis focused on transaction-based measures. The structure of the paper is as follows: the following section depicts a methodological approach; the next section is devoted to presenting the empirical results; finally, the study is discussed and concluded in the last section.

Data and methods

Research sample

The present study encompasses a data set from 2014 to 2021, which encompasses periods of high and low market liquidity with an unprecedented COVID-19 pandemic resulting in a significant, though temporary drop in liquidity and extreme uncertainty and volatility. Concurrently, the period was characterised by several regulatory shifts in the WSE, including the rescission of short-selling restrictions in June 2015, the implementation of MiFID II in 2018, and the reduction of minimum tick size in April 2019. The study period and the range of market conditions and regulatory frameworks it encompasses facilitate a more comprehensive analysis of the properties of our measure. The research sample comprises stocks included in three indices: WIG20, mWIG40, and sWIG80. The selection of these indices is driven by the necessity to guarantee a minimum level of liquidity and the availability of continuous data on the orders executed in the market. In contrast, smaller companies that do not feature in these three indices encounter infrequent order flow, which may result in the absence of time-variability in the LOB properties. The ability to access the full limit order book is a prerequisite for computing the OFN representation of a LOB. Consequently, the study's sample encompasses data on 140 stocks (out of approximately 450 listed during the period under scrutiny) in each period. On average, these 140 companies account for over 98% of turnover and approximately 95% of the WSE capitalisation, thereby ensuring the representativeness of the sample for the entire market. Given that indices were subject to changes, the study's total scope encompasses 259 companies. The data employed in this study, specifically the limit order book (LOB) data comprising all buy and sell orders placed within a specified time interval, were obtained directly from the Warsaw Stock Exchange.

Liquidity measures

In order to calculate liquidity measures based on the data from the limit order book, LOB snapshots are taken at 10-minute intervals. Given that trading on the WSE occurs between 9 AM and 5 PM and that opening and closing auctions are excluded due to specific regulations governing trading then, 47 LOB snapshots per day are obtained, amounting to approximately 235 snapshots per week. For each snapshot, the respective values are computed and subsequently averaged (using an arithmetic average with series stability) across each week to derive the weekly values for each liquidity measure and winsorised at 1st and 99th percentile to account for outliers.

In the study, we aim to analyse whether the LOB representation by ordered fuzzy numbers (OFNs) developed by Marszałek and Burczyński (2024) reflects stock liquidity. In this measure's construction, the LOB snapshot is represented by an ordered fuzzy number, i.e. an ordered pair of continuous real functions on the interval:

$$A=(f,g)\ with\ f,g:[0,1]\rightarrow R \tag{1}$$

LOB data are transformed into OFNs following Marszałek and Burczyński (2024), who defined an OFN representation of a LOB in moment t (A_t) as the pair of functions $f_t(x)$ and $g_t(x)$ as follows:

$$f_t(x)=-\begin{cases}0 & if\ (1-x)p_r(t)>\mu_a^1(t),\\ \sum_{i=1}^{L_a}v_a^i(t) & if\ (1-x)p_r(t)\le\mu_a^{L_a}(t),\\ (\sum_{i=1}^{l_a}v_a^i(t))(1+\dfrac{\mu_a^{l_a}(t)-(1-x)p_r(t)}{(1-x)p_r(t)-p_a^{l_a+1}(t)}) & otherwise\end{cases} \tag{2}$$

$$f_t(x)=-\begin{cases}0 & if\ (1-x)p_r(t)>\mu_b^1(t),\\ \sum_{i=1}^{L_b}v_b^i(t) & if\ (1-x)p_r(t)\le\mu_b^{L_b}(t),\\ (\sum_{i=1}^{l_b}v_b^i(t))(1+\dfrac{\mu_b^{l_b}(t)-(1-x)p_r(t)}{(1-x)p_r(t)-p_b^{l_b+1}(t)}) & otherwise\end{cases} \tag{3}$$

Where:

$p^i_a(t)$ = ask price for ith price level at time t,

$p^i_b(t)$ = bid price for ith price level at time t,

$v^i_a(t)$ = ask volume for ith price level at time t,

$v^i_b(t)$ = bid volume for ith price level at time t,

L_a = number of (nonzero) ask price levels,

L_b = number of (nonzero) bid price levels,

$\mu^i_a(t)$ = volume-weighted average ask price at time t from level 1 to i,

$\mu^i_b(t)$ = volume-weighted average bid price at time t from level 1 to i,

l_a = lowest i that satisfies the relation $\mu^i_a < (1 - x)\ p_r(t)$,

l_b = lowest i that satisfies the relation $\mu^i_b > (1 + x)\ p_r(t)$,

$p_r(t)$ = mid-price at time t.

The values of |f(x)| and g(x) represent the sizes of trades that can be executed at different levels of potential cost of (1 – x), which is the percentage difference between the actual average transaction price and the reference price. The higher the value of |f(x)| (g(x)) the more shares can be sold (purchased) at the potential cost (1 – x) when a market sell (buy) order is placed, indicating higher liquidity. The liquidity measure LIQ^{OFN} is then computed by employing the expected value defuzzification operator of A_t (Marszałek & Burczyński, 2021):

$$LIQ^{OFN} = E(|A_t|) = E(|f_{A_t}|, |g_{A_t}|) = \frac{1}{2}\int_0^1 [|f_{A_t}(s)| + |g_{A_t}(s)|]ds \quad (4)$$

In order to ascertain whether LIQ^{OFN} reflects stock liquidity, it is necessary to compare it with other commonly used measures of liquidity. To ensure that the results are not driven by varying data demand for calculations, a comparison is made between LIQ^{OFN} and conventional liquidity measures relying solely on data from the LOB, with particular attention paid to those measures that rely exclusively on best buy and sell orders. The following liquidity measures are considered:

1) Næs and Skjeltorp's (2006) LOB slope ($LIQ^{LOBslope}$),
2) Volume depth (Brockman & Chung, 2000)(LIQ^{Depth}),
3) Value depth (Brockman & Chung, 2000)($LIQ^{\$Depth}$),
4) Bid-ask spread (Amihud & Mendelson, 1986)(LIQ^{BAS}),
5) Effective spread (Chordia et al., 2001)(LIQ^{EffS}),
6) Quote slope (Hasbrouck & Seppi, 2001)(LIQ^{QS}),
7) Composite liquidity (Chordia et al., 2001)(LIQ^{CL}),
8) Order ratio (Ranaldo, 2001)(LIQ^{OR}).

As posited by Sarr and Lybek (2002), a select number of measures encompass three liquidity dimensions: tightness (LIQ^{BAS} and LIQ^{EffS}), depth (LIQ^{Depth}, $LIQ^{\$Depth}$ and $LIQ^{LOBslope}$) and resiliency (LIQ^{QS}, LIQ^{CL} and LIQ^{OR}). It is imperative to note that all the measures under scrutiny require static LOB data, which renders them accurate for ex-ante liquidity measurement.

Methods

We compare our measure, LIQ^{OFN}, with other LOB based liquidity measures in several areas. Firstly, an examination of the distributional properties of the measures is conducted, with particular attention paid to variability and non-normality. Additionally, an investigation is made into the correlation among the measures. This approach aligns with prevailing practices in the evaluation of novel liquidity measures (Abdi & Ranaldo, 2017; Corwin & Schultz, 2012; Fong et al., 2017). In addition, in line with Będowska-Sójka and Kliber (2021), principal component analysis (PCA) is employed to ascertain whether the representation of the LOB by OFNs provides information that is absent in other proxies. To complement these results, factor analysis with varimax rotation (Hair et al., 2019) is also employed.

Results

Descriptive statistics

The first area of comparison is that of the measures' distributional properties. To this end, a comparison is made of the means, standard deviations, coefficients of variation, skewness, and kurtosis of the measures' distributions. Table 1 presents the time-series averages of the cross-sectional statistics, and Table 2 presents the cross-sectional averages of the time-series statistics. The information provided by both tables is different, thus allowing inferences to be made about the distributions of the measures for other purposes.

Cross-sectional statistics (Table 1) are of particular importance for example in the context of asset pricing. The coefficient of variation for LIQ^{OFN} is comparable to that of the dollar depth, placing its cross-sectional variability in the middle of the pile. However, as evidenced by the Breusch-Pagan (1979) test, the cross-sectional variance is not constant over time for half of the measures, including our LIQ^{OFN}. This property is generally desirable for liquidity measures, as it allows for the capture of the flight-to-liquidity phenomenon (Rösch & Kaserer, 2013). This phenomenon occurs when market liquidity declines and investors move their capital from less to more liquid shares, thereby making more liquid stocks even more liquid and less liquid stocks increasingly illiquid. Consequently, the liquidity spread between high and low liquid assets widens as a reaction to increased market uncertainty, as evidenced by increased cross-sectional variability of liquidity.

All the measures are right-skewed, indicating that their means are higher than their medians. Intriguingly, LIQ^{OFN}'s skewness is the third lowest among the measures. All measures exhibit elevated cross-sectional kurtosis, indicating a greater extremity of outliers than would be expected in a normal distribution, with LIQ_{OFN} again having the second lowest kurtosis. The Jarque-Bera (1980) test rejects the null hypothesis that skewness and kurtosis match a normal distribution for all liquidity measures under investigation. The average values of the test statistics range between 2,751 for LIQ^{BAS} and 72,163 for LIQ^{OR}. The Shapiro-Wilk (1965) test further validates the non-normality of the measures' distributions, with the average values of the test statistics ranging from 6.83 to 10.221. Very similar conclusions may be drawn about time-series properties, with LIQ^{OFN}'s time-series volatility being of an intermediate value, the same as its skewness and kurtosis.

Table 1: Time-series averages of cross-sectional statistics

Measure	Mean	Standard deviation	Breusch-Pagan	Coefficient of variation
LIQ^{OFN}	0.2291	0.5666	3.991**	2.4599
$LIQ^{LOBslope}$	503.6200	622.8400	1,792.000***	1.2258
LIQ^{Depth}	45,342.0000	431,578.0000	0.082	6.1891
$LIQ^{\$Depth}$	60,125.0000	226,366.0000	0.692	2.4278
LIQ^{BAS}	0.0093	0.0086	3,426.000***	0.9030
LIQ^{EffS}	0.0065	0.0179	538.000***	1.8182
LIQ^{QS}	0.0805	0.2982	7.496***	3.4376
LIQ^{CL}	14.5050	142.6800	0.008	1.7806
LIQ^{OR}	369.5000	4,139.6000	0.034	7.8723
Measure	**Skewness**	**Kurtosis**	**Jarque-Bera**	**Shapiro-Wilk**
LIQ^{OFN}	4.1091	19.8530	4,211.000***	9.2480***
$LIQ^{LOBslope}$	3.9704	23.3870	5,423.000***	8.0280***
LIQ^{Depth}	9.1493	93.0200	61,547.000***	10.1060***
$LIQ^{\$Depth}$	6.0314	49.3720	23,559.000***	9.2370***
LIQ^{BAS}	2.6862	13.4410	2,751.000***	6.8300***
LIQ^{EffS}	4.7554	39.2780	21,878.000***	7.8670***
LIQ^{QS}	7.2654	62.0630	27,349.000***	9.7520***
LIQ^{CL}	4.2348	29.6540	12,278.000***	8.1430***
LIQ^{OR}	9.7299	102.5600	72,163.000***	10.2210***

Note: ***, ** and * denote statistical significance at 1%, 5% and 10% respectively

Source: Author's own calculations based on the data from Capital IQ and WSE.

Table 2: Cross-sectional averages of time-series statistics

Measure	Mean	Standard deviation	Breusch-Pagan	Coefficient of variation
LIQ^{OFN}	0.1556	0.1023	7e+06***	0.8750
$LIQ^{LOBslope}$	400.7300	170.5700	5e+06***	0.4267
LIQ^{Depth}	140,333.0000	215,000.000	20188***	0.9862
$LIQ^{\$Depth}$	69,796.0000	124,169.0000	12751***	0.9202
LIQ^{BAS}	0.0117	0.0054	1e+06***	0.4300
LIQ^{EffS}	0.0102	0.0128	47232.00***	0.8219
LIQ^{QS}	0.1217	0.0677	8e+05***	0.4958
LIQ^{CL}	19.7200	180.3200	7.53***	0.9085
LIQ^{OR}	1,538.0000	3,360.7000	4622.00***	2.3484
Measure	**Skewness**	**Kurtosis**	**Jarque-Bera**	**Shapiro-Wilk**
LIQ^{OFN}	2.1968	11.2760	14155.00***	6.5940***
$LIQ^{LOBslope}$	1.3209	3.9555	836.35***	5.3940***
LIQ^{Depth}	2.9926	20.3630	30729.00***	7.3760***
$LIQ^{\$Depth}$	2.9662	20.3260	32539.00***	7.2330***
LIQ^{BAS}	1.2979	4.2768	1519.60***	5.1640***
LIQ^{EffS}	4.4558	51.4190	151970.00***	7.2200***
LIQ^{QS}	1.1307	2.7888	465.66***	5.1470***
LIQ^{CL}	2.5098	14.1220	12950.00***	7.0120***
LIQ^{OR}	5.8356	56.3450	115817.00***	9.3370***

Note: ***, ** and * denote statistical significance at 1%, 5% and 10% respectively

Source: Author's own calculations based on the data from Capital IQ and WSE.

Correlation analysis

In the following analysis, the capacity of the measure based on the OFN representation of the LOB to capture liquidity is investigated. To this end, LIQ^{OFN} is compared with other proxies through the analysis of the correlations among the measures. The methodology employed is similar to that used by Corwin and Schultz (2012), Abdi and Ranaldo (2017) and Fong et al.

(2017). This involves two key steps: 1) calculating the time-series averages of the cross-sectional Pearson correlation among liquidity proxies and 2) calculating the cross-sectional averages of the time-series Pearson correlation among them. Given that LIQ^{BAS}, LIQ^{EffS}, LIQ^{QS}, LIQ^{CL} and LIQ^{OR} are measures of illiquidity, with higher values denoting lower liquidity, for the purposes of correlation they are multiplied by -1 to ensure that a positive value of the correlation coefficient denotes a positive correlation of liquidity. The results are presented in Table 3 and Table 4 respectively.

Table 3: Time-series averages of cross-sectional Pearson correlations

Measure	$LIQ^{LOBslope}$	LIQ^{Depth}	$LIQ^{\$Depth}$	LIQ^{BAS}	LIQ^{EffS}	LIQ^{QS}	LIQ^{CL}	LIQ^{OR}
LIQ^{OFN}	0.7285	0.0579	0.4655	0.3613	0.2736	0.0781	0.2568	0.0197
$LIQ^{LOBslope}$		-0.0155	0.3337	0.4431	0.2755	0.0838	0.3204	0.0603
LIQ^{Depth}			0.6175	-0.1204	-0.0841	0.0594	0.0955	-0.5813
$LIQ^{\$Depth}$				0.1444	0.0617	0.0425	0.2458	-0.2755
LIQ^{BAS}					0.7791	0.2073	0.6585	0.1733
LIQ^{EffS}						0.1721	0.5438	0.1290
LIQ^{QS}							0.0894	-0.0453
LIQ^{CL}								-0.0473

Source: Author's own calculations based on the data from Capital IQ and WSE.

LIQ^{OFN} exhibits the highest cross-sectional correlation with Næs and Skjeltorp's (2006) LOB slope, yet the correlation is approximately 0.7, suggesting that LIQ^{OFN} encompasses information not captured by $LIQ^{LOBslope}$. The correlation of LIQ^{OFN} with other considered measures is less than 0.5, with the correlation with LIQ^{QS} and LIQ^{OR} equalling only several per cent. Consequently, the cross-sectional Pearson correlation suggests that our liquidity measure behaves similarly to other measures capturing the entire LOB but potentially captures a different liquidity dimension than other LOB-based liquidity measures.

Table 4: Cross-sectional averages of time-series Pearson correlations

Measure	$LIQ^{LOBslope}$	LIQ^{Depth}	$LIQ^{\$Depth}$	LIQ^{BAS}	LIQ^{EffS}	LIQ^{QS}	LIQ^{CL}	LIQ^{OR}
LIQ^{OFN}	0.5497	0.3660	0.6241	0.5593	0.4213	0.2432	0.5124	-0.0304
$LIQ^{LOBslope}$		0.0678	0.2327	0.6269	0.4531	0.2928	0.3829	0.0433
LIQ^{Depth}			0.8147	-0.0033	-0.0127	0.3888	0.2930	-0.3831
$LIQ^{\$Depth}$				0.1874	0.1446	0.1862	0.4623	-0.2555
LIQ^{BAS}					0.7334	0.4918	0.6180	0.1247
LIQ^{EffS}						0.3328	0.4638	0.0989
LIQ^{QS}							0.4322	0.1113
LIQ^{CL}								0.0062

Source: Author's own calculations based on the data from Capital IQ and WSE.

A weaker correlation is observed between LIQ^{OFN} and $LIQ^{LOBslope}$ when the time-series Pearson correlation is considered. Conversely, a stronger (than cross-sectional) time-series correlation of LIQ^{OFN} with other measures, except LIQ^{OR}, is observed. The findings indicate that time-series correlations of LIQ^{OFN} are stronger than cross-sectional ones, suggesting that LIQ^{OFN} comoves with other variables commonly recognised as liquidity measures. This indicates that LIQ^{OFN} reflects stock liquidity. Conversely, low cross-sectional correlations support the conjecture that LIQ^{OFN} contains distinct information and reflects a different dimension of stock liquidity compared to the other measures under scrutiny.

To further analyse the correlations among the scrutinised measures, a multicollinearity test was also applied using the Variance Inflation Factor (VIF). This allows for the assessment of the extent to which a given variable is linearly related to the other variables in the model (James et al., 2023). The analysis showed that all VIF values remain well below the critical threshold of 10, indicating no significant multicollinearity. The highest values were observed for the variables $LIQ^{LOBslope}$ (2.88), LIQ^{Depth} (2.77), and LIQ^{OFN} (2.70), suggesting only a moderate degree of dependency between these and the other variables. The remaining indicators exhibited very low VIF values (close to 1), indicating their near-complete independence. Consequently, it can be concluded that the set of variables does not contain significant redundancies and is suitable for further modelling analyses without the need to eliminate any components due to excessive multicollinearity. Nevertheless, the highest values of VIFs for $LIQ^{LOBslope}$, LIQ^{Depth} and LIQ^{OFN} suggest that our LIQ^{OFN} measure potentially captures more than just one liquidity dimension.

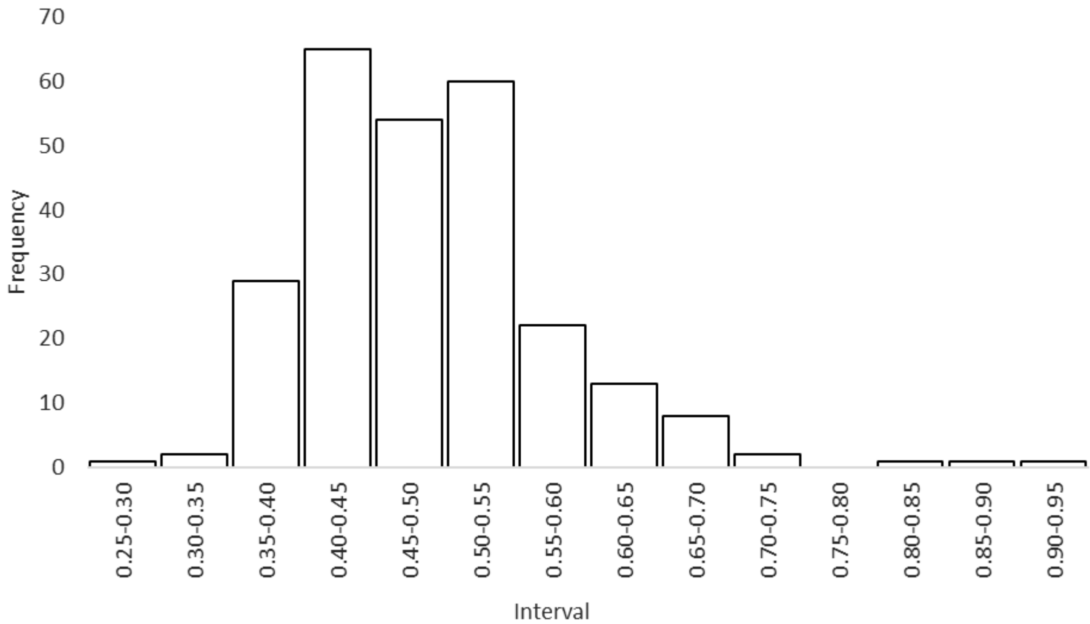
Altogether, these findings further corroborate the notion of stock liquidity as a multidimensional concept, which cannot be encapsulated by a single measure (Sarr & Lybek, 2002). Furthermore, correlation analysis offers more insights into the LIQ^{OFN} as a measure of stock liquidity, with time-series correlations of LIQ^{OFN} with other measures proving to be much stronger than cross-sectional. The former's ability to reflect the co-movements of liquidity trends over time suggests its susceptibility to common macroeconomic variables or seasonality, akin to other measures. Conversely, cross-sectional correlations are related to differences between companies, which can be an advantage in portfolio allocation. These findings collectively indicate the potential usefulness in asset pricing and portfolio management.

Principal component analysis

The results presented thus far indicate a correlation between the OFN representation of a limit order book and other liquidity measures, albeit a relatively weak correlation. This suggests that LIQ^{OFN} measures stock liquidity, yet it relates to a different dimension than other measures under scrutiny. The subsequent analysis aims to test the conjecture that our measure based on the OFN contains distinct information compared to measures. To this end, a principal component analysis (PCA) was performed, and the proportion of variability in all liquidity measures explained by the first component was determined. First, PCA was conducted for all nine measures company by company. The distribution of the amount of the whole variability of all liquidity measures explained by the first component is presented in Figure 1.

In the majority of cases, the first component explains only 0.35 to 0.60 of the variability of the whole system, with an average value of 0.49. However, for a select few companies, the first component explains over 70% of all variability amongst the measures. Furthermore, Figure 2 presents the distributions of weights in the first principal component for individual measures. The majority of these distributions resemble asymmetric bimodal distributions with two main clusters around -0.4 and 0.4 (the exception being the LIQ^{OR} measure, which has a unimodal distribution concentrated near values close to zero). These results indicate that almost every measure (except LIQ^{OR}) plays a significant role in the first principal component, suggesting that the analysed measures contain distinct information and reflect different dimensions of liquidity. Będowska-Sójka and Kliber (2021) found similar for low-frequency liquidity measures in the Warsaw Stock Exchange, i.e. Amihud's (2002) illiquidity ratio, closing quoted spread (CQS) of Chung and Zhang (2014), Corwin and Schultz (2012) spread estimator and Fong's et al. (2017) volatility-over-volume measure. The average (absolute value of) factor loadings for the first component (data not reported) are roughly equal for the majority of the measures. Except for LIQ^{OR} (0.148) and LIQ^{Depth} (0.267) average factor loadings range between 0.3 and 0.4, which denotes they explain a similar share in a common variability of all liquidity measures.

Figure 1: The distribution of the amount of the whole variability of liquidity measures explained by the first principal component (by company)

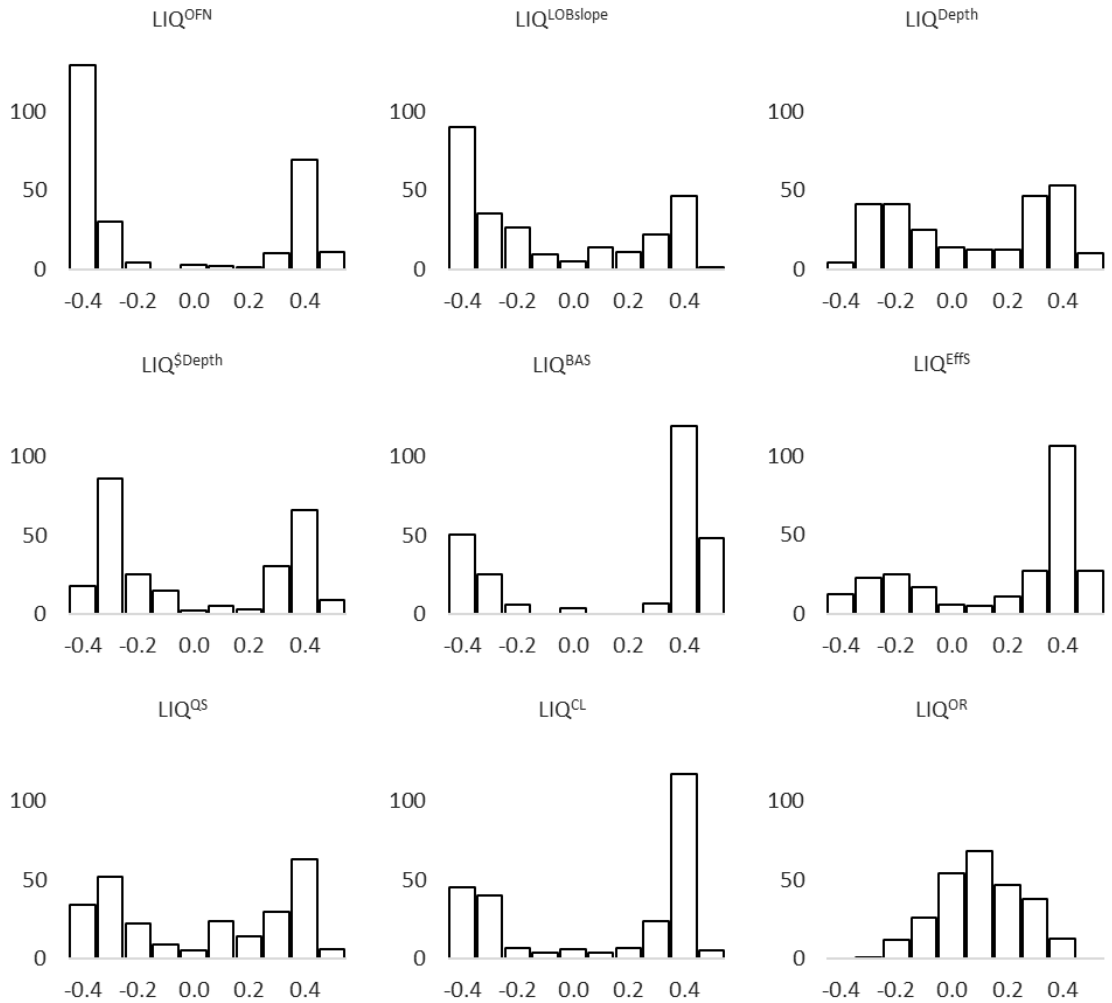


Source: Author's own calculations based on the data from Capital IQ and WSE.

Figure 3 presents the distribution of the amount of the whole variability of all liquidity measures explained by the first component when we redo the PCA week by week. The results obtained are comparable to those presented in Figure 1. However, it can be observed that the shares of the cross-sectional liquidity measures' variability explained by the first component are much more concentrated and generally lower in comparison to the shares of the time-series variability. The share of the variability explained by the first component ranges from 0.25 to 0.5, with an average of 0.375, which is of much lower magnitude than the variability explained by the first component in Figure 1. This is consistent with the results of the correlation analysis presented in the previous section. It can be posited that considered liquidity measures contain distinct information and reflect different liquidity dimensions (see Figure 4). Similar to the results for company-by-company PCA, week-by-week PCA (the absolute value of) factor loadings range from 0.3 to 0.45 for the majority of measures, with LIQ^{Depth}, LIQ^{QS} and LIQ^{OR} being the exceptions. This finding indicates that the measures under scrutiny contribute approximately equally to explaining common variability within the system.

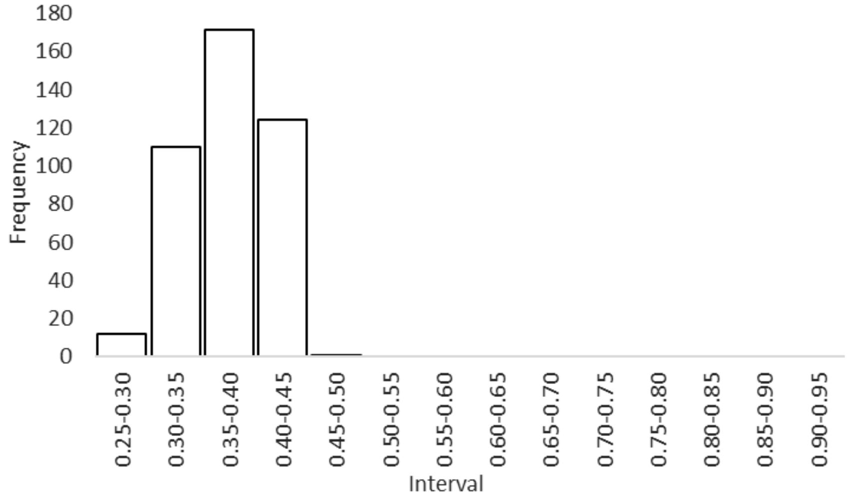
The proportion of variation in all measures of variability explained by the first principal component is relatively low (0.49 for time-series and 0.375 for cross-sectional variations), indicating that a significant amount of the variance remains unexplained by that component. However, this finding aligns with the concept of the multidimensionality of liquidity by indicating that each measure contributes a different type of information and does not undermine the usefulness of LIQ^{OFN} in measuring stock liquidity. The findings of the principal component analysis suggest that LIQ^{OFN} may serve as a measure of stock liquidity, yet it contains distinct information from other liquidity measures based on the LOB data, reflecting a different dimension of stock liquidity. According to Sarr and Lybek's (2002) classification, LIQ^{OFN} likely reflects market depth, akin to $LIQ^{LOBslope}$, LIQ^{Depth} and $LIQ^{\$Depth}$. However, LIQ^{OFN} reflects depth in a more generalised manner as it is not limited to specific price levels. Conversely, LIQ^{BAS} and LIQ^{EffS} are the measures of market tightness, while the remaining measures (LIQ^{QS}, LIQ^{CL} and LIQ^{OR}) reflect market resiliency.

Figure 2: The distribution of the factor loadings of each liquidity measure in the first principal component (by company)



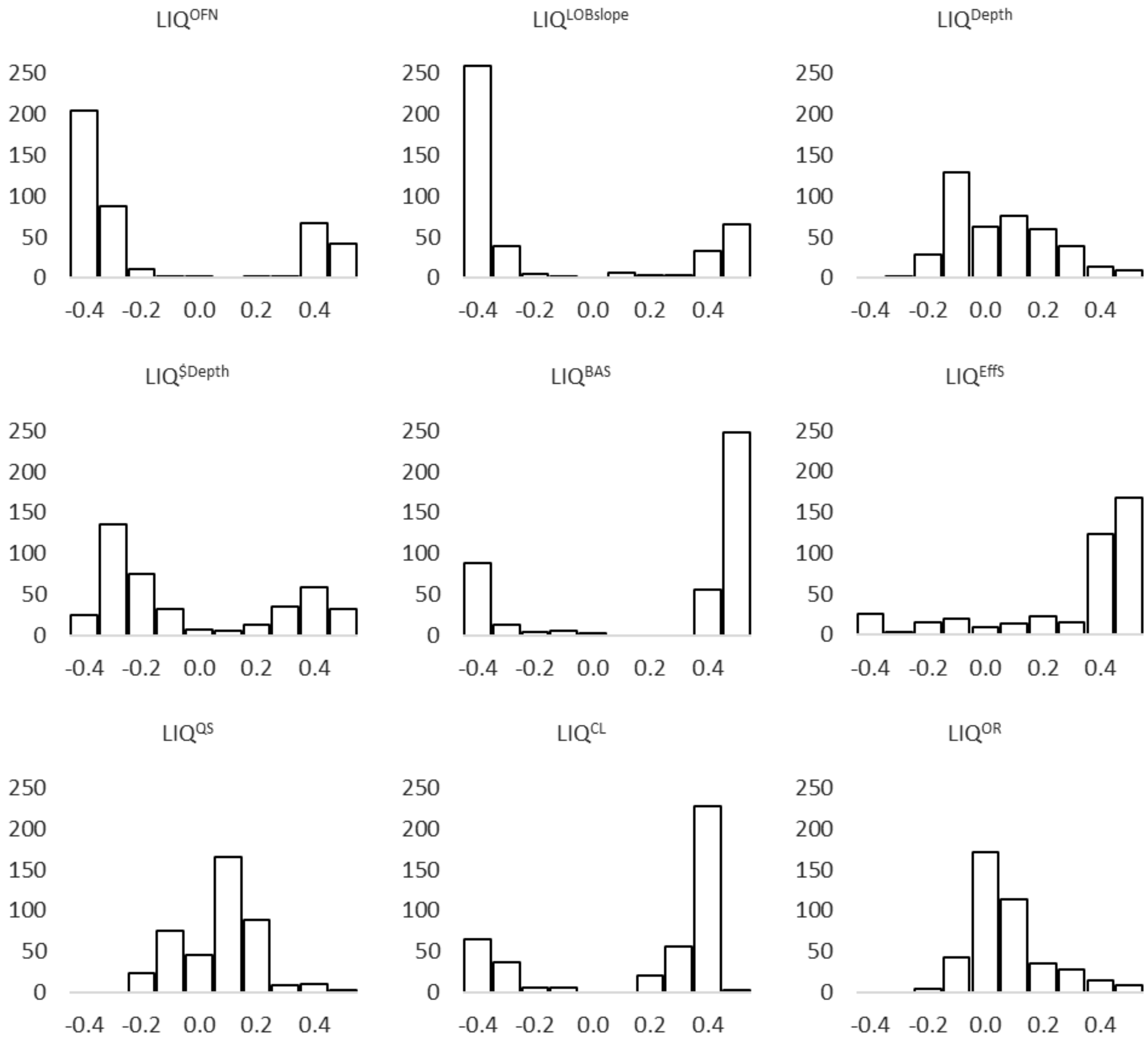
Source: Author's own calculations based on the data from Capital IQ and WSE.

Figure 3: The distribution of the amount of the whole variability of liquidity measures explained by the first principal component (by week)



Source: Author's own calculations based on the data from Capital IQ and WSE.

Figure 4: The distribution of the factor loadings of each liquidity measure in the first principal component (by week)



Source: Author's own calculations based on the data from Capital IQ and WSE.

Factor analysis

To further explore the informational structure of the analysed liquidity measures and their relation to LIQ^{OFN}, a factor analysis with varimax rotation (Hair et al., 2019) was carried out. The analysis was applied to all liquidity measures except LIQ^{OFN}, which was subsequently correlated with the extracted latent factors. As the remaining liquidity measures reflect three liquidity dimensions (tightness, depth, and resiliency), three factors are used to rotate the dataset. First, the factor analysis was conducted company by company, resulting in the identification of three factors consisting of:
F1: LIQ^{BAS}, LIQ_{Effs}, LIQ^{QS} and LIQ^{CL}.
F2: LIQ^{Depth} and $LIQ^{\$Depth}$.
F3: LIQ^{LOB} and LIQ^{OR}.

It is evident that the first factor appears to represent the tightness dimension of liquidity. Meanwhile, the second factor is predominantly shaped by LIQ^{Depth} and $LIQ^{\$Depth}$, which are commonly interpreted as market depth proxies, and the third factor comprises measures that reflect resiliency. The correlation results indicate that LIQ^{OFN} exhibits the strongest correlation with the first and second rotated factors, with average correlation values of –0.42 and 0.31, respectively. This finding suggests that LIQ_{OFN} is predominantly associated with latent dimensions that, although not dominant in terms of overall variance, carry important diagnostic content. The positive correlation between LIQ^{OFN} and the factor related to market depth implies that LIQ^{OFN} is also positively associated with the market's capacity to absorb volume without significant price changes. These findings underscore the notion that LIQ^{OFN} reflects a multi-dimensional view of liquidity, bridging both cost -related and volume-based aspects of market functioning.

An analogous factor analysis with varimax rotation was conducted in the week-by-week dimension to complement the company-by-company analysis. In this case, the factor extraction was repeated for each weekly cross-section of the data, with LIQ^{OFN} excluded from the initial factor structure, and subsequently, the correlation of this with the extracted components was calculated. The following cross-sectional liquidity factors have been distinguished:
F1: LIQ^{BAS}, LIQ_{Effs} and LIQ^{CL}.
F2: LIQ^{Depth} and $LIQ^{\$Depth}$.
F3: LIQ^{LOB}, LIQ^{QS} and LIQ^{OR}.

The findings indicate that, in contrast to the firm-level setting, LIQ^{OFN} exhibits a significant correlation with only a single factor - the first one - with an average correlation of –0.36. This factor is predominantly characterised by substantial loadings from LIQ^{BAS}, LIQ^{EffS}, and LIQ^{CL}, which are conventional proxies of market tightness. This finding suggests that LIQ^{OFN} predominantly captures cost-based aspects of liquidity, such as transaction costs and spreads, rather than volume-based or resiliency-related features in the cross-section. Compared to the company-level results, the reduced dimensional overlap observed here underscores the contextual sensitivity of LIQ^{OFN}'s informational content. Furthermore, this finding lends support to the multi-dimensional nature of liquidity as a market characteristic and corroborates our conjecture that LIQ^{OFN} captures more than just one liquidity dimension.

Conclusions

In this paper, the objective was to examine whether the recently developed ordered fuzzy numbers (OFN) representation of a limit order book (LOB), as proposed by Marszałek and Burczyński (2024), may serve as a measure of stock liquidity. To this end, an analysis was conducted in which the OFN representation was compared with eight distinct measures derived from the LOB data, with a particular focus on those that relied exclusively on best buy- and sell orders. The study encompassed a total of 259 companies, which were included in one of the three indices: WIG20, mWIG40 and sWIG80, over an eight-year period from 2014 to 2021.

Among the liquidity measures that were subject to analysis, our LIQ^{OFN} exhibited a medium degree of variation, both in the cross-section and in the time series. Although high cross-sectional variation is a desirable feature, e.g. for asset pricing purposes, high time-series volatility is rather unfavourable, as it renders LIQ^{OFN} more difficult to predict in comparison to other liquidity proxies. High time-series correlations of LIQ^{OFN} with other considered liquidity measures indicate a strong co-movement, suggesting that our measure based on the OFN representation of the LOB reflects stock liquidity. However, lower cross-sectional correlations suggest the presence of distinct information. This is presumably because it captures a different liquidity dimension from the other measures.

This conjecture is supported by the principal component analysis, which revealed that the first component explains on average only about 50% of the time-series variation of all scrutinised measures. When the cross-sectional variation is considered, the average share of the variation explained by the first component falls to 0.375. This finding lends support to the notion of stock liquidity's multidimensionality and the limitations of a single measure in fully capturing it (Sarr & Lybek, 2002). However, as demonstrated by the factor analysis provided, our LIQ^{OFN} measure is likely to capture more than just one liquidity dimension, i.e. tightness and depth. In general, based on the study's findings, we can assert that the OFN representation of a LOB can serve as a measure of stock liquidity. This is due to the fact that it is based on the data from the entire LOB, not only from the best buy- and sell orders. This makes it useful in the context of high-volume order execution, especially where the LOB is shallow or fragmented.

Despite the indications from time-series correlations that LIQ^{OFN} co-moves with other liquidity measures, shares with them trends over time, and is susceptible to common macroeconomic variables or seasonality, further studies are required to test the usefulness of the LIQ^{OFN} in capturing stock liquidity in varying conditions. In particular, it is important to assess the performance of this measure in periods of extreme illiquidity or volatility, such as the pandemic caused by the SARS-CoV-2 virus, and in dynamic regulatory frameworks, including various regulatory shifts. Subsequent research could also encompass out-of-sample analysis, including out-of-sample predictive power, and testing LIQ^{OFN} in different markets.

Future research could also concentrate on the practical applications of Marszałek and Burczyński's (2024) LIQ^{OFN} measure, in order to test its usefulness for investors not only in terms of capturing stock liquidity. Given relatively weak cross-sectional correlations among LIQ^{OFN} and other scrutinised measures, and the distinct information they provide, one such potential application include but is not limited to testing the capability of the measure to capture stock liquidity premium (Amihud & Mendelson, 1986), which would likely result in the possibility of developing a profitable investment strategy. Another potential application of the measure is in portfolio management and optimal liquidity trading as defined by Huberman and Stanzl (2005). One should analyse the potential of integrating LIQ^{OFN} into high-frequency trading algorithms as an alternative to classic indicators.

We perceive some regulatory and supervisory applications of our LIQ^{OFN}, which, by virtue of its design can serve as a tool to detect sudden drops in market depth before the occurrence of a liquidity shock. Given its resilience to minor perturbations from market manipulations, LIQ^{OFN} also facilitates the monitoring of dynamic changes in the structure of the order book, which can serve as an early warning of attempts at market manipulation, such as spoofing or quote stuffing. Consequently, LIQ^{OFN} can be employed by the supervisory authorities (e.g. the Polish Financial Supervision Authority or the European Securities and Markets Authority) as a surveillance instrument to ensure market stability.

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